FRESH PRODUCE INVENTORY FOR TIME-PRICE AND STOCK DEPENDENT DEMAND

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ABSTRACT
The market demand for fresh produce product is rising regularly in today’s health priority life. Basically, the demand for fresh produce products are also based on the measurement of freshness and the size of its shelf space for displaying the products which obviously fascinate more and more customers to purchase the product. Moreover, the expiration date and selling price for any deteriorating item plays a major role in the purchasing decision of a customer. Therefore, in this article, the market demand for fresh produce product is assumed to be a quadratic varying time function of its freshness, the selling price of item, the displayed stock in its shelf space and the expiration date. It may be gainful to preserve a high stock level at the end of the replenishment cycle, with this freshness-and-price-stock dependent demand. Therefore, varying the traditional way of zero ending inventory level to non-zero ending inventory is analyzed. So, the main objective of this article is to maximize the total profit by estimating the optimal selling price of the item, the optimal ending inventory level and the optimal replenishment cycle length, the optimal time to vacant the backroom and the optimal ordered quantity. The classical optimization technique is utilized for calculating the optimal values. Thereafter, using the concept of eigen-values of a Hessian matrix, we have proved the concave nature of the profit function. Finally, a numerical example along with the sensitivity analysis of decision variables by varying various inventory parameters are presented to validate the derived model and extracts significant managerial insights.

KEYWORDS: Deteriorating inventory, fresh produce products, shelf space, expiration date, stock-price-time-dependent demand.

MSC: 90B05

1. INTRODUCTION

Basically, the consumer’s demand for fresh produce products like bread, milk, milk products, blood banks etc. are dependent on the age of the inventory which plays a major role in decision making of consumer’s purchase, can be negatively impacted due to the damage of consumer’s confidence on the product quality. Hence, the measurement of freshness of the product and the size of its shelf space for displaying the products which obviously fascinate more and more customers to purchase the product. Also, the expiration date

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intensely affects the product’s demand rate, so it is to be treated as a main factor to estimate the freshness of a product. As the product approaches the expiration date, the customer’s demand rate declines and tends to zero.

An inventory model was derived by Sarker et al. (1997) demonstrating the negative effect of ageing of stock on demand. Further an inventory model for perishable products was constructed by Hsu et al. (2006) including the expiration date. An inventory model dealing with the fresh produce products with stock-freshness condition dependent demand rate was developed by Bai and Kendall (2008). A model for deteriorating products by considering the measurement of product freshness in terms of the time remained until the expiration date was estimated by Herbon (2014). Many other researchers contributed significantly in the same field like Wang et al. (2014), Wu and Chan (2014), and Wu et al. (2014).

In practical situation, there are many aspects which influences the market demand rate like; selling price, stock, quality, time, and efforts in terms of either service and/or advertisement. Levin (1972) highlighted on maintaining greater displayed stock level could motivate to purchase more and more. Baker (1988) demonstrated an inventory model by expressing the demand pattern as a power function of displayed stock level. This happens due to the product’s popularity and/or variety and its visibility to the consumer’s. An inventory model with EOQ concept in which the total profit is maximized on fulfilling the constraints namely, budget and capacity of storage was developed by Sana and Chaudhari (2004) with the rate of demand together based on item obtainability and expenditures on advertisements. For deteriorating items with the concept of partial backlogging and with a level of stock based rate of demand, and a bound on the extreme level of inventory, a model on inventory was represented by Min and Zhou (2009). Other model on inventory with rate of demand based on level of stock for items which are deteriorating in nature was proposed by Yang et al. (2010), permitting back-logging partially and including the inflation effect. An EOQ concept consisting of back-logging partially, in which the rate of demand is based on level of stock, and a manageable rate of deterioration determines the strategies of preservation and lot order size in optimum manner to rise the total profit to maximum, was formulated by Lee et al. (2012).

Firstly, Silver et al. (1969) have attempted for time varying demand rate, then after many scholars like Silver (1979), Chung et al. (1993, 1994), Bose et al. (1995), Hariga (1995), Lin, et al. (2000), Mehta et al. (2003, 2004), Shah et al. (2009), and Shah et al. (2014) have expressed rate of demand as time varying in terms of linear, exponential or quadratic, etc. in nature.

