

PREFACE



For a long time Variational Analysis has been a main research topic. One of the most important contributors to this area is Professor Boris Mordukhovich. On the occasion of his 70th birthday, we are editing this special issue on Variational Analysis and Optimization. Professor Mordukhovich derived a huge variety of important results in this field. There are more than 400 articles and books authored by him, in particular his famous books *Variational Analysis and Generalized Differentiation, I: Basic Theory; II: Applications*, which are essential references within this topic. Furthermore, there are thousands of references and forthcoming papers based on his contributions. His great ideas and fundamental results have been recognized worldwide. Professor Mordukhovich is Doctor Honoris Causa of six universities in Europe and Asia. He is a fellow at least of the following prestigious scientific associations: the Society for Industrial and Applied Mathematics, the American Mathematical Society, the SIAM Special Group on Control and Systems Theory, the International Federation of Nonlinear Analysis, the Mathematical Optimization Society and the International Working Group on Generalized Convexity. Moreover, he is a Corresponding Member of the Accademia Peloritana dei

Pericolanti (Italy), a Lifetime Scholar of the WSU Academy of Scholars and a member of the editorial boards of many important journals in the area of optimization and analysis as well as guest editor of more than 30 special issues. Due to his important scientific achievements, the Program Committee of the X International Conference on Operations Research in Havana 2012 contacted Professor Mordukhovich and invited him to give a plenary talk as opening lecture of this event. We were very happy when he accepted our invitation although knowing that our resources were modest. Of course, his lecture was brilliant and very insightful presenting new research approaches. It was a fruitful opportunity to discuss topics of variational analysis, especially on applications of Mordukhovich derivatives to optimization problems. In addition, the participation of Professor Mordukhovich at this conference was wonderful from a scientific and personal point of view. In Cuba, his quality as a scientist has been well-known and his visit showed us the friendly, open and kind person he is. He was always open-minded for discussions with the participants of this conference. As a result, collaborations with him, his students and the research group at Martin-Luther-University Halle-Wittenberg in Germany were fostered. Professor Mordukhovich participated at the Colloquium in Wittenberg 2015 and further contacts were established.

In the following year 2016 at the XII International Conference on Operations Research, we, the editors, agreed to dedicate a special issue of the Journal Investigación Operacional to topics of Variational Analysis and Optimization recognizing Professor Mordukhovich's contributions to this topic. Therefore, we will be part of the celebrations for his 70th-Birthday. He accepted to send a contribution to this journal, honoring this issue. His contribution is concerned with studies on the sweeping (Moreau) processes with controlled moving sets. As pointed out in his article, this model has applications in elastoplasticity, hysteresis, electric circuits and traffic equilibriums. This remarkable paper includes new results on a class of optimal control problems using variational analysis, and it opens this special issue.

This issue consists of research papers, which are related to the topics of Professor Mordukhovich's research. They present e.g. recent advances in multi-objective and generalized semi-infinite optimization models, fields where variational analysis has been applied.

It follows a brief overview on the contributions included in this issue:

- Extended real-valued functions, a unified approach by P. Weidner. The paper characterizes continuity, semi-continuity, convexity in a new way.
- The development of models with economic application appears in the contribution of S. D. Flam. How to share risk is discussed if inf-convolution admits a sub-gradient at the aggregate risk.
- Necessary and sufficient optimality conditions for vector generalized semi-infinite programming problems are presented in the paper by F. Guerra-Vázquez, L. Hernández-Rebollar and J.-J. Rückmann.

The following six papers are concerned with multi-objective problems, a common research topic of the editors.

- The paper by C. Günther studies generalized-convex multi-objective optimization problems involving forbidden regions defined by convex sets. A complete characterization of the set of (strictly, weakly) Pareto efficient points is provided. For a special location problem with forbidden regions given by Euclidean balls, the results are illustrated.
- Overshooting methods are applied to the solution of differential-algebraic equations optimizing two criteria via scalarizing functions in the paper by E. Köbis and M.A. Köbis.
- In the paper by V. Tuan, T.T. Le and C. Tammer, vector optimization problems with a cone-convex objective function and a non-convex set of feasible solutions are studied. The paper establishes the existence of Lagrange multipliers for Pareto points and natural extensions of former results by Durea and Tammer. Tools of generalized differentiation by Boris Mordukhovich are used in this paper.
- An application of optimization with respect to variable ordering structures to radiotherapy treatment is discussed in the paper by T.T. Le. Via variational analysis, necessary optimality conditions for the corresponding non-dominated solutions are obtained.
- B. Zargini and A. R. Doagooei establish a relation between inverse variational inequalities and multi-objective optimization problem with respect to variable domination structure. Using scalarizations, a known approach is adapted to this framework.
- As in the case of optimization problems, convex functions with respect to variable ordering structures enjoy important properties. A method for generating test convex functions under variable ordering and hence, optimization convex problems is developed in the paper by G. Bouza, D. Hernández and J.-J. Rückmann.
- This special issue concludes with a review on global convergence properties of inexact Newton theory on the perturbed KKT-conditions by V. Baryamureeba, T. Steihaug and M. El Gham.

We consider this current issue as our modest homage to the 70th birthday of this great scientist and friend. We hope that this special issue encourages the development of forthcoming new results as well as international scientific cooperation in the area of variational analysis.

The Editors

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