

# USE OF LINEAR AND POWER TRANSFORMATIONS FOR ESTIMATING THE POPULATION MEAN IN TWO-OCCASION SUCCESSIVE SAMPLING

Surya K. Pal<sup>1\*</sup>, Housila P. Singh<sup>\*\*</sup> and Vishal Mehta<sup>\*\*\*</sup>

\*University School of Business, Chandigarh University, Mohali-140413, Punjab, India.

\*\*School of Studies in Statistics, Vikram University, Ujjain-456010, India.

\*\*\*Department of Mathematics, Jaypee University of Information Technology (JUIT), Wakanaghat-173234, Salon, Himachal Pradesh, India

## ABSTRACT

The paper advocates the problem of estimating population mean on the current (second) occasion using auxiliary information in successive sampling over two-occasions. A class of estimators based on transformed auxiliary variable is derived. The bias and mean square error of the proposed estimator have been obtained. The suggested estimator has been compared with simple mean estimator when there is no matching and the optimum estimator, which is a combination of the means of the matched and un-matched portion of the sample at the second occasion. Optimum replacement policy and the efficiency of the suggested estimator have been discussed. Numerical Illustration is given in support of the present study.

**KEYWORDS:** Auxiliary variable, study variable, successive sampling, mean square error.

**MSC:** 62D05

## RESUMEN

Este paper trata el problema de estimar la media de la población una clase de estimadores basada en la variable transformada. Son obtenidos el sesgo y el error cuadrático medio del estimador propuesto. El estimador ha sido comparado con el común estimador de la media cuando no hay solapamiento con el estimador óptimo, el que es una combinación de las medias de la porción en la actual, (segunda) ocasión, usando información auxiliar en el muestreo sucesivo en dos-ocasiones solapada y no solapada de la muestra en la segunda ocasión. Se sugiere una política de reemplazo óptimo y se discute sobre la eficiencia del estimador sugerido. Una ilustración numérica es dada para soportar el presente estudio

**PALABRAS CLAVE:** variable auxiliar, variable de estudio, muestreo sucesivo, error cuadrático medio.

## 1. INTRODUCTION

In successive (rotation) sampling, it is not uncommon in practice to use the information collected on the preceding occasions to improve the precision of the estimates on the current occasion. The problem of sampling on two successive occasions with a partial replacement of sampling units was first introduced by Jessen (1942) in the analysis of a survey that collected from data. After Jessen (1942) several authors including Patterson (1950), Eckler (1955), Rao and Graham (1964), Singh et al. (1992) and among others have developed the theory of successive sampling. Feng and Zou (1997) and Biradar and Singh (2001) used the auxiliary information on both the occasions for estimating the current population mean in the successive sampling. Singh (2005), Singh and Vishwakarma (2007a, 2007b, 2009), Singh and Pal (2014), Singh and Pal(2015a, 2015b, 2015c), Singh and Pal (2016a, 2016b, 2016c) Singh and Pal (2017a, 2017b, 2017c, 2017d, 2017e), Pal and Singh (2017a, 2017b) have used the auxiliary information on both the occasion and envisaged several estimators for the estimating the population mean on current occasion in two- occasion successive (rotation) sampling. The procedure discussed in the above studies have used information only on the

---

<sup>1</sup> Corresponding author: Dr. Surya K. Pal ([suryakantpal6676@gmail.com](mailto:suryakantpal6676@gmail.com))

population mean  $\bar{Z}$  of the auxiliary variable  $z$ , while in various survey situations information on other parameters of the auxiliary variable  $z$  such as coefficient of variation  $C_z$ , population standard deviation  $S_z$ , population coefficients of skewness  $\beta_1(z)$  and kurtosis  $\beta_2(z)$ ; and the correlation coefficients between study variable  $y$  and  $z$  ( $\rho_{yz}$ ); and the auxiliary variable  $x$  and  $z$  ( $\rho_{xz}$ ) are known, for instance see Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh and Tailor (2003), Kadilar and Cingi (2004), Chandra and Singh (2005), Koyuncu and Kadilar (2009), Sousa et al.(2010) and Singh and Solanki (2013a, 2013b). Keeping this in view we have suggested an effective and efficient estimation procedure, which provides a cost effective estimate of the population mean on the current (second) occasion in two-occasion successive (rotation) sampling. Let  $U = (U_1, U_2, \dots, U_N)$  be the finite population of size  $N$  units, which has been sampled over two occasions. Let  $x$  ( $y$ ) be the variable under study on the first (second) occasion respectively. It is assumed that information on an auxiliary variable  $z$  (stable over occasion) is readily available for the both the occasions. It is assumed that the population under investigation is large, and the sample size is constant on each occasion. A simple random sample of  $n$  units is drawn without replacement (WOR) on the first occasion. A random sub sample of  $m (= n\lambda)$  units is retained (soprelapada) from the sample drawn on the first occasion for its use on the current (second) occasion, while a fresh sample of size  $u = (n-m) = n\mu$  units is drawn on the current (second) occasion, from the entire population by simple random sampling without replacement (SRSWOR) procedure so that the sample size on the current (second) is also  $n$ . The fractions of the soprelapada and fresh samples are respectively designated by  $\lambda$  and  $\mu$  such that  $\lambda + \mu = 1$ .

In what follows we shall use the following notations throughout this paper.  $\bar{X}, \bar{Y}, \bar{Z}$ : The population means of the variables  $x, y$  and  $z$  respectively.  $\bar{x}_m, \bar{x}_n, \bar{y}_u, \bar{y}_m, \bar{z}_u, \bar{z}_n$ : The sample means of the respective variables based on the sample sizes indicated in suffices.  $C_x, C_y, C_z$ : The coefficients of variation of the variables  $x, y$  and  $z$  respectively,  $\rho_{yx}, \rho_{yz}, \rho_{xz}$ : The correlation coefficients between the variables

shown in suffices.  $S_x^2 = (N-1)^{-1} \sum_{i=1}^N (x_i - \bar{X})^2, S_y^2 = (N-1)^{-1} \sum_{i=1}^N (y_i - \bar{Y})^2, S_z^2 = (N-1)^{-1} \sum_{i=1}^N (z_i - \bar{Z})^2$  are the population mean squares of  $x, y$  and  $z$  respectively,  $f = n/N$ : Sampling fraction. *fda*: fist degree of approximation.

The rest part of the paper is prepared as follows: In Section 2, the estimators of population mean have been suggested and the expressions of their bias and the mean square error are obtained. Section 3 addresses the problem of optimal replacement policy while Section 4 has focused on efficiency comparisons and empirical study of proposed estimators. Section 5 dealt with the optimum estimator along with its properties. Concluding remarks are given in Section 6.