Moreover, an inventory system dealing with stock level based rate of demand, it may be profitable to maintain orders of more quantities and end with the positive on-hand stocks. An EOQ model with non-ending inventory and stock based demand was firstly developed by Urban (1992) by relaxing the terminal condition of zero. Same way, Yang (2014) investigated the gain in the total profit by relaxing the assumption of zero ending inventory in Pando et al. (2012) to non-zero ending inventory.

An inventory model for fresh produce with shelf space allocation and freshness condition based demand rate was established by Bai and Kendall (2008). Wu et al. (2016) proposed an inventory model for fresh produce increasing time varying trend demand depends on freshness, stock-level and expiration date. This article overcomes the limitations of the previous research work by introducing quadratic nature of time varying trended demand depending on freshness, stock level, and expiration-date as well as selling price of the fresh produce. This type of demand increases initially then after some time, it tends to decrease. Selling price is also a decision variable in the derived model. These two factors highlights the novelty of this paper.

This article deals with an inventory management problem of fresh product inventory having time-price and stock dependent demand rate. As such a fresh produce product is having a very small shelf-life and its usefulness or condition of freshness gradually deteriorates throughout its lifetime. In practical scenario, there is a limitation to display stocks in a shelf space, so this constraint is included in this article. The market demand rate of fresh produce product is assumed to be based on displayed stocks, selling price, and shelf space and expiration date. It is considered to be profitable to maintain a higher stock level at the end of the cycle for the stock dependent demand for a product. Therefore, varying the traditional assumption of zero ending inventory to a constant stock is considered.

Finally, an EOQ model for fresh produce product with quadratic time varying demand dependent on product freshness, selling price, stock level and expiration date is analyzed. The main objective of this article is to maximize the total profit function by estimating the optimal ending inventory level at the replenishment cycle length, optimal selling price, optimal replenishment cycle length, optimal order quantity and optimal time at which the backroom became vacant. The classical optimization technique is utilized for calculating the optimal values. Thereafter, using the concept of eigen-values of a Hessian matrix, we have proved the concave nature of the profit function. Finally, a numerical example along with the sensitivity analysis of
decision variables by varying various inventory parameters are presented to validate the derived model and extracts significant managerial insights.
In section-2, notations and assumptions are considered, formulation of mathematical model is done in section-3, Numerical examples with sensitivity analysis are done in section-4 and finally conclusion and future scope are discussed in section-5.

2. NOTATIONS AND ASSUMPTIONS

a. Notations:

Parameters

\( A \)  
Fixed ordering cost (dollar/order)

\( C \)  
Unit purchasing price (in dollars)

\( H \)  
Unit inventory holding cost per year (in dollars)

\( m \)  
Expiration date (in years)

\( t \)  
Time (in years)

\( s \)  
Salvage value per unit (in dollars)

\( W \)  
Available shelf space (in units)

Decision variables

\( T \)  
Replenishment cycle length (in years)

\( E \)  
Ending inventory level at time \( T \) (in units)

\( P \)  
Unit selling price (in dollars)

\( Q \)  
Optimum order size (in units)

\( t^\star \)  
Time period at which no more stock remains in the backroom (in years)

Functions

\( D(t, p) \)  
Demand rate per unit at any time \( t \) and at any price \( p \), which is a polynomial form in \( t \), and concavely increasing in the number of displayed stocks

\( I(t) \)  
Inventory level at time \( t \) (in units).

\( TP(E, T, p) \)  
The total profit in each period (in dollars)

\( Q \)  
Order size (in units)

Optimal values

\( T^\star \)  
Optimal replenishment cycle length (in years)

\( E^\star \)  
Optimal ending inventory level in units at time \( T \) (in units)

\( P^\star \)  
Optimal unit selling price (in dollars)

\( Q^\star \)  
Optimum order size (in units)

\( t^\star \)  
Optimal Time period at which no more stock remains in the backroom (in years)

\( TP^\star \)  
Optimal total profit per year (in dollars)

