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ON COMPARISON OF TWO GENERALIZED CLASSES OF RATIO -TYPE ESTIMATORS IN SAMPLE SURVEYS

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ABSTRACT
In this paper some generalized classes of minimum mean square error estimators of the finite population mean in the presence of a single auxiliary variable are proposed and their efficiencies are compared both theoretically and with numerical illustrations.

KEYWORDS:Classes of estimators,Ratio estimator, Regression estimator, Simple random sampling, Bias, Mean square error, Efficiency.

MSC: 62D05

RESUMEN
En este paper son propuestas algunas clases de estimadores mínimo cuadráticos de la media de poblaciones finitas, en la presencia de una variable auxiliar, y sus eficiencias son comparadas teóricamente y usando ilustraciones numéricas.

1. INTRODUCTION

In sample surveys the availability of auxiliary information is often exploited to improve the efficiency of estimators of the finite population mean/total /variance of the study variable in question. Ratio and regression estimators are two classical estimators making use of the information on auxiliary variable correlated with the study variable in the estimation procedure. Ratio estimator in large samples is always less efficient than the linear regression estimator unless the line of regression passes through the origin in which case both the estimators are equally efficient. When regression line does not pass through the origin the researchers in sampling theory have suggested certain modifications to the ratio estimator , so that its asymptotic efficiency, that is , efficiency in large samples equals to that of the linear regression estimator. I t may be pointed out here that the linear regression theory does not quite fit in the sample survey situation because of the inherent restrictive assumptions such as (i) linearity of regression,(ii)constancy of the residual(error)variance and (iii) infinite nature of population ,which are rarely satisfied in sampling from finite populations (Cochran,1977). Srivastava(1971) has shown that if we attach any function of the ratio of the sample mean of the auxiliary variable to its population mean to the sample mean of the study variable the resultant estimator has the minimum mean square error (to first order of approximation) equal to that of the linear regression estimator subject to certain regularity conditions to be satisfied by the assumed function and also this minimum mean square error cannot be reduced further. This has encouraged the sampling theory research workers to define the function differently to generate a variety of ratio-type and product-type estimators to be used in practice. Let $U=\left(U_1,U_2,...U_N\right)$ be a finite population of size N. To each unit $U_i,\left(i=1,2,...,N\right)$ in the population the paired values $\left(y_i,x_i\right)$ corresponding to study variable y and an auxiliary variable x, correlated with y are attached.
Now, define the population means of the study variable y and auxiliary variable x as

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$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i\,, \quad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Thus, the population ratio is defined as $R = \frac{\bar{Y}}{\bar{X}}$.

Further, define the finite population variances of y and x and their covariance as

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2$$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2$$

and $S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_i - \bar{X}\right)$ respectively.

Also, the coefficient of variations of y, x and their coefficient of covariation are defined by

$C_y = \frac{S_y}{\bar{Y}}, C_x = \frac{S_x}{\bar{X}},\ \&\ C_{yx} = \frac{S_{yx}}{\bar{Y}\bar{X}} = \rho_{yx} C_y C_x$ respectively,

where ρ is the simple correlation between y and x.

A simple random sample‘s’ of size n is selected from U without replacement and the values $\left(y_i, x_i\right), i = 1,2,...,n$ are observed on the sampled units.

The classical ratio estimator of the population mean $\bar{Y}$, using auxiliary information on x is give by

$$\bar{y}_R = \frac{\bar{y}}{\bar{x}}\bar{X} \tag{1.1}$$

where $\bar{y}$ and $\bar{x}$ are sample means of y and x respectively, defined by $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \ \ and \ \ \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

The ratio estimator $\bar{y}_R$ envisages advance knowledge of $\bar{X}$. The bias (B) and mean square error (MSE) of $\bar{y}_R$ to $O\left(\frac{1}{n}\right)$ are given by

$$B\left(\bar{y}_R\right) = \theta\bar{Y}\left(C_x^2 - C_{yx}\right) \quad \text{(See Sukhatme, et al 1970)} \tag{1.2}$$

and $$MSE\left(\bar{y}_R\right) = \theta\bar{Y}^2\left(C_y^2 + C_x^2 - 2C_{yx}\right) \text{(See Sukhatme, et al 1970)} \tag{1.3}$$

where $\theta = \left(\frac{1}{n} - \frac{1}{N}\right)$.