## 2. DEVELOPMENT OF THE ESTIMATOR

To develop the estimator of the population mean  $\bar{Y}$  on the current (second) occasion in two-occasion successive sampling, motivated by Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh and Agnihotri (2008) and Srivastava (1967) we have given two estimators using linear as well as power transformation simultaneously over auxiliary variable  $z$ . One is based on a sample of size  $u = n\mu$  drawn, afresh on the second occasion and is defined by

$$P_u = \bar{y}_u \left( \frac{a\bar{Z} + b}{a\bar{z}_u + b} \right)^\alpha, \quad (2.1)$$

where  $\alpha$  being a constant which takes real values +ive (-ive) for generating ratio-type (product-type) estimators and '0 (zero)' for the usual unbiased estimator which does not utilize information on auxiliary variable  $z$ ; and  $(a, b)$  are suitably chosen scalars. The scalars  $(a, b)$  may assume real values as well as parametric values  $C_z$  (coefficient of variation of the auxiliary variable  $z$ ),  $\rho_{xz}$  (correlation coefficient between  $x$  and  $z$ ),  $\beta_1(z)$  (coefficient of skewness of  $z$ ),  $\beta_2(z)$  (coefficient of kurtosis of  $z$ ),  $\bar{Z}$  (population mean of  $z$ ) and  $S_z$  (standard deviation of  $z$ ) etc.

Motivated by Singh et al. (2004), we consider the second estimator  $P_m$  based on the sample of size  $m$  common with both the occasions, as

$$P_m = \bar{y}_m \left( \frac{\bar{x}_n}{\bar{x}_m} \right)^{\alpha_1} \left( \frac{a\bar{Z} + b}{a\bar{z}_n + b} \right)^{\alpha_2} \left( \frac{a\bar{Z} + b}{a\bar{z}_m + b} \right)^{\alpha_3} \quad (2.2)$$

where  $(a, b)$  are same as defined earlier and  $\alpha_i$ 's ( $i = 1, 2, 3$ ) are unknown scalar to be suitably determined (or  $\alpha_i$ 's ( $i = 1, 2, 3$ ) may take real values +ive (-ive) and '0'). Now combining the two estimators  $P_u$  and  $P_m$ , we propose the final estimator of the population mean  $\bar{Y}$  on the current occasion, as

$$P = \omega P_u + (1 - \omega) P_m, \quad (2.3)$$

where  $\omega$  is an unknown constant to be determined under certain criterion.

It is interesting to mention that for  $\alpha = 1$  in (2.1) and  $(\alpha_1, \alpha_2, \alpha_3) = (1, 1, 0)$  in (2.2) the estimators  $P_u$  and  $P_m$  respectively reduce to the Singh and Majhi (2013) estimators and hence combined estimator 'P' is more general than that of the Singh and Majhi (2013) estimator.

### 3. BIAS AND MEAN SQUARED ERROR (MSE) OF THE PROPOSED ESTIMATOR

We note that the estimators  $P_u$  and  $P_m$  are rectified versions of ratio and chain ratio- type estimators respectively, they are biased estimators of the population mean  $\bar{Y}$ . Thus the combined estimator  $P$  in (2.3) would be a biased estimator of the population mean  $\bar{Y}$ . The bias and mean squared error (MSE) of the estimator 'P' are obtained up to the first order approximation. To obtain the bias and mean square error of suggested class of estimators 'P' we define following quantities

$\bar{y}_u = \bar{Y}(1 + e_{0u})$ ,  $\bar{y}_m = \bar{Y}(1 + e_{0m})$ ,  $\bar{x}_n = \bar{X}(1 + e_{1n})$ ,  $\bar{x}_m = \bar{X}(1 + e_{1m})$ ,  $\bar{z}_u = \bar{Z}(1 + e_{2u})$ ,  $\bar{z}_m = \bar{Z}(1 + e_{2m})$  and  $\bar{z}_n = \bar{Z}(1 + e_{2n})$  such that

$$E(e_{0u}) = E(e_{0m}) = E(e_{1n}) = E(e_{1m}) = E(e_{2u}) = E(e_{2n}) = 0$$

and

$$\begin{aligned} E(e_{0u}^2) &= \eta C_y^2, E(e_{0m}^2) = \Delta C_y^2, E(e_{1n}^2) = \Delta C_x^2, E(e_{1m}^2) = \lambda C_x^2, E(e_{2u}^2) = \eta C_z^2, \\ E(e_{2n}^2) &= \lambda C_z^2, E(e_{2m}^2) = \Delta C_z^2, E(e_{0u}e_{0m}) = -\wp C_y^2, E(e_{0u}e_{1m}) = -\wp \rho_{yx} C_y C_x, \\ E(e_{0u}e_{1n}) &= -\wp \rho_{yx} C_y C_x, E(e_{0u}e_{2u}) = \eta \rho_{yz} C_y C_z, E(e_{0u}e_{2m}) = -\wp \rho_{yz} C_y C_z, \\ E(e_{0u}e_{2n}) &= -\wp \rho_{yz} C_y C_z, E(e_{0m}e_{1m}) = \Delta \rho_{yx} C_y C_x, E(e_{0m}e_{1n}) = \lambda \rho_{yx} C_y C_x, \\ E(e_{0m}e_{2u}) &= -\wp \rho_{yz} C_y C_z, E(e_{0m}e_{2m}) = \Delta \rho_{yz} C_y C_z, E(e_{0m}e_{2n}) = \lambda \rho_{yz} C_y C_z, \\ E(e_{1m}e_{1n}) &= \lambda \rho_{yx} C_y C_x, E(e_{1m}e_{2u}) = -\wp \rho_{xz} C_x C_z, E(e_{1m}e_{2n}) = \lambda \rho_{xz} C_x C_z, \\ E(e_{1m}e_{2m}) &= \Delta \rho_{xz} C_x C_z, E(e_{1n}e_{2u}) = -\wp \rho_{xz} C_x C_z, E(e_{1n}e_{2n}) = \lambda \rho_{xz} C_x C_z, \\ E(e_{1n}e_{2m}) &= \lambda \rho_{xz} C_x C_z, E(e_{2u}e_{2n}) = -\wp C_z^2, E(e_{2u}e_{2m}) = -\wp C_z^2, E(e_{2m}e_{2n}) = \Delta C_z^2. \end{aligned}$$

Where  $\lambda = (n^{-1} - N^{-1})$ ,  $\eta = (u^{-1} - N^{-1})$ ,  $\Delta = (m^{-1} - N^{-1})$ ,  $\wp = N^{-1}$ .

#### 3.1 The Bias and MSE of the Estimator 'P<sub>u</sub>'

We now express  $P_u$  at (2.1) in terms of e's as

$$P_u = \bar{Y} \frac{(1 + e_{0u})}{(1 + \phi e_{2u})^\alpha} = \bar{Y}(1 + e_{0u})(1 + \phi e_{2u})^{-\alpha}, \quad (3.1)$$

where  $\phi = a\bar{Z}/(a\bar{Z} + b)$ .

