AN EPQ MODEL FOR DETERIORATING ITEMS WITH PRICE DEPENDENT DEMAND AND TWO LEVEL TRADE CREDIT FINANCING

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ABSTRACT
In practice, trade credit induces more sales over time by allowing customers to purchase without immediate cash. More often manufacturers offer a permissible delay on total purchase amount to the credit-worthy retailers. This is called up-stream full trade credit. On the other hand, the retailers frequently request its credit-risk customers to pay a fraction of the total purchase amount at the time of purchase and the remaining balance must be paid after the permissible delay. This is called down-stream partial trade credit. The purpose of this paper is to establish an economic production quantity (EPQ) model for deteriorating items with both up-stream and down-stream trade credits. The associated profit function is maximized with respect to selling price and cycle time using classical optimization. At the end of the paper, a numerical example and sensitivity analysis are provided to illustrate the features of the proposed model.

KEYWORDS: Deteriorating items, EPQ, Trade credit, Inventory, Permissible delay in payments

MSC: 90B05

1. INTRODUCTION
Trade credit financing refers to the practice of suppliers allowing retailers to place and receive orders without making immediate payment. In a competitive market, trade credit from the point of view of the supplier act as a promotional tool to stimulate the demand. They do not charge any interest on the purchase amount within the permissible delay period. So the retailers can earn some interest from the sales revenue during the allowable delay period. Similar to the supplier, the retailer can also pass on trade credit period to his end customers in order to generate more demand. Soni et al. (2010) gave a very fine review article for inventory models under trade credit.

Min et al. (2010) developed an inventory model for deteriorating items under stock-dependent demand and delay in payments. Lin et al. (2012) developed an inventory model with trade credit financing in which the retailer gets defective items from the supplier. Sheng-Chih Chen et al. (2014) developed an economic
production quantity (EPQ) model for deteriorating items with two-level trade credit. Researchers like Jaggi et al. (2013), Giri et al. (2015), Shah and Cardenas-Barron (2015), Shah et al. (2015), Wu and Zhao (2015), etc. and their cited references have done inspiring work in the area of inventory modelling with trade credit. Recently, Lashgari et al. (2016) developed an inventory model for deteriorating items with two levels of trade credit linked to order quantity. In the present market scenario, the selling price of a product is a big factor for customers in selecting the item. In practice, a higher selling price decreases demand of the product, whereas low price has the reverse effect. Shah et al. (2013) developed an inventory model for deteriorating items with a price dependent demand under biddable two-part trade credit. Mondal et al. (2003), You (2005), Teng et al. (2005), Maiti et al. (2009), Shastri et al. (2014) have used price dependent demands in their research.

In this paper, we have developed an inventory model for a single supplier (or manufacturer), single retailer and single item. The supplier offers a full trade credit to its credit-worthy retailers while on the other hand, the retailer requests its credit-risky customers to pay the fraction of the total purchase amount in advance and the remaining balance must be settled after the permissible delay period. The item considered in the study deteriorates with constant rate and the demand rate is dependent on time and selling price. Under the above assumptions, our objective is to maximize profit function with respect to cycle time and selling price. The paper is organised as follows: In section 2, basic notation and assumptions of the proposed problem are given. Section 3 includes the derivation of the mathematical model of the proposed problem. In section 4, a numerical example is given to support the proposed model and sensitivity analysis is carried out followed by conclusion in section 5.