![Figure 1: Graphical representation of the system](image)
b. **Assumptions:**

1. Shortages and quantity discounts are impermissible.
2. As such the process of deterioration of fresh product is continuous in nature with its expiration date. Therefore, the freshness index is one at zero time period and then gradually decays over time periods, and approaches closer to zero when it is tending to the expiration date $m$. Hence, assuming the freshness index at time $t$ is given by,

$$f(t) = \frac{m-t}{m}, \quad 0 \leq t \leq m \quad (1)$$

3. The retailer obtains $Q$ units and displays $W$ units on the shelf with the rest of the products (ie. $Q-W$ units) stored in the backroom at time zero. When sales are made, stocks in the backroom are shifted to the shelf space until no more stocks in the backroom at time $t_i$ as demonstrated in figure-1. Hence, during this time period $[0, t_i]$, the shelf space is occupied fully and the demand rate depends on the quadratic trend, price and the freshness index. Let us assume the demand rate at time $t$, with $P$ as the selling price per unit, where, $a > 0$ is a scale demand, $0 \leq b < 1$ denotes the linear rate of change of demand with respect to time, $0 \leq c < 1$ denotes the quadratic rate of change of demand and $\eta, \lambda$ are the mark up for selling price and available shelf space respectively, is given by,

$$D(t, p) = a(1 + bt - ct^2)W^p f(t); \quad 0 \leq t \leq t_i < T$$

where, $a > 0, 0 \leq b \leq 1, 0 \leq c \leq 1, \eta > 1, 0 \leq \lambda < 1 \quad (2)$

4. When the demand depends on the stock level and time-varying freshness, it would be more desirable and profitable to keep fresh products and higher on hand displayed stocks. (i.e., non-zero ending inventory). Therefore, assuming that the ending inventory level $E \geq 0$.

5. The shelf space is only partially stocked and the demand rate depends on its freshness and the displayed units for the time period $t_i$ to $T$. Once the ending inventory level reaches to $E$ units at the replenishment cycle time $T$, the retailer sells those $E$ units at a salvage price $s$ per unit, receives a new order quantity $Q$ units, and starts a new replenishment cycle. Assuming the demand rate at time $t$, during this time period as,

$$D(t, p) = a(1 + bt - ct^2)N^p f(t); \quad t_i \leq t \leq T$$

where, $a > 0, 0 \leq b \leq 1, 0 \leq c \leq 1, \eta > 1, 0 \leq \lambda < 1 \quad (3)$

6. Without loss of generality, we may assume that the order quantity $Q$ is greater than or equal to the shelf space $W$ (i.e. $Q \geq W$) or reducing the shelf space to $Q$ units to fully utilize the shelf space as $t_i \geq 0$.

7. Replenishment occurs instantaneously.

3. **MATHEMATICAL MODEL FORMULATION:**

On the basis of the above mentioned assumptions, the inventory level $I(t)$, at time $t$, during the time period $[0, t_i]$ is computed by the following differential equation:
\[
\frac{dl(t)}{dt} = - \left[a \left(1 + bt - ct^2 \right) w^{-\eta} \left( \frac{m-t}{m} \right) \right] ; \quad 0 \leq t \leq t_1 \tag{4}
\]

With the boundary condition \( I(t_1) = w \). Solving the differential equation (4) with \( I(0) = Q \) we obtain,

\[
I(t) = \frac{-ap^{-\eta} w^{\lambda} \left[ \frac{1}{4} ct^4 + \frac{1}{3} (-cm-b)t^3 + \frac{1}{2} (bm-1)t^2 + mt \right]}{m} + Q, \quad 0 \leq t \leq t_1 \tag{5}
\]

Substituting \( t_1 \) into equation (5), and using \( I(t_1) = w \) and \( t_1 < T \leq m \), we get \( t_1 \) as the positive root of the following equation,

\[
\left[ \frac{1}{4} ct_1^4 + \frac{1}{3} (-cm-b)t_1^3 + \frac{1}{2} (bm-1)t_1^2 + mt_1 \right] - \frac{(Q-w)m}{w^{\lambda} p^{-\eta} a} = 0 \tag{6}
\]

Therefore, the order quantity is,

\[
Q = I(0) = w + \frac{ap^{-\eta} w^{\lambda} \left[ \frac{1}{4} ct_1^4 + \frac{1}{3} (-cm-b)t_1^3 + \frac{1}{2} (bm-1)t_1^2 + mt_1 \right]}{m} \geq w \tag{7}
\]

In the same way, the inventory level \( I(t) \) at time \( t \) during the time period \([t_1, T]\) is computed by the following differential equation:

\[
\frac{dl(t)}{dt} = - \left[a \left(1 + bt - ct^2 \right) \left[ I(t) \right]^{\lambda} w^{-\eta} \left( \frac{m-t}{m} \right) \right] ; \quad t_1 \leq t \leq T, \tag{8}
\]