We assume that  $|\phi e_{2u}| < 1$ , so that  $(1 + \phi e_{2u})^{-\alpha}$  is expandable. Now expanding the right hand side of (3.1), multiplying out and neglecting terms of e's having power greater than two we have

$$P_u \cong \bar{Y} \left[ 1 + e_{0u} - \alpha \phi e_{2u} - \alpha \phi e_{0u} e_{2u} + \frac{\alpha(\alpha + 1)}{2} \phi^2 e_{2u}^2 \right], \text{ or}$$

$$(P_u - \bar{Y}) \cong \bar{Y} \left[ e_{0u} - \alpha \phi e_{2u} - \alpha \phi e_{0u} e_{2u} + \frac{\alpha(\alpha+1)}{2} \phi^2 e_{2u}^2 \right]. \quad (3.2)$$

Taking expectation on both sides of (3.2), we get the bias of the proposed class of estimators  $P_u$  up to first

degree of approximation as  $B(P_u) = \left( \frac{\bar{Y}\alpha\phi}{2} \right) \eta C_z^2 \{ \phi(\alpha+1) - 2k_{yz} \}$ , where  $k_{yz} = \rho_{yz} C_y / C_z$ .

Squaring both sides of (3.2) and neglecting terms of e's having power greater than two, we have

$$\begin{aligned} (P_u - \bar{Y})^2 &\cong \bar{Y}^2 [e_{0u} - \alpha \phi e_{2u}]^2 \\ &\cong \bar{Y}^2 [e_{0u}^2 + \alpha^2 \phi^2 e_{2u}^2 - 2\alpha \phi e_{0u} e_{2u}]. \end{aligned} \quad (3.2)$$

Taking expectation on both sides of (2.7), we get the *MSE* of the proposed class of estimators  $P_u$  up to first degree of approximation as

$$MSE(P_u) = \bar{Y}^2 \eta [C_y^2 + \alpha \phi C_z^2 (\alpha \phi - 2k_{yz})] = \bar{Y}^2 \eta \eta_0,$$

where  $\eta_0 = [C_y^2 + \alpha \phi C_z^2 (\alpha \phi - 2k_{yz})]$ .

Thus we state the following theorem.

**Theorem 3.1** To the first degree of approximation the bias and *MSE* of the estimator  $P_u$  are respectively, given by

$$B(P_u) = \left( \frac{\bar{Y}\alpha\phi}{2} \right) \eta C_z^2 \{ \phi(\alpha+1) - 2k_{yz} \}. \quad (3.4)$$

$$\begin{aligned} MSE(P_u) &= \bar{Y}^2 \eta [C_y^2 + \alpha \phi C_z^2 (\alpha \phi - 2k_{yz})] \\ &= \bar{Y}^2 \eta \eta_0, \end{aligned} \quad (3.5)$$

where  $\eta_0 = [C_y^2 + \alpha \phi C_z^2 (\alpha \phi - 2k_{yz})]$ .

### 3.2 The Bias and *MSE* of the Estimator ' $P_m$ '

To obtain the bias and *MSE* of the estimator  $P_m$  at (2.2), we express  $P_m$  in terms of e's:

$$P_m = \bar{Y} (1 + e_{0m}) (1 + e_{1n})^{\alpha_1} (1 + e_{1m})^{-\alpha_1} (1 + \phi e_{2n})^{-\alpha_2} (1 + \phi e_{2m})^{-\alpha_3}, \quad (3.6)$$

We suppose that  $|e_{1n}| < 1$ ,  $|e_{1m}| < 1$ ,  $|\phi e_{2n}| < 1$  and  $|\phi e_{2m}| < 1$ , so that  $(1 + e_{1n})^{\alpha_1}$ ,  $(1 + e_{1m})^{-\alpha_1}$ ,  $(1 + \phi e_{2n})^{-\alpha_2}$  and  $(1 + \phi e_{2m})^{-\alpha_3}$  are expandable. Expanding the right hand side of (2.9), multiplying out and neglecting terms of e's having power greater than two, we have

$$\begin{aligned} P_m &\cong \bar{Y} \left[ 1 + e_{0m} + \alpha_1 (e_{1n} - e_{1m}) - \phi (\alpha_2 e_{2n} + \alpha_3 e_{2m}) + \alpha_1 (e_{0m} e_{1n} - e_{0m} e_{1m}) - \phi (\alpha_2 e_{0m} e_{2n} \right. \\ &\quad + \alpha_3 e_{0m} e_{2m}) + \alpha_1 \alpha_2 \phi (e_{1m} e_{2n} - e_{1n} e_{2n}) + \alpha_1 \alpha_3 \phi (e_{1m} e_{2m} - e_{1n} e_{2m}) + \phi^2 \alpha_2 \alpha_3 e_{2m} e_{2n} \\ &\quad \left. + \alpha_1 \left\{ \frac{(\alpha_1 - 1)}{2} e_{1n}^2 + \frac{(\alpha_1 + 1)}{2} e_{1m}^2 - \alpha_1 e_{1m} e_{1n} \right\} + \frac{\phi^2}{2} \left\{ \alpha_2 (\alpha_2 + 1) e_{2n}^2 + \alpha_3 (\alpha_3 + 1) e_{2m}^2 \right\} \right] \end{aligned}$$

or

$$\begin{aligned} (P_m - \bar{Y}) &\cong \bar{Y} \left[ e_{0m} + \alpha_1 (e_{1n} - e_{1m}) - \phi (\alpha_2 e_{2n} + \alpha_3 e_{2m}) + \alpha_1 (e_{0m} e_{1n} - e_{0m} e_{1m}) - \phi (\alpha_2 e_{0m} e_{2n} \right. \\ &\quad + \alpha_3 e_{0m} e_{2m}) + \alpha_1 \alpha_2 \phi (e_{1m} e_{2n} - e_{1n} e_{2n}) + \alpha_1 \alpha_3 \phi (e_{1m} e_{2m} - e_{1n} e_{2m}) + \phi^2 \alpha_2 \alpha_3 e_{2m} e_{2n} \\ &\quad \left. + \alpha_1 \left\{ \frac{(\alpha_1 - 1)}{2} e_{1n}^2 + \frac{(\alpha_1 + 1)}{2} e_{1m}^2 - \alpha_1 e_{1m} e_{1n} \right\} + \frac{\phi^2}{2} \left\{ \alpha_2 (\alpha_2 + 1) e_{2n}^2 + \alpha_3 (\alpha_3 + 1) e_{2m}^2 \right\} \right] \end{aligned} \quad (3.7)$$

Taking expectation on both sides of (3.7), we get the bias of the proposed class of estimators  $P_m$  up to first degree of approximation as

$$B(P_m) = (\bar{Y} / 2) \left[ \Gamma \alpha_1 C_x^2 (\alpha_1 - 2k_{yx} + 1) + \lambda C_z^2 \{ \alpha_2 \phi (\phi (\alpha_2 + 1) - 2k_{yz}) \} \right]$$

$$+ \alpha_3 \phi(\alpha_2 \phi - 2k_{xz} \alpha_1) + \Delta \alpha_3 \phi C_z^2 \{ \phi(\alpha_3 + 1) + 2\alpha_1 k_{xz} - 2k_{yz} \}$$