The ratio estimator $\bar{y}_R$ is a biased estimator and the bias decreases with increase in sample size.

In large samples $\bar{y}_R$ is more efficient than the simple mean per unit estimator $\bar{y}$ if $K > \frac{1}{2}$. where $K = \rho\frac{C_y}{C_x}$.

Srivastava(1967) proposed a class of estimators given by

$$\bar{y}_{SR} = \bar{y}(\frac{\bar{X}}{\bar{x}})^{\alpha} \ , \tag{1.4}$$

where α is a real constant to be suitably chosen. $\bar{y}_{SR}$ belongs to Srivastava's (1971)generalized class $\bar{y}_{SRG} = \bar{y}h(u),$ where $u = \bar{x} / \bar{X}$ .The well-known ratio and product estimators belong to this class.
Swain (2012) suggested a generalized class of ratio type estimator as

$$t_g = \bar{y}\left[\alpha\left(\frac{\bar{X}}{\bar{x}}\right)^g + (1-\alpha)\left(\frac{\bar{x}}{\bar{X}}\right)^h\right]^{\delta}, \tag{1.5}$$

where α, g, h and δ are real numbers to be suitably chosen.
Bahl and Tuteja (1991) proposed a ratio type exponential estimator as

$$\bar{y}_{BTR} = \bar{y}\exp(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}) \tag{1.6}$$

Swain(2014) showed that to $O(1/n)$,

$$\bar{y}_{SQR} = \bar{y}(\frac{\bar{X}}{\bar{x}})^{1/2} \tag{1.7}$$

and $\bar{y}_{BTR} = \bar{y}\exp(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}})$ have equal bias and efficiency.
Swain(2013) proposed a generalized exponential ratio type estimator as

$$\bar{y}_{SW} = \bar{y}e^{\alpha\left(\frac{\bar{X}-\bar{x}}{\bar{X}}\right)} \tag{1.7a}$$

where α is a real number to be suitably chosen.
To $O(1/n)$, the bias and mean square error of $\bar{y}_{sw}$ are given by

$$B(\bar{y}_{sw}) = \theta\bar{Y}(\frac{\alpha^2}{2}C_x^2 - \alpha C_{yx}) \tag{1.8}$$

and

$$MSE(\bar{y}_{sw}) = \theta\bar{Y}^2(C_y^2 + \alpha^2 C_x^2 - 2\alpha C_{yx}) \tag{1.9}$$

To $O(1/n)$, $\bar{y}_{SR}$ and $\bar{y}_{SW}$ are equally efficient and $\bar{y}_{SR}$ is less biased than $\bar{y}_{SW}$ if $K > \frac{(2\alpha+1)}{4}$

For optimum α the above inequality reduces to $K > 1/2$.
In the following we suggest a generalized exponential ratio type estimator and compare it with generalized power transformation estimator as proposed by Swain (2012) as regards bias and efficiency.

2. GENERALIZED EXPONENTIAL RATIO -TYPE ESTIMATOR

We propose a generalized exponential ratio type estimator as

$$t_g^* = \bar{y}\left[\alpha e^{g\left(\frac{\bar{X}-\bar{x}}{\bar{X}}\right)} + (1-\alpha)e^{h\left(\frac{\bar{x}-\bar{X}}{\bar{X}}\right)}\right]^{\delta} \tag{2.1}$$

where α, g, h and δ are free real constants to be suitably chosen.

In this regard we may fix g, h and δ and minimize the approximate mean square error of t_g^* to $O\left(\frac{1}{n}\right)$ with respect to α .In fact we may fix any three of the constants g, h, δ and α and minimize the mean square error with respect to the remainder. expression for mean square error of t_g^* we obtain the optimum approximate mean square error of t_g^* and also the optimum approximate bias of t_g^*.