With the boundary condition \( I(t_1) = w \). Solving the differential equation (8) with \( I(T) = E \) we obtain,

\[
I(t) = \left\{ \left(1 - \lambda \right) \left[ \frac{ap^{-\eta}}{m} \left[ \frac{1}{4} c(T^4 - t^4) + \frac{1}{3} (-cm-b)(T^3 - t^3) \right] \right] + \frac{m(w^{\lambda-\eta} - E^{1-\lambda})}{ap^{-\eta} (1-\lambda)} \right\} \frac{1}{(1-\lambda)} , \quad t_1 \leq t \leq T, \tag{9}
\]

Refer to Appendix 1, for the detailed derivation. Substituting \( t_1 \) into equation (9), and using \( I(t_1) = w \) and \( t_1 < T \leq m \), and rearranging terms, we get \( t_1 \) as the positive root of the following equation,

\[
\left[ \frac{1}{4} ct_1^4 + \frac{1}{3} (-cm-b)t_1^3 + \frac{1}{2} (bm-1)t_1^2 + mt_1 \right] + \frac{m(w^{\lambda-\eta} - E^{1-\lambda})}{ap^{-\eta} (1-\lambda)} = 0 \tag{10}
\]

From equation (6) and equation (10), we obtain the order quantity as,

\[
Q = w + \frac{w^{\lambda} p^{-\eta} a}{m} \left[ \frac{1}{4} cT^4 + \frac{1}{3} (-cm-b)T^3 + \frac{1}{2} (bm-1)T^2 + mT \right] - \frac{m(w^{\lambda-\eta} - E^{1-\lambda})}{ap^{-\eta} (1-\lambda)} \geq w \tag{11}
\]

Assuming \( t_1 = \alpha T \) where \( \alpha \) be the proportionality constant.
The total profit of the system is computed with the following components:

1. **Sales revenue**:  
   The sales revenue per unit time is given by,  
   \[
   SR = \frac{p(Q-E)}{T}
   \]  
   (12)

2. **Salvage value**:  
   The salvage value per unit time is given by,  
   \[
   SV = \frac{1}{T}sE
   \]  
   (13)

3. **Purchasing cost**:  
   The purchasing cost per unit time is given by,  
   \[
   PC = \frac{CQ}{T}
   \]  
   (14)

4. **Ordering cost**:  
   The ordering cost per unit time is given by,  
   \[
   OC = \frac{A}{T}
   \]  
   (15)

5. **Holding cost**:  
   The holding cost during \([0, t_1]\) is,  
   \[
   H_1 = h \int_0^{t_1} I(t) \, dt
   \]
   \[
   H_1 = h \int_0^{t_1} \left[ -\frac{ap^{-\eta}w^\lambda}{m} \left[ \frac{1}{4}ct^4 + \frac{1}{3}(-cm-b)t^3 + \frac{1}{2}(bm-1)t^2 + mt \right] \right] dt + hQ_{t_1}
   \]  
   (16)

The holding cost during \([t_1, T]\) is,  
\[
H_2 = h \int_{t_1}^{T} I(t) \, dt
\]
\[
H_2 = h \int_{t_1}^{T} \left\{ (1-\lambda) \left[ \frac{ap^{-\eta}}{m} \left[ \frac{1}{4}c(T^4-t^4) + \frac{1}{3}(-cm-b)(T^3-t^3) \right] \right] + E^{1-\lambda} \right\} \frac{1}{(1-\lambda)} dt
\]  
(17)

The integration of (17) seems to be too complicated to get an explicitly analytical solution. However, the holding cost of \(H_2\) is relatively small to the overall profit. For simplicity, we may use a simple approximation to calculate it as given below. The average inventory level during \([t_1, T]\) approximately equals to \(\frac{w+E}{2}\).