Squaring both sides of (3.7) and neglecting terms of e's having power greater than two, we have

$$\begin{aligned} (P_m - \bar{Y})^2 &\cong \bar{Y}^2 [e_{0m} + \alpha_1(e_{1n} - e_{1m}) - \phi(\alpha_2 e_{2n} + \alpha_3 e_{2m})]^2 \\ &\cong \bar{Y}^2 [e_{0m}^2 + \alpha_1^2(e_{1n} - e_{1m})^2 + \phi^2(\alpha_2 e_{2n} + \alpha_3 e_{2m})^2 \\ &\quad + 2\alpha_1(e_{0m}e_{1n} - e_{0m}e_{1m}) - 2\phi(\alpha_2 e_{0m}e_{2n} + \alpha_3 e_{0m}e_{2m}) \\ &\quad - 2\alpha_1 \phi \{ \alpha_2(e_{1n}e_{2n} - e_{1m}e_{2n}) + \alpha_3(e_{1n}e_{2m} - e_{1m}e_{2m}) \}]. \end{aligned} \quad (3.8)$$

Taking expectation on both sides of (3.8), we get the *MSE* of the proposed class of estimators  $P_m$  up to first degree of approximation as  $MSE(P_m) = \bar{Y}^2 (m^{-1}\eta_1 + n^{-1}\eta_2 - N^{-1}\eta_3)$ , where

$$\eta_1 = [C_y^2 + \alpha_1 C_x^2 (\alpha_1 - 2k_{yx}) + \alpha_3 \phi C_z^2 (\alpha_3 \phi - 2k_{yz} + 2\alpha_1 k_{xz})],$$

$$\eta_2 = [\alpha_1 C_x^2 (2k_{yx} - \alpha_1) + \phi C_z^2 \{ \alpha_2 (\alpha_2 \phi - 2k_{yz}) + 2\alpha_3 (\alpha_2 \phi - \alpha_1 k_{xz}) \}],$$

and

$$\eta_3 = (\eta_1 + \eta_2) = [C_y^2 + \phi C_z^2 (\alpha_2 + \alpha_3) \{ \phi(\alpha_2 + \alpha_3) - 2k_{yz} \}].$$

Thus we state the following theorem.

**Theorem 3.2** To the first degree of approximation the bias and *MSE* of the estimator  $P_m$  are respectively, given by

$$\begin{aligned} B(P_m) &= \left( \frac{\bar{Y}}{2} \right) \left[ \Gamma \alpha_1 C_x^2 (\alpha_1 - 2k_{yx} + 1) + \lambda C_z^2 \{ \alpha_2 \phi (\phi(\alpha_2 + 1) - 2k_{yz}) + \alpha_3 \phi (\alpha_2 \phi - 2k_{xz} \alpha_1) \} \right. \\ &\quad \left. + \Delta \alpha_3 \phi C_z^2 \{ \phi(\alpha_3 + 1) + 2\alpha_1 k_{xz} - 2k_{yz} \} \right] \end{aligned} \quad (3.9)$$

$$MSE(P_m) = \bar{Y}^2 (m^{-1}\eta_1 + n^{-1}\eta_2 - N^{-1}\eta_3) \quad (3.10)$$

where  $\eta_i$ 's ( $i = 1, 2, 3$ ) are same as defined earlier.

### 3.3 The Covariance between $P_u$ and $P_m$

The covariance between  $P_u$  and  $P_m$  to the first degree of approximation is obtained as follows:

$$\begin{aligned} Cov(P_u, P_m) &= E[(P_u - \bar{Y})(P_m - \bar{Y})] \\ &\cong \bar{Y}^2 E[(e_{ou} - \alpha \phi e_{2u}) \{ e_{0m} - \alpha_1(e_{1m} - e_{1n}) - \phi(\alpha_2 e_{2n} + \alpha_3 e_{2m}) \}] \\ &\cong \bar{Y}^2 E[e_{ou}e_{0m} - \alpha_1(e_{ou}e_{1m} - e_{ou}e_{1n}) - \phi(\alpha_2 e_{ou}e_{2n} + \alpha_3 e_{ou}e_{2m}) \\ &\quad - \alpha \phi e_{ou}e_{0m} + \phi \alpha \alpha_1 (e_{1m}e_{2u} - e_{1n}e_{2u}) + \phi^2 \alpha (\alpha_2 e_{2u}e_{2n} + \alpha_3 e_{2u}e_{2m})] \\ &= - \left( \frac{\bar{Y}^2}{N} \right) [C_y^2 + \phi C_z^2 \{ \alpha \phi (\alpha_2 + \alpha_3) - k_{yz} (\alpha + \alpha_2 + \alpha_3) \}] \end{aligned} \quad (3.11)$$

Thus we state the following theorem.

**Theorem 3.2** To the first degree of approximation the covariance between of the estimators  $P_u$  and  $P_m$  is given by

$$Cov(P_u, P_m) = - \left( \frac{\bar{Y}^2}{N} \right) \eta_4 \quad (3.12)$$

where  $\eta_4 = [C_y^2 + \phi C_z^2 \{ \alpha \phi (\alpha_2 + \alpha_3) - k_{yz} (\alpha + \alpha_2 + \alpha_3) \}]$ .

The *MSE* of the combined estimator 'P' given by

$$\begin{aligned} MSE(P) &= MSE(P_m) + \omega^2 [MSE(P_u) + MSE(P_m) - 2Cov(P_u, P_m)] \\ &\quad - 2\omega [MSE(P_m) - Cov(P_u, P_m)] \end{aligned} \quad (3.13)$$

Substituting the values of  $MSE(P_u)$ ,  $MSE(P_m)$  and  $Cov(P_u, P_m)$  from (3.5), (3.10) and (3.12), we get the *MSE* of  $P$  to the first degree of approximation, as

$$MSE(P) = \bar{Y}^2 \left[ (m^{-1}\eta_1 + n^{-1}\eta_2 - N^{-1}\eta_3) + \omega^2 \left\{ u^{-1}\eta_0 + m^{-1}\eta_1 + n^{-1}\eta_2 - \left( \frac{\phi^2 C_z^2}{N} \right) (\alpha_2 + \alpha_3 - \alpha) \right\} \right. \\ \left. - 2\omega \left\{ m^{-1}\eta_1 + n^{-1}\eta_2 - \left( \frac{\phi C_z^2}{N} \right) (\alpha_2 + \alpha_3 - \alpha) (\phi(\alpha_2 + \alpha_3) - k_{yz}) \right\} \right] \quad (3.14)$$

Now we make the following assumption:

**Assumption 3.1**

- (i) Population size  $N$  is large enough (i.e.  $N \rightarrow \infty$ ) so that the finite population correction (fpc) terms are ignored.
- (ii) Since  $x$  and  $y$  denoted the same study variable over two occasions and  $z$  is the stable auxiliary variable correlated to  $x$  and  $y$ , therefore we assume that the coefficients of variation of  $x$ ,  $y$ ,  $z$  are approximately equal (i.e.  $C_y \cong C_x \cong C_z$ ) for instance see Murthy (1967), Reddy (1978), Cochran(1977) and Fen and Zou (1977).
- (iii) “ $\rho_{xz} = \rho_{yz}$ ”. This is an intuitive assumption, which has been considered by Cochran (1977), Fen and Zou (1977) and Singh and Priyanka (2008).