2. NOTATION AND ASSUMPTIONS

2.1. NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$A$</td>
<td>Ordering cost per order (in $)</td>
</tr>
<tr>
<td>$C$</td>
<td>Purchase cost per unit (in $)</td>
</tr>
<tr>
<td>$h$</td>
<td>Holding cost per item per unit time ($/unit)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Rate of deterioration ($0 &lt; \theta &lt; 1$)</td>
</tr>
<tr>
<td>$p$</td>
<td>Selling price per unit (in $) (decision variable)</td>
</tr>
<tr>
<td>$R(p,t)$</td>
<td>Demand rate ($t \geq 0$) units</td>
</tr>
<tr>
<td>$P$</td>
<td>Production rate in units per year ($P = l \cdot R(p,t)$, $l \geq 1$)</td>
</tr>
<tr>
<td>$I(t)$</td>
<td>Inventory level at time $t \geq 0$ (units)</td>
</tr>
<tr>
<td>$T$</td>
<td>Length of production inventory cycle (years) (decision variable)</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Point of time at which production stops (years)</td>
</tr>
<tr>
<td>$Q$</td>
<td>Maximum inventory level when production stops at $t = T_i$</td>
</tr>
<tr>
<td>$I_c$</td>
<td>Rate of interest charged per dollar per year</td>
</tr>
<tr>
<td>$I_e$</td>
<td>Rate of interest earned per dollar per year</td>
</tr>
<tr>
<td>$M$</td>
<td>Trade credit period offered by the manufacturer to the retailer (years)</td>
</tr>
<tr>
<td>$N$</td>
<td>Trade credit period offered by the retailer to his end customers (years)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Fraction of the total purchase amount paid by customer ($0 \leq \varepsilon \leq 1$)</td>
</tr>
<tr>
<td>$1-\varepsilon$</td>
<td>Fraction of the total purchase amount remains to pay by the customer within the permissible delay period $N$</td>
</tr>
</tbody>
</table>
The deterioration rate is considered as constant and there is no repair or replacement of deteriorated items during cycle time.
2. The replenishment instantaneous and time horizon is infinite.
3. Lead time is negligible or zero and shortages are not allowed.
4. Demand rate $R(t)$ is considered as $R(t) = \alpha(1 + \beta t)\eta^\alpha$ where, $\alpha > 0$ is the scale demand, $\beta > 0$ and $\eta > 0$ is the price elasticity.
5. The manufacturer offers a trade credit period $M$ to the retailer. In credit-linked demand situation, when a retailer gets delayed period for payments from the manufacturer, the retailer also tries to pass on similar offers to his end customers. In our problem retailer offers partial trade credit period $N$ to his customers who needs to pay $\epsilon$ portion of the total purchase amount at the time of purchase and remaining balance must be settled within a permissible delay of $N$ years.
6. During the credit period offered by the manufacturer, the retailer sells the item and uses the sales revenue to earn interest from the bank at the rate $I_e$. After the end of credit period the retailer pays the purchase amount to the manufacturer. Now the retailer loses some money due to capital opportunity cost at a rate of $c$ for the items left in stock and the items already sold but not yet paid by the customers.

3. MATHEMATICAL MODEL

During the period $[0,T_1]$, inventory is consumed due to demand rate $R(t)$ and constant deterioration rate $\theta$. The production is also going on in this period so it also affects the inventory level. The governing differential equation for inventory level $I_1(t)$ at any time $t$, where $0 \leq t \leq T_1$ is given by

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = P - R = (l-1)R(t), \quad 0 \leq t \leq T_1$$

with the initial condition $I_1(0) = 0$.

Now, during the period $[T_1,T]$, the inventory is consumed by the demand and deterioration only so the governing differential equation in this non-production period is given by

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -R(t), \quad T_1 \leq t \leq T$$

with the end inventory level $I_2(T) = 0$.

On solving the differential equations (1) and (2) with given conditions, we get the solutions as

$$I_1(t) = \frac{\alpha p^{-\eta}(l-1)}{\theta^2} \left[\left(1-e^{-\theta t}\right)(\theta - \beta) + \theta \beta t\right], \quad 0 \leq t \leq T_1$$

and

$$I_2(t) = \frac{\alpha p^{-\eta}}{\theta^2} \left[\left(\theta - \beta + \theta \beta T\right)e^{\theta(T-1)} - \left(\theta - \beta + \theta \beta t\right)\right], \quad T_1 \leq t \leq T$$

As the function $I_1(t)$ is continuous in nature, we can establish a relationship between $T_1$ and $T$. This relation in simplified form is obtained as

$$T_1 = \frac{1}{2} \frac{T(\beta + T \theta + 2)}{l-1}$$

Here, our objective is to maximize retailer’s total profit. For the considered inventory problem the general expression of retailer’s profit is given by
\[ \pi(T, p) = SR - PC - OC - HC + IC + IE \] (6)

Where,

- \( SR \) = Net sales revenue = \( \frac{p}{T} \int_0^T R(p, t) dt \)
- \( PC \) = Purchase cost = \( \frac{CQ}{T} \)
- \( OC \) = Ordering cost = \( \frac{A}{T} \)
- \( HC \) = Holding cost = \( \frac{h}{T} \left[ \int_0^T I_1(t) dt + \int_{I_2(t)}^T \right] \)
- \( IE \) = Interest earned per cycle
- \( IC \) = Interest charged per cycle

To derive retailer’s total profit, mathematical expression for \( IE \) and \( IC \) is required. For this up-stream and down-stream trade credit periods should be taken into consideration. From the values of \( M \) and \( N \), there are two cases possible: (1) \( N < M \) and (2) \( N \geq M \). These two main cases are then divided into sub-cases based on the values of \( M, T \) and \( T + N \).