Thus, the holding cost during the time period \([t_1, T]\) approximately equals to,  
\[
H_2 = \frac{h}{2}(w+E)(T-t_1)
\]  
(18)
Therefore, the total holding cost from equation (16) and equation (18) is given by,

\[
H = H_1 + H_2 = \left[ h \int_0^t \left[ - \frac{ap^n w^k}{m} \left( \frac{1}{4} ct^4 + \frac{1}{3} (-cm - b) t^3 + \frac{1}{2} (bm - 1) t^2 + mt \right) \right] dt + hQt \right] + \frac{h}{2} (w + E) (T - t_i) \tag{19}
\]

Now the total profit per unit time of the inventory system with freshness, selling price and stock dependent demand is calculated as,

\[
TP(T, E, p) = \frac{1}{T} \left( SR + SV - PC - OC - HC \right)
\]

Therefore, from equation (12) to equation (15) and equation (19). We get,

\[
TP(T, E, p) = \frac{1}{T} \left[ \left( p \left( Q - E \right) + sE - CQ - A \right) \right] \left[ h \int_0^t \left[ - \frac{ap^n w^k}{m} \left( \frac{1}{4} ct^4 + \frac{1}{3} (-cm - b) t^3 + \frac{1}{2} (bm - 1) t^2 + mt \right) \right] dt + hQt \right] + \frac{h}{2} (w + E) (T - t_i) \tag{20}
\]

Now, to maximize the total profit stated in equation (20), we apply the below stated necessary and sufficient condition:

\[
\frac{\partial TP}{\partial T} = 0, \frac{\partial TP}{\partial E} = 0, \frac{\partial TP}{\partial p} = 0 \tag{21}
\]

To check the concavity of the total profit function of obtained solution, we adopt the below stated algorithm,

Step 1: Assigning the inventory parameters some specific hypothetical values.

Step 2: Obtaining the solutions by solving simultaneous equations stated in equation (21), utilizing the mathematical software Maple XVIII.

Step 3: Computing all the Eigen values of below stated hessian matrix \( H \) at the optimal point obtained from equation (21),

\[
H = \begin{bmatrix}
\frac{\partial^2 TP}{\partial T^2} & \frac{\partial^2 TP}{\partial T \partial E} & \frac{\partial^2 TP}{\partial T \partial p} \\
\frac{\partial^2 TP}{\partial E \partial T} & \frac{\partial^2 TP}{\partial E^2} & \frac{\partial^2 TP}{\partial E \partial p} \\
\frac{\partial^2 TP}{\partial p \partial T} & \frac{\partial^2 TP}{\partial p \partial E} & \frac{\partial^2 TP}{\partial p^2}
\end{bmatrix}
\]

If all of the eigenvalues are negative, it is said to be a negative-definite matrix. Then the profit function is concave down then stop.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

4.1 Numerical example

In this section, a numerical example is provided to illustrate the derived mathematical model.

Example: Assuming the following values in developed model.

\[
a = 50000 \text{ units}, b = 15\%, c = 1\%, h = $0.2 / \text{unit}, C = $20 / \text{unit}, \lambda = 0.1, m = 0.25 \text{ years}, \\
a = $100 / \text{unit}, s = $15 / \text{unit}, w = 20 \text{units}, \alpha = 0.2, \eta = 1.1.
\]
Solution: By following the above stated algorithm

For checking the concavity of total profit function, we check the nature of the Eigen values of hessian matrix.

\[
H = \begin{bmatrix}
\frac{\partial^2 TP}{\partial T^2} & \frac{\partial^2 TP}{\partial TE} & \frac{\partial^2 TP}{\partial E}\frac{\partial T}{p} \\
\frac{\partial^2 TP}{\partial ET} & \frac{\partial^2 TP}{\partial E^2} & \frac{\partial^2 TP}{\partial E}\frac{\partial p}{p} \\
\frac{\partial^2 TP}{\partial pT} & \frac{\partial^2 TP}{\partial pE} & \frac{\partial^2 TP}{\partial p^2}
\end{bmatrix}
\]

\[
H = \begin{bmatrix}
-2699394.473 & -2.331983877 & 19.63846150 \\
-2.331983877 & -24.01892908 & 0.5175479438 \\
19.63846150 & 0.5175479438 & -0.09533938026
\end{bmatrix}
\]

All the three Eigen values are computes as,

\[
\lambda_1 = -2.6993*10^6 < 0, \lambda_2 = -24.0301 < 0, \lambda_3 = -0.0840 < 0
\]

Therefore, all the three Eigen values of Hessian matrix are negative. So, the profit function is concave down in nature as shown in the figure-2.
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4.2. Sensitivity analysis on the optimal inventory policy

In this part, the sensitivity analysis of the decision variables with respect to various inventory parameters is carried out. Table-1 demonstrates the values of decision variables on varying the various inventory parameters from case-2 and case-4 respectively in the range -20% to 20%. From table-1 the following observations are extracted;

(a) Sensitivity analysis of basic market scale demand (\(\alpha\)):

In order to maintain the freshness of the products and adequate quantity to be displayed for higher sales, the ending inventory level at time \(T\), ordered quantity increases gradually along with simultaneous shortening of replenishment cycle length with a slight drop in the time period point at which no more stock remains in the backroom is observed with a bit hike in selling price resulting in the total profit gain of the firm, with the variation of scale demand.