Under the assumption 3.1, the  $MSE$  of  $P$  at (3.14) reduce to:

$$MSE(P) = [m^{-1}\eta_1^* + n^{-1}\eta_2^* + \omega^2 (u^{-1}\eta_0^* + m^{-1}\eta_1^* + n^{-1}\eta_2^*) - 2\omega(m^{-1}\eta_1^* + n^{-1}\eta_2^*)],$$

$$\text{where } \eta_0^* = [1 + \alpha\phi(\alpha\phi - 2\rho_{yz})], \eta_1^* = [1 + \alpha_1(\alpha_1 - 2\rho_{yx}) + \alpha_3\phi(\alpha_3\phi - 2\rho_{yz} + 2\alpha_1\rho_{yz})]$$

$$\text{and } \eta_2^* = [\alpha_1(2\rho_{yx} - \alpha_1) + \phi\{\alpha_2(\alpha_2\phi - 2\rho_{yz}) + 2\alpha_3(\alpha_2\phi - \alpha_1\rho_{yz})\}].$$

We express the  $MSE(P)$  at (3.15) in terms  $\mu (= u/n)$  as

$$MSE(P) = \frac{S_y^2}{n\mu(1-\mu)} [\mu(\eta_1^* + \eta_2^*) - \mu^2\eta_2^* + \omega^2\{\eta_0^* + \mu(\eta_1^* + \eta_2^* - \eta_0^*) - \mu^2\eta_2^*\} \\ - 2\omega\mu\{\mu(\eta_1^* + \eta_2^*) - \mu\eta_2^*\}] \quad (3.16)$$

$$\text{This is minimized when } \omega_{opt} = \frac{\mu(\eta_1^* + \eta_2^* - \mu\eta_2^*)}{\{\eta_0^* + \mu(\eta_1^* + \eta_2^* - \eta_0^*) - \mu^2\eta_2^*\}}$$

$$\text{Thus the resulting minimum } MSE \text{ of } d \text{ is given by } \min.MSE(P) = \frac{S_y^2\eta_0^*(\eta_1^* + \eta_2^* - \mu\eta_2^*)}{n\{\eta_0^* + \mu(\eta_1^* + \eta_2^* - \eta_0^*) - \mu^2\eta_2^*\}}$$

Thus we state the following theorem.

**Theorem 3.4** The optimum value of  $\omega$  [which minimizes the  $MSE(P)$  in (3.16)] and minimum  $MSE$  of  $P$  are respectively given by

$$\omega_{opt} = \frac{\mu(\eta_1^* + \eta_2^* - \mu\eta_2^*)}{\{\eta_0^* + \mu(\eta_1^* + \eta_2^* - \eta_0^*) - \mu^2\eta_2^*\}}, \quad (3.17)$$

and

$$\min.MSE(P) = \frac{S_y^2\eta_0^*(\eta_1^* + \eta_2^* - \mu\eta_2^*)}{n\{\eta_0^* + \mu(\eta_1^* + \eta_2^* - \eta_0^*) - \mu^2\eta_2^*\}}. \quad (3.18)$$

where  $\eta_i$ 's ( $i = 1, 2, 3$ ) are same as defined earlier.

**Corollary 3.1** If there is complete matching i.e.  $\mu = 0$ , then

$$\min.MSE(P) = \frac{S_y^2\eta_0^*}{n}. \quad (3.19)$$

**Corollary 3.2** If there is no matching i.e.  $\mu = 1$ , then

$$\min.MSE(P) = \frac{S_y^2\eta_0^*}{n}. \quad (3.20)$$

In both the cases,  $\min.MSE(P)$  has the same value. This provides an implication that there must be an optimum choice of  $\mu$ , other than extreme values so that  $\min.MSE(P)$  will be fewer than the quantity given in (3.19) or (3.20). Thus, for obtaining current estimate (neither the case of “complete matching” nor the case of “no matching”) more precise, it is always advisable to replace the sample partially.

#### 4. OPTIMUM REPLACEMENT POLICY

To determine the value of  $\mu$  so that the population mean  $\bar{Y}$  on current (second) occasion may be estimated with maximum efficiency, we minimize  $\min.MSE(P)$  in equation (3.18) with respect to  $\mu$  and hence we get

$$\hat{\mu} = \frac{(\eta_1^* + \eta_2^*) \pm \sqrt{\eta_0^* \eta_1^*}}{\eta_2^*} = \mu_0(\text{say}). \quad (4.1)$$

The real values of  $\hat{\mu}$  exists iff the quantity under square root is greater than or equal to zero i.e.  $\eta_0^* \eta_1^* \geq 0$ .

For any combination of  $(\alpha, \alpha_1, \alpha_2, \alpha_3, \phi, \rho_{yx}$  and  $\rho_{yz})$  which satisfies the condition  $\eta_0^* \eta_1^* \geq 0$ , two real values of  $\hat{\mu}$  are possible, hence to select a value of  $\hat{\mu}$ , it should be noted that  $0 \leq \hat{\mu} \leq 1$ , all other values of  $\hat{\mu}$  are inadmissible. Putting the admissible value of  $\hat{\mu}$  say  $\mu_0$  from equation (4.1) into equation (3.18), we get the optimum value of  $\min.MSE(P)$ , which is shown in equation (4.2):

$$\min.MSE(P)_{opt} = \frac{S_y^2 \eta_0^* (\eta_1^* + \eta_2^* - \mu_0 \eta_2^*)}{n \{ \eta_0^* + \mu_0 (\eta_1^* + \eta_2^* - \eta_0^*) - \mu_0^2 \eta_2^* \}}. \quad (4.2)$$

#### 5. EFFICIENCY COMPARISON

The percent relative efficiencies of the proposed estimator ‘ $P$ ’ with respect to (i)  $\bar{y}_n$ , when there is no matching, and (ii)  $\hat{Y} = \psi \bar{y}_u + (1 - \psi) \bar{y}_d$ , when no auxiliary information is used at any occasion, where  $\bar{y}_d = \bar{y}_m + \beta_{yx}(\bar{x}_n - \bar{x}_m)$ , have been obtained for different values of  $(\alpha, \alpha_1, \alpha_2, \alpha_3, \rho_{yx}, \rho_{yz})$  and  $\beta_{yx}$  being the population regression coefficients of  $y$  on  $x$ . Since  $\bar{y}_n$  and  $\hat{Y}$  are unbiased estimators of the population  $\bar{Y}$ , the variance of  $\bar{y}_n$  and optimum variance of  $\hat{Y}$  for large  $N$  (i.e.  $N \rightarrow \infty$ ) are, respectively given