**Case 1.1: \( N < M \) and \( M \leq T \)**

As shown in figure 1, for the case 1.1 the retailer earns interest for the portion of instant payment during the period \([0, M]\) and from the portion of delayed payment during the period \([N, M]\). Therefore, the total interest earned in case-1.1 is given by

\[
IE_1 = \frac{pL}{T} \left[ \varepsilon \int_0^M (R(p, t) \cdot t) dt + (1 - \varepsilon) \int_0^{M-N} (R(p, t) \cdot t) dt \right] \] (7)

At the end of the manufacturer’s credit period \( M \), the retailer pays the purchasing cost to the manufacturer. Now the retailer incurs an opportunity cost at the rate of \( I_c \) for, (1) all the items sold after \( M \) for the portion of instant payment and (2) all the items sold after \( M - N \) for the portion of delayed payments. Therefore, the total interest charged in case-1.1 is given by

\[
IC_1 = \frac{C_l}{T} \left[ \varepsilon \int_0^{T-M} (R(p, t) \cdot t) dt + (1 - \varepsilon) \int_0^{T-M-N} (R(p, t) \cdot t) dt \right] \] (8)
As a result, the retailer’s total profit in case 1.1 using equations (6)-(8) is given by

\[
\pi_1(T, p) = \frac{p}{T} \int_0^T R(p,t)dt - \frac{CQ}{T} - \frac{A}{T} \int_0^T I_1(t)dt + \frac{T}{T} \int_T^T I_2(t)dt
\]

\[
- \frac{CI_c}{T} \left[ \int_0^{T-M} e \left( R(p,t) \right) dt + (1-e) \int_0^{T+N-M} \left( R(p,t) \right) dt \right]
\]

\[
+ \frac{pI_c}{T} \left[ \int_0^M e \left( R(p,t) \right) dt + (1-e) \int_0^{M-N} \left( R(p,t) \right) dt \right] \quad (9)
\]

**Case 1.2: N < M and T \leq M \leq T+N**

As shown in figure 2, for this case the retailer earns interest for the portion of instant payment during period \([0, M]\) and from the portion of delayed payment during the period \([N, M]\). Also since \(T \leq M\), the retailer does not need to pay any interest for the portion of instant payment. However, for the period \([M - N, T]\) the retailer must be charged for the items sold in this duration. Therefore, the total interest earned and interest charged in case-1.2 is given by

\[
IE_2 = \frac{pI_c}{T} \left[ \int_0^T e \left( R(p,t) \right) dt + e (M - T) \int_0^T R(p,t)dt + (1-e) \int_0^{M-N} \left( R(p,t) \right) dt \right] \quad (10)
\]

\[
IC_2 = \frac{CI_c}{T} \left[ (1-e) \int_0^{T+N-M} \left( R(p,t) \right) dt \right] \quad (11)
\]

As a result, the retailer’s total profit in this case using equations (6), (10) and (11) is given by

\[
\pi_2(T, p) = \frac{p}{T} \int_0^T R(p,t)dt - \frac{CQ}{T} - \frac{A}{T} \int_0^T I_1(t)dt + \frac{T}{T} \int_T^T I_2(t)dt
\]

\[
+ \frac{pI_c}{T} \left[ \int_0^T e \left( R(p,t) \right) dt + e (M - T) \int_0^T R(p,t)dt + (1-e) \int_0^{M-N} \left( R(p,t) \right) dt \right] \quad (12)
\]

\[
- \frac{CI_c}{T} \left[ (1-e) \int_0^{T+N-M} \left( R(p,t) \right) dt \right]
\]
Case 1.3: $N < M$ and $T + N \leq M$