(b) Sensitivity analysis of quadratic rate of change of demand (\(c\)):

With the variation in linear rate of change of demand, there are increments in ending inventory level at time \(T\) as well as in ordered quantity. Lengthening of replenishment cycle length as well as the enlargement of time period point at which no more stock remains in the backroom are witnessed with a higher selling price, the total profit rises.

(c) Sensitivity analysis of linear rate of change of demand (\(b\)):

From table-1 the following observations are extracted;

Table 1: Sensitivity Analysis of the decision variables on varying various inventory parameters

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<th>(t_i)</th>
<th>(TP)</th>
<th>(E)</th>
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<td>0.0109</td>
<td>20.5480</td>
<td>29189.9563</td>
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</table>

| \(W\) | 12.3094  | 28172.8431 | 208.4152       | 0.0500         | 20.6453        | 0.0100        | 28172.8431| 28372.2051 |
|       | 13.8274  | 28538.5363 | 207.3404       | 0.0509         | 24.2450        | 0.0101        | 28538.5363| 28538.5363 |
|       | 15.3416  | 16.8522    | 206.3119       | 0.0519         | 26.0264        | 0.0103        | 15.3416  | 28538.5363 |
|       | 18.3592  | 18.3592    | 205.3247       | 0.0528         | 27.7991        | 0.0105        | 18.3592  | 28538.5363 |
|       | 204.3745 | 204.3745   | 204.7347       | 0.0537         | 27.7991        | 0.0107        | 204.3745 | 28796.2928 |
| \(\alpha\) | 15.3418 | 28538.5742 | 206.3094       | 0.0519         | 24.2442        | 0.0083        | 28538.5742| 28538.5552 |
|       | 15.3417  | 28538.5363 | 206.3107       | 0.0519         | 24.2440        | 0.0093        | 28538.5363| 28538.5363 |
|       | 15.3416  | 15.3416    | 206.3119       | 0.0519         | 24.2439        | 0.0103        | 15.3416  | 28538.5363 |
|       | 15.3415  | 15.3415    | 206.3130       | 0.0519         | 24.2437        | 0.0114        | 15.3415  | 28538.5363 |
|       | 206.3142 | 206.3142   | 206.3142       | 0.0519         | 24.2436        | 0.0124        | 206.3142 | 28538.50098|
There is a decrement in each decision variable with the variation in quadratic rate of change of demand. The ending inventory level, the ordered quantity decreases with the decline in replenishment cycle length as well as in time period at which no more stock remains in the backroom with the drop in selling price, therefore, the total profit of the firm falls.

Sensitivity analysis of holding cost \( (h) \):

With the variation in holding cost, each decision variable values remains unchanged. Therefore, firm’s total profit remains unaltered.

Sensitivity analysis of unit purchasing cost \( (C) \):

With the variation of the purchasing cost, there is a decrement in ending inventory level at time \( T \) as well as in ordered quantity, along with the lengthening of replenishment cycle length, a hike in time period to vacant the backroom is seen with vastly uplifting the selling price but finally, the firm’s total profit declines.

Sensitivity analysis of available shelf space \( (\lambda) \):

If mark-up for available shelf space is varied, the ending inventory level and ordered quantity increases but shortening of replenishment cycle length as well as fall in time period to vacant the backroom is observed. But a hike in selling price results in higher profit margin of the firm.

Sensitivity analysis of expiration date \( (m) \):

With the variation in expiration date of product, there are increments in ending inventory level at time \( T \) as well as in ordered quantity. Lengthening of replenishment cycle length as well as the enlargement of time period point to vacant the backroom are witnessed with a higher selling price results the total profit gain of the firm.