$$V(\bar{y}_n) = \frac{S_y^2}{n}, \quad (5.1)$$

and

$$V(\hat{Y}) = [1 + \sqrt{(1 - \rho_{yx}^2)}] \frac{S_y^2}{n}, \quad (5.2)$$

The percent relative efficiencies  $E_1$  and  $E_2$  of  $P$  (under optimal condition) with respect to  $\bar{y}_n$  and  $\hat{Y}$  are respectively given by

$$E_1 = \frac{V(\bar{y}_n)}{\min.MSE(d)_{opt}} \times 100 = \frac{[\eta_0^* + \mu_0 (\eta_1^* + \eta_2^* - \eta_0^*) - \mu_0^2 \eta_2^*]}{\eta_0^* (\eta_1^* + \eta_2^* - \mu_0 \eta_2^*)} \times 100 \quad (5.3)$$

and

$$E_2 = \frac{V(\hat{Y})_{opt}}{\min.MSE(d)_{opt}} \times 100 = \frac{[1 + \sqrt{(1 - \rho_{yx}^2)}][\eta_0^* + \mu_0 (\eta_1^* + \eta_2^* - \eta_0^*) - \mu_0^2 \eta_2^*]}{2\eta_0^* (\eta_1^* + \eta_2^* - \mu_0 \eta_2^*)} \times 100 \quad (5.4)$$

To illustrate our results we have computed the percent relative efficiencies of  $E_1$  and  $E_2$  for

(i)  $\alpha = \alpha_1 = 0.50, \alpha_2 = \alpha_3 = 0.25$ , (ii)  $\alpha = 0.75, \alpha_1 = \alpha_2 = 0.50, \alpha_3 = 0.25$ ,  
 (iii)  $\alpha = \alpha_1 = 0.75, \alpha_2 = 0.50, \alpha_3 = 0.25$ , and (iv)  $\alpha = 1.00, \alpha_1 = 0.75, \alpha_2 = 0.50, \alpha_3 = 0.25$   
 and different values of  $\rho_{yx}$  and  $\rho_{yz}$ ; and findings are shown in Tables 5.1 and 5.2.

**Table 5.1: Optimum values  $\mu_0$  and percent relative efficiencies of the estimator  $P$**

$\rho_{yx}$	$\rho_{yz}$	$\phi$	$\alpha = \alpha_1 = 0.50, \alpha_2 = \alpha_3 = 0.25$			$\alpha = 0.75, \alpha_1 = \alpha_2 = 0.50, \alpha_3 = 0.25$		
			$\mu_0$	$E_1$	$E_2$	$\mu_0$	$E_1$	$E_2$
0.5	0.5	0.2	0.53	116.03	108.26	0.52	119.82	111.79
		0.3	0.52	120.19	112.14	0.52	125.34	116.95
		0.4	0.52	124.05	115.74	0.51	129.96	121.25
		0.5	0.52	127.52	118.98	0.51	133.39	124.46
		0.6	0.52	130.52	121.78	0.51	135.41	126.34
		0.7	0.51	132.96	124.05	0.51	135.88	126.77
	0.6	0.2	0.53	118.21	110.29	0.52	123.25	115.00
		0.3	0.52	123.74	115.45	0.51	131.11	122.33
		0.4	0.52	129.17	120.52	0.51	138.42	129.15
		0.5	0.51	134.39	125.39	*	-	-
		0.6	0.51	139.29	129.96	0.50	149.82	139.78
		0.7	0.50	143.74	134.11	0.49	153.13	142.87
	0.7	0.2	0.52	120.48	112.41	0.52	126.90	118.40
		0.3	0.52	127.52	118.98	0.51	137.47	128.27
		0.4	0.51	134.76	125.73	0.50	148.16	138.23
		0.5	0.51	142.09	132.57	0.49	158.51	147.89
		0.6	0.50	149.39	139.39	0.48	167.97	156.72
		0.7	0.50	156.53	146.05	0.48	175.88	164.10
0.6	0.5	0.2	0.55	120.07	108.07	0.54	124.04	111.63
		0.3	0.54	124.46	112.02	0.54	129.87	116.88
		0.4	0.54	128.55	115.69	0.53	134.75	121.28
		0.5	0.54	132.23	119.01	0.53	138.39	124.55
		0.6	0.53	135.41	121.87	0.53	140.54	126.49
		0.7	0.53	137.99	124.19	0.53	141.06	126.95
	0.6	0.2	0.54	122.37	110.14	0.54	127.66	114.89
		0.3	0.54	128.23	115.40	0.53	135.96	122.36
		0.4	0.54	133.97	120.58	0.52	143.70	129.33
		0.5	0.53	139.51	125.56	0.52	150.46	135.41
		0.6	0.53	144.71	130.24	0.52	155.80	140.22
		0.7	0.52	149.43	134.49	0.51	159.33	143.40
	0.7	0.2	0.54	124.77	112.29	0.53	131.51	118.36
		0.3	0.54	132.23	119.01	0.52	142.68	128.41
		0.4	0.53	139.90	125.91	0.52	154.00	138.60
		0.5	0.53	147.68	132.91	0.51	164.99	148.49
		0.6	0.52	155.45	139.90	0.50	175.06	157.55
		0.7	0.52	163.05	146.74	0.50	183.50	165.15



0.7	0.5	0.2	0.57	124.78	106.94	0.56	128.96	110.52
		0.3	0.56	129.46	110.96	0.56	135.17	115.85
		0.4	0.56	133.82	114.70	0.55	140.38	120.31
		0.5	0.56	137.75	118.06	0.55	144.27	123.65
		0.6	0.56	141.15	120.98	0.55	146.59	125.64
		0.7	0.56	143.92	123.35	0.55	147.16	126.13
	0.6	0.2	0.57	127.23	109.05	0.56	132.80	113.82
		0.3	0.56	133.48	114.40	0.55	141.65	121.40
		0.4	0.56	139.62	119.66	0.55	149.91	128.48
		0.5	0.55	145.55	124.74	0.54	157.14	134.68
		0.6	0.55	151.12	129.52	0.54	162.88	139.60
		0.7	0.55	156.18	133.86	0.54	166.70	142.87
	0.7	0.2	0.56	129.79	111.24	0.56	136.90	117.33
		0.3	0.56	137.75	118.06	0.55	148.81	127.54
		0.4	0.55	145.96	125.10	0.54	160.90	137.91
		0.5	0.55	154.30	132.25	0.53	172.69	148.01
		0.6	0.54	162.65	139.40	0.53	183.51	157.28
		0.7	0.54	170.82	146.41	0.52	192.62	165.09

Note: ‘\*’ indicates  $\mu_0$  does not exist.