As shown in figure 3, the total interest earned and interest charged in this sub-case is given by

$$IE_3 = \frac{pI_\varepsilon}{T} \left[ \varepsilon \int_0^T (R(p,t) \cdot t) dt + \varepsilon (M - T) \int_0^T R(p,t) dt \right]$$

$$+ \frac{pI_\varepsilon}{T} \left[ (1 - \varepsilon) \int_0^T (R(p,t) \cdot t) dt + (1 - \varepsilon)(M - T - N) \int_0^T R(p,t) dt \right]$$

(13)

As a result, the retailer’s total profit using equations (6), (13) and (14) is given by

$$\pi_3(T, p) = \frac{p}{T} \int_0^T R(p,t) dt - \frac{CQ}{T} - \frac{A}{T} \left[ \int_0^T \frac{I_1(t)}{T} dt + \int_0^T \frac{T}{T_1} \int_0^T R(p,t) dt \right]$$

$$+ \frac{pI_\varepsilon}{T} \left[ \varepsilon \int_0^T (R(p,t) \cdot t) dt + \varepsilon (M - T) \int_0^T R(p,t) dt \right]$$

$$+ \frac{pI_\varepsilon}{T} \left[ (1 - \varepsilon) \int_0^T (R(p,t) \cdot t) dt + (1 - \varepsilon)(M - T - N) \int_0^T R(p,t) dt \right]$$

(15)

Next we discuss two sub-cases for the second case $N \geq M$

Case 2.1: $N \geq M$ and $M \leq T$

Figure 4: $N \geq M$ and $M \leq T$
As shown in figure 4, the total interest earned and interest charged in this sub-case is given by

\[ IE_4 = \frac{pl_e}{T} \left[ \varepsilon \int_0^M R(p,t) \cdot t \, dt \right] \]  

(16)

\[ IC_4 = \frac{Cl_e}{T} \left\{ \varepsilon \left[ \int_0^{T-M} R(p,t) \cdot t \, dt + (1-\varepsilon)(T+2(N-M))\int_0^T R(p,t) \, dt \right] \right\} \]  

(17)

As a result, the retailer’s total profit using equations (6), (16) and (17) is given by

\[ \pi_4(T, p) = \frac{p}{T} \int_0^T R(p,t) \, dt - \frac{CQ}{T} - \frac{A}{T} - \frac{h}{T} \left[ \int_0^T I(t) \, dt + \int_{T_i}^T I_2(t) \, dt \right] - \frac{Cl_e}{T} \left\{ \varepsilon \left[ \int_0^{T-M} R(p,t) \cdot t \, dt + (1-\varepsilon)(T+2(N-M))\int_0^T R(p,t) \, dt \right] \right\} + \frac{pl_e}{T} \left[ \varepsilon \int_0^M R(p,t) \cdot t \, dt \right] \]  

(18)

**Case 2.2:** \( N \geq M \) and \( M \geq T \)

As shown in figure 5, the total interest earned and interest charged in this sub-case is given by

\[ IE_5 = \frac{pl_e}{T} \left[ \varepsilon \int_0^T (R(p,t) \cdot t) \, dt + \varepsilon (M-T)\int_0^T R(p,t) \, dt \right] \]  

(19)

\[ IC_5 = \frac{Cl_e}{T} \left\{ (1-\varepsilon)\int_0^T (R(p,t) \cdot t) \, dt + (1-\varepsilon)(N-M)\int_0^T R(p,t) \, dt \right\} \]  

(20)

As a result, the retailer’s total profit using equations (6), (19) and (20) is given by

\[ \pi_5(T, p) = \frac{p}{T} \int_0^T R(p,t) \, dt - \frac{CQ}{T} - \frac{A}{T} - \frac{h}{T} \left[ \int_0^T I(t) \, dt + \int_{T_i}^T I_2(t) \, dt \right] - \frac{Cl_e}{T} \left\{ (1-\varepsilon)\int_0^T (R(p,t) \cdot t) \, dt + (1-\varepsilon)(N-M)\int_0^T R(p,t) \, dt \right\} + \frac{pl_e}{T} \left[ \varepsilon \int_0^T (R(p,t) \cdot t) \, dt + \varepsilon (M-T)\int_0^T R(p,t) \, dt \right] \]  

(21)
The retailer’s total profit in each case is function of two variables $T$ and $p$. To maximize total profit with respect to selling price $p$ and cycle time $T$, the necessary conditions are

$$\frac{\partial \pi_i(T,p)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial \pi_i(T,p)}{\partial p} = 0, \quad i = 1,2,..,5$$