Sensitivity analysis of fixed ordering cost \( (A) \):

With respect to the increment in ordering cost, there are increments in ending inventory level as well as in ordered quantity. Lengthening of replenishment cycle length as well as the elaboration of time period point to vacant the backroom are seen with a higher selling price but finally declines the total profit gain of the firm.

Sensitivity analysis of proportionality constant of replenishment cycle length \( (\alpha) \):

There is a decrement in ending inventory level at time \( T \) as well as in ordered quantity, with shortening slightly of replenishment cycle length and a hike in time period to vacant the backroom, but selling price increases. Finally, the firm’s total profit declines with the variation of proportionality constant of replenishment cycle length.

Sensitivity analysis of Salvage value \( (s) \):

High increment in ending inventory level and ordered quantity with shortening in replenishment cycle length and a fall in time period to vacant the backroom are observed. With a hike in selling price, the firm’s total profit grows, with the variant salvage values.

Sensitivity analysis of available shelf space \( (w) \):

If available shelf space is varied, the ending inventory level and ordered quantity increases highly but shortening of replenishment cycle length as well as fall in time period to vacant the backroom is observed. But a hike in selling price results in higher profit margin of the firm.

4. CONCLUSION

The market demand for fresh produce product is rising significantly in today’s health priority life. Basically, the demand for fresh produce products are also based on the measurement of freshness and the size of its shelf space for displaying the products which obviously fascinate more and more customers to purchase the product. Moreover, the expiration date and selling price for any deteriorating item plays a major role in the purchasing decision of a customer. Therefore, in this article, the market demand for fresh produce product is assumed to be a quadratic varying time function of its freshness, the selling price of item, the displayed stock in its shelf space and the expiration date. It may be gainful to preserve a high stock level at the end of the replenishment cycle, with this freshness-and-price-stock dependent demand.

Therefore, varying the traditional way of zero ending inventory level to non-zero ending inventory. So, then fulfilling the main objective of this article to maximize the total profit by estimating the optimal selling price of the item, the optimal ending inventory level and the optimal replenishment cycle length, the optimal time to vacant the backroom and the optimal ordered quantity. The classical optimization technique is utilized for calculating the optimal values. Thereafter, using the concept of eigen-values of a Hessian matrix, we have proved the concave nature of the profit function. Finally, a numerical example along with the sensitivity analysis of decision variables by varying various inventory parameters are presented to validate the derived model and extracts significant implications.
managerial insights. The illustration shows the gain in total profit by variation of inventory parameters $\bar{a}$, $b$, $\lambda$, $m$, $s$, $W$ and loss in profit gain by variation in $c$, $C$, $A$, $\alpha$.

The derived model can be further extended by utilizing the concept of trade credit and/or including the constraint of shortages, partial backlogging; also to hike the total profit of the firm efforts for advertising and/or service investment can be used.

**REFERENCES**


### Appendix-1: Solution of differential equation (8):

Re-arranging (8) we get,

\[
\left[ I(t) \right]^{-\lambda} dI(t) = -\left[ a(1+bt-ct^2) \right]^{\rho_{n-1}} \left( \frac{m-t}{m} \right) dt
\]  

\text{(A1)}

Taking integration on both sides of (A1) yields,

\[
\frac{1}{1-\lambda} \left[ I(t) \right]^{-\lambda} = -\frac{ap_{n-1}}{m} \left[ \frac{1}{4} ct^4 + \frac{1}{3} (-cm-b)t^3 + \frac{1}{2} (bm-1)t^2 + mt \right] + C
\]

\text{(A2)}

Substituting \( I(T) = E \) into (A2), and re-arranging terms, we have

\[
C = \frac{1}{1-\lambda} \left[ E \right]^{-\lambda} + \frac{ap_{n-1}}{m} \left[ \frac{1}{4} T^4 + \frac{1}{3} (-cm-b)T^3 + \frac{1}{2} (bm-1)T^2 + mT \right]
\]

\text{(A3)}

Finally, substituting (A3) into (A2), re-arranging the terms, we get,

\[
I(t) = \left( 1-\lambda \right) \left[ \frac{ap_{n-1}}{m} \left[ \frac{1}{4} (T^4 - t^4) + \frac{1}{3} (-cm-b)(T^3 - t^3) \right] + \frac{1}{2} (bm-1)(T^2 - t^2) + m(T - t) \right]^{\frac{1}{1-\lambda}} + E^{1-\lambda}
\]

\text{(A4)}