**Table 5.2: Optimum values  $\mu_0$  and percent relative efficiencies of the estimator  $P$**

$\rho_{yx}$	$\rho_{yz}$	$\phi$	$\alpha = \alpha_1 = 0.75, \alpha_2 = 0.50, \alpha_3 = 0.25$			$\alpha = 1.00, \alpha_1 = 0.75, \alpha_2 = 0.50, \alpha_3 = 0.25$		
			$\mu_0$	$E_1$	$E_2$	$\mu_0$	$E_1$	$E_2$
0.5	0.5	0.2	0.51	117.01	109.17	0.74	119.78	111.75
		0.3	0.50	122.10	113.92	*	-	-
		0.4	0.50	126.28	117.82	*	-	-
		0.5	0.49	129.31	120.64	<b>0.24</b>	<b>130.30</b>	<b>121.57</b>
		0.6	0.49	130.96	122.19	0.58	130.26	121.53
		0.7	0.49	131.13	122.34	0.90	126.51	118.04
	0.6	0.2	0.51	120.23	112.18	1.00	125.00	116.63
		0.3	0.50	127.49	118.95	*	-	-
		0.4	0.49	134.17	125.19	<b>0.07</b>	<b>136.93</b>	<b>127.76</b>
		0.5	0.48	139.90	130.53	0.26	143.26	133.66
		0.6	0.48	144.31	134.64	0.38	146.58	136.76
		0.7	0.47	147.05	137.20	0.49	146.55	136.73
	0.7	0.2	0.50	123.65	115.37	*	-	-
		0.3	0.49	133.44	124.50	*	-	-
		0.4	0.48	143.23	133.63	<b>0.15</b>	<b>148.61</b>	<b>138.66</b>
		0.5	0.47	152.61	142.39	0.24	159.47	148.78
		0.6	0.46	161.05	150.27	0.31	167.83	156.59
	0.7	0.45	167.96	156.71	0.38	172.56	161.00	
0.6	0.5	0.2	0.54	122.92	110.63	0.60	125.45	112.90

		0.3	0.53	128.38	115.54	0.62	131.53	118.37
		0.4	0.52	132.87	119.58	0.62	135.74	122.16
		0.5	0.52	136.11	122.50	0.58	137.61	123.85
		0.6	0.52	137.88	124.09	0.49	137.22	123.50
		0.7	0.52	138.05	124.24	0.34	135.19	121.67
	0.6	0.2	0.53	126.38	113.74	0.63	130.05	117.05
		0.3	0.52	134.18	120.76	0.70	139.61	125.65
		0.4	0.52	141.36	127.22	0.79	147.95	133.16
		0.5	0.51	147.52	132.76	0.91	153.95	138.56
		0.6	0.50	152.25	137.02	1.00	156.25	140.63
	0.7	0.7	0.50	155.19	139.67	<b>0.07</b>	<b>154.89</b>	<b>139.40</b>
		0.2	0.53	130.05	117.05	0.67	135.17	121.65
		0.3	0.52	140.57	126.51	0.85	149.73	134.76
		0.4	0.51	151.11	136.00	*	-	-
		0.5	0.50	161.22	145.10	*	-	-
0.7	0.5	0.6	0.49	170.32	153.29	<b>0.02</b>	<b>174.67</b>	<b>157.20</b>
		0.7	0.48	177.76	159.99	0.30	182.06	163.86
		0.2	0.57	130.32	111.70	0.60	132.92	113.92
		0.3	0.56	136.27	116.79	0.61	139.46	119.52
		0.4	0.56	141.15	120.98	0.60	144.04	123.45
	0.6	0.5	0.55	144.68	124.00	0.58	146.21	125.32
		0.6	0.55	146.59	125.64	0.54	145.87	125.02
		0.7	0.55	146.75	125.77	0.49	143.30	122.82
		0.2	0.56	134.09	114.92	0.61	137.81	118.11
		0.3	0.56	142.59	122.21	0.63	147.86	126.72
	0.7	0.4	0.55	150.42	128.92	0.64	156.40	134.05
		0.5	0.54	157.14	134.68	0.63	162.45	139.23
		0.6	0.54	162.30	139.11	0.59	165.31	141.68
		0.7	0.53	165.50	141.85	0.52	164.95	141.38
		0.2	0.56	138.09	118.36	0.63	143.15	122.69
0.7	0.3	0.55	149.57	128.19	0.66	157.66	135.13	
	0.4	0.54	161.09	138.07	0.71	172.00	147.41	
	0.5	0.53	172.16	147.56	0.76	184.70	158.31	
	0.6	0.52	182.14	156.10	0.80	193.81	166.11	
	0.7	0.51	190.29	163.09	0.76	197.41	169.19	

**Note:** ‘\*’ indicates  $\mu_0$  does not exist.

It is observed from Tables 5.1 and 5.2 that the values of percent relative efficiencies  $E_1$  and  $E_2$  are larger than 100 for the parametric values considered here. Thus the proposed estimator ‘ $P$ ’ is more efficient than the usual unbiased estimator  $\bar{y}_n$  and  $\hat{Y}$ . It is interesting to observe from Table 5.2 that the minimum value  $\mu_0$  is 0.02, which shows that the fraction of fresh sample to be replaced at the current occasion is as low as about 2 percent of the total sample size, which highly price is saving. Also for this value of  $\mu_0$  (i.e.  $\mu_0=0.02$ ) the proposed estimator ‘ $P$ ’ is more efficient than  $\bar{y}_n$  and  $\hat{Y}$  with substantial gain in efficiency.

On the whole we conclude that there is enough scope of selecting the values of scalars  $(\alpha, \alpha_1, \alpha_2, \alpha_3, a, b)$  with different choices of correlations  $\rho_{yx}$  and  $\rho_{yz}$ ; for obtaining better estimators than  $\bar{y}_n$  and  $\hat{Y}$  with considerable gain in efficiency.

## 6. CONCLUSIONS

In the present paper we have made the use of linear and power transformations simultaneously over the auxiliary variable. We have suggested a class of estimators 'P' for estimating the population mean on current (second) occasion in two-occasion successive sampling. We have studied the properties of suggested the class of estimator 'P' under the large sample approximation. Optimum replacement policy relevant to the suggested estimation procedures has been discussed. The proposed estimator is compared with usual estimators numerically.

It is evident from Tables 5.1 and 5.2 the use of information on a transformed auxiliary variable is highly rewarding in improving the precision of the suggested estimator. The principal interesting thing in this study is that the proposed linear and power transformations are quite effective in reducing the cost of the survey [for instance see Table 5.2, the values of  $\mu_0$  are like 0.02, 0.07, 0.15, 0.24 etc]. Thus the suggested estimators may be recommended to the survey statisticians / practitioners for their practical applications.