The critical point $(T^*, p^*)$ can be found by solving $\frac{\partial \pi_i(T,p)}{\partial T} = 0$ and $\frac{\partial \pi_i(T,p)}{\partial p} = 0$ simultaneously for each case and for maximum profit following condition should be satisfied for critical point.

$$\frac{\partial^2 \pi_i(T,p)}{\partial T^2} > 0 \quad \text{and} \quad \frac{\partial^2 \pi_i(T,p)}{\partial p^2} < 0$$

Using this approach, we calculate maximum profit in each case for the numerical example provided in following section.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

4.1. NUMERICAL EXAMPLE

Example 1: We consider an inventory system with following parameters in appropriate units:

- $\alpha = 5000, \quad \beta = 0.18, \quad \theta = 3\%, \quad M = 0.3 \text{ year}, \quad N = 0.2 \text{ year}, \quad A = $100, \quad h = $0.5, \quad C = $10, \quad I_e = 12\%, \quad I_e = 9\%, \quad l = 1.5, \quad \epsilon = 0.3$ \ and \ $\eta = 1.62$.

We have calculated maximum profit in each case using Maple 18 software for the above parameters and the comparison of profit values for every case is shown in following chart.

As shown in figure 6, it is clear that optimal result is obtained in each case and profit function attains maximum value in case-1.3 $(N < M$ and $T + N \leq M)$. The optimal cycle time and the optimal selling price for this case are $(T^*, p^*) = (3.873, 10.2143)$ and the maximum profit is $\pi_{\text{max}} = $816.8372. The concave nature of the profit function in this case is shown in following figure.

Figure 6: Profit comparison in each case

Figure 7: Concavity of profit function in case 1.3
4.2. SENSITIVITY ANALYSIS

In this section, we analyze the effect of inventory parameters on the profit function and the decision variables of the proposed model. We change each parameter in between -20% to 20% at a time keeping other parameters untouched. We perform this analysis only for the sub-case 1.3 where highest profit is achieved. Results of sensitivity analysis are shown in following figure (8)-(10).

From figure 8, we can observe that

- Selling price increases significantly when purchase cost \( C \) and parameter \( l \) associated with production rate increases.
- Price elasticity \( \eta \) drastically reduces selling price.
- Parameters \( \beta \) moderately increases selling price and rate of interest earned \( I_e \) decreases selling price marginally.
- For other parameters, a very minor effect is observed on selling price.

![Figure 8: Effect of inventory parameters on selling price](image)

From figure 9, we can observe that

- A heavy decrease in cycle time \( T \) is observed when rate of interest earned \( I_e \) and price elasticity \( \eta \) increases.
- By increasing parameters \( \beta, C \) and \( l \) a noticeable increase is observed in cycle time. Also a noticeable decrease in cycle time occurs when parameters \( \alpha \) and \( h \) are increased.
- Other parameters have a very minor effect on cycle time.

![Figure 9: Effect of inventory parameters on cycle time](image)
From figure 10, we can observe that

- Retailer’s total profit decreases heavily, when $l$ and $\eta$ increases.
- Significant increase is observed in profit when demand parameters $\alpha$ and $\beta$ increases.
- For the increased value of the purchase cost $C$ and rate of interest earned $I_e$, the profit value decreases significantly.
- Other parameters have a very minor effect on profit.

![Figure 10: Effect of inventory parameters on profit](image)

5. CONCLUSION

In this paper, we have studied an inventory model for deteriorating items with two-level trade credit. The demand is assumed to be dependent on time and selling price. We have derived seller’s total profit function and maximized it with respect to cycle time and selling price in five different cases, depending upon trade credit period offered by the manufacturer and retailer. Using the numerical example, we have shown that out of all cases the seller’s total profit is highest when $N < M$ and $T + N \leq M$. The sensitivity analysis is carried out for different inventory parameters.

In this competitive market, many firms and companies work on a low profit margin. Due to this they have to use promotional tools like trade credit in order to attract more players and to stimulate demand. In this type of scenario the proposed model is highly applicable. For further research, one can extend the model by considering partial backlogging, shortages, effect of inflation, etc. We can also go for integrated solution for both the seller and the buyer.

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