RECEIVED: NOVEMBER, 2018

REVISED: JULY, 2019

## REFERENCES

- [1] BIRADAR, R. S. and SINGH, H.P. (2001): Successive sampling using auxiliary information on both the occasions. **Calcutta Statistical Association Bulletin**, 51, 243-251.
- [2] CHANDRA, P. and SINGH, H. P. (2005): A family of estimators for population variance using knowledge of kurtosis of an auxiliary variable in sample surveys. **Statistics in Transition**, 7, 27-34.
- [3] COCHRAN, W.G. (1977): **Sampling Techniques**. Third Edition. John Wiley.
- [4] ECKLER, A.R. (1955): Rotation sampling. **The Annals of Mathematical Statistics**, 26, 664-685.
- [5] FENG, S. and ZOU, G. (1997): Sampling rotation method with auxiliary variable. **Communication in Statistics-Theory and Methods**, 26, 1497-1509.
- [6] JESSEN, R.J. (1942): **Statistical investigation of a sample survey for obtaining form facts**. Iowa Agricultural Experiment Station Road Bulletin no. 304, Ames, USA.
- [7] KADILAR, C. and CINGI, H. (2004): Ratio estimators in simple random sampling. **Applied Mathematics and Computation**, 151, 893-902.
- [8] KOYUNCU, N. and KADILAR, C. (2009): Ratio and product estimators in stratified random sampling. **Journal of Statistical Planning and Inference**, 139, 2552-2558.
- [9] MURTHY, M.N. (1967): **Sampling theory and Method**. Statistical Publishing Society, Calcutta.
- [10] PAL, S.K. and SINGH, H.P. (2017a): Efficient utilization of auxiliary information on estimation of population mean using exponential type estimators in successive sampling. **Journal of Statistical Theory and Applications**, 16, 476-489.
- [11] PAL, S.K., and SINGH, H.P. (2017b): Use of power transformation for estimating the population mean in presence of non-response in successive sampling. **Journal of Applied Mathematics, Statistics and Informatics**, 13, 109-140.
- [12] PATTERSON, H.D. (1950): Sampling on successive occasions with partial replacement of units. **Journal of the Royal Statistical Society, B**, 12, 241- 255.
- [13] RAO, J.N.K. and GRAHAM, J.E. (1964): Rotation design for sampling on repeated occasions. **Journal of the American Statistical Association**, 59, 492-509.
- [14] REDDY, V.N. (1978): A study on the use of prior knowledge on certain population parameters in estimation. **Sankhya C**, 40, 29-37.
- [15] REDDY, V.N. (1973): A study on the use of prior knowledge on certain population parameter in estimation. **Sankhya, C**, 40, 29-37.
- [16] REDDY, V.N. (1974): On a transformed ratio method of estimation. **Sankhya, B**, 36, 59 -70.
- [16] SINGH, G.N. (2005): On the use of chain type ratio estimator in successive Sampling. **Statistics in Transition - New Series**, 7, 21-26.

- [17] SINGH, H. P. and SOLANKI, R. S. (2013B): Improved estimation of finite population variance using auxiliary information. **Communication in Statistics-Theory and Methods**, 42, 2718-2730.
- [18] SINGH, H. P. and SOLANKI, R. S. (2013A): A new procedure for variance estimation in simple random sampling using auxiliary information. **Statistical Papers**, 54, 479 - 497.
- [19] SINGH, H. P. and VISHWAKARMA, G.K. (2007A): Modified exponential ratio product estimators for finite population mean in double sampling. **Austrian Jour. Statist.**, 36, 217 – 225.
- [20] SINGH, H.P. and PAL, S.K. (2015): Improved estimation of current population mean over two occasions. **Sri Lankan Journal of Applied Statistics**, 16, 1-19.
- [21] SINGH, H.P. and PAL, S.K. (2015): On the estimation of population mean in current occasion in two-occasion rotation patterns. **Journal of Statistics Applications and Probability**, 4, 305-313.
- [22] SINGH, H.P. and PAL, S.K. (2015): On the estimation of population mean in rotation sampling. **Journal of Statistics Applications and Probability**, 2, 131-136.
- [23] SINGH, H.P. and PAL, S.K. (2015): On the estimation of population mean in rotation sampling. **Journal of Statistics Applications and Probability Letters**, 2, 131-136.
- [24] SINGH, H.P. and PAL, S.K. (2015): On the estimation of population mean in successive sampling. **International Journal of Mathematical Sciences and Applications**, 5, 179-185.
- [25] SINGH, H.P. and PAL, S.K. (2016): An efficient effective rotation pattern in successive sampling over two occasions. **Communication in Statistics-Theory and Methods**, 45, 5017-5027.
- [26] SINGH, H.P. and PAL, S.K. (2016): Search of good rotation patterns through exponential type regression estimator in successive over two occasions. **Italian Journal of Pure and Applied Mathematics**, 36, 567-582.
- [27] SINGH, H.P. and PAL, S.K. (2016): Use of several auxiliary variables in estimating the population mean in two occasion rotation pattern. **Communication in Statistics-Theory and Methods**, 45, 6928-6942. .
- [28] SINGH, H.P. and PAL, S.K. (2017): A review of chain type estimators and a new chain type multivariate regression approach for population under two occasion successive sampling. **Proceedings of the National Academy of Sciences, India Section A: Physical Sciences**, 87, 31-56.
- [29] SINGH, H.P. and PAL, S.K. (2017): A modified procedure for estimating the population mean in two occasion successive sampling. **Afrika Statistika**, 12, 1347-1365.
- [30] SINGH, H.P. and PAL, S.K. (2017): An exponential method for estimating the population mean in successive sampling. **Journal of Probability and Statistical Science**, 15, 79-97.
- [31] SINGH, H.P. and PAL, S. K. (2017A): Estimation of population mean in two occasion successive sampling. **Communication in Statistics-Simulation and Computation**, 46, 4908-4973.
- [32] SINGH, H.P. and PAL, S.K. (2017): Search of good rotation pattern using exponential method of estimation in two occasion successive sampling. **Communication in Statistics-Theory and Methods**, 46, 5466-5486.
- [33] SINGH, H.P. and TAILOR, R. (2003): Use of known correlation coefficient in estimating the finite population mean. **Statistics in Transitions**, 6, 555-560.
- [34] SINGH, H.P. and VISHWAKARMA, G.K. (2007B): A general class of estimators in successive sampling. **Metron**, 65, 201-227.
- [35] SINGH, H.P. and VISHWAKARMA, G.K. (2009): A general procedure for estimating population mean in successive sampling. **Communication in Statistics-Theory and Methods**, 38, 293-308.
- [36] SINGH, H.P. SINGH, H.P. and SINGH, V.P. (1992): A generalized efficient class of estimators of population mean in two-phase and successive Sampling. **International Journal of Management Systems**, 8, 173-183.
- [37] SISODIA, B.V.S. and DWIVEDI, V. K. (1981): A modified ratio estimator using coefficient of variation of auxiliary variable. **Journal of the Indian Society of Agricultural Statistics**, 33, 13-18.
- [38] SOUSA, R., SHABBIR, J., REAL, P.C. and GUPTA, S. (2010): Ratio estimation of the population mean of a sensitive variable in the prance of auxiliary information. **Journal of Statistics Theory and Practice** , 4, 495-507.
- [39] SUKHATME, P. V., B. V. SUKHATME, and C. ASHOK. (1984): **Sampling theory of surveys with applications**, 3rd Iowa State University Press, Ames.
- [40] UPADHYAYA, L.N. and SINGH, H. P. (1999): Use of transformed auxiliary variable in estimating the finite population mean. **Biometrical Journal**, 41, 627- 636.