Now define $\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$ with :

$E(e_0) = E(e_1) = 0$, $V(e_0) = \theta C_y^2$, $V(e_1) = \theta C_x^2$ a $COV(e_0 e_1) = \theta C_{yx}$.

Bias and Mean Square Error of t_g^* :

Expanding t_g^* in power series and keeping up to second-degree terms we have

$$t_g^* = \bar{Y}(1+e_0)\left[\alpha\left(1 - ge_1 + \frac{g^2}{2}e_1^2 + \dots\right) + (1-\alpha)\left(1 + he_1 + \frac{h^2 e_1^2}{2} + \dots\right)\right]^{\delta}$$

$$= \bar{Y}\left[1 + \delta\{h - \alpha(g+h)\}e_1 + \delta\left\{\frac{h^2}{2} + \alpha\left(\frac{g^2 - h^2}{2}\right)\right\}e_1^2 \right.$$

$$\left. + \frac{\delta(\delta-1)}{2}\{h - \alpha(g+h)\}^2 e_1^2 + e_0 + \delta\{h - \alpha(g+h)\}e_0 e_1\right] \quad (2.2)$$

Thus the bias of t_g^* to $O\left(\frac{1}{n}\right)$ is given by

$$B\left(t_g^*\right) = \theta\bar{Y}\left[\delta\{h - \alpha(g+h)\}C_{yx}\right.$$

$$\left. + \delta\left\{\frac{h^2}{2} + \alpha\left(\frac{g^2 - h^2}{2}\right)\right\}C_x^2 + \frac{\delta(\delta-1)}{2}\{h - \alpha(g+h)\}^2 C_x^2\right] \quad (2.3)$$

The mean square error (MSE) of t_g^* to $O\left(\frac{1}{n}\right)$ is given by

$$MSE\left(t_g^*\right) = \theta\bar{Y}^2\left[C_y^2 + \delta^2\{h - \alpha(g+h)\}^2 C_x^2 + 2\delta\{h - \alpha(g+h)\}C_{yx}\right] \quad (2.4)$$

Minimizing (2.4) with respect to α gives

$$\alpha_{opt} = \frac{K + \delta h}{\delta(g+h)} \quad (2.5)$$

Substituting the optimum value of α in (2.4) we have

$$MSE\left(t_g^*\right)_{opt} = \theta\bar{Y}^2 C_y^2\left(1 - \rho^2\right) \quad , \quad (2.6)$$

which equals the large sample mean square error of the linear regression estimator

$$\bar{y}_{\ell r} = \bar{y} + b\left(\bar{X} - \bar{x}\right) \quad (2.7)$$

where b is the sample regression coefficient of y on x.

It may be verified that we obtain the same minimum mean square error as given by (2.6),if we fix any three of the unknown constants and minimize (2.4) with respect to the remainder.

$$B\left(t_g^*\right)_{opt}=\theta\bar{Y}\Big[\delta\left\{h-\alpha_{opt}(g+h)\right\}C_{yx}+\delta\left\{\frac{h^2}{2}+\alpha_{opt}\left(\frac{g^2-h^2}{2}\right)\right\}C_x^2$$

$$+\frac{\delta(\delta-1)}{2}\left\{h-\alpha_{opt}(g+h)\right\}^2C_x^2$$

$$=\theta\bar{Y}\left[\delta\left\{h-\frac{K+\delta h}{\delta(g+h)}(g+h)\right\}C_{yx}+\delta\left\{\frac{h^2}{2}+\frac{K+\delta h}{\delta(g+h)}\left(\frac{g^2-h^2}{2}\right)\right\}C_x^2\right.$$

$$+\frac{\delta(\delta-1)}{2}\left\{h-\frac{k+\delta h}{\delta(g+h)}(g+h)\right\}^2C_x^2$$

$$=\theta\bar{Y}\left[\delta\left\{h-\frac{K+\delta h}{\delta}\right\}C_{yx}+\delta\left\{\frac{h^2}{2}+\frac{(k+\delta h)}{\delta}\left(\frac{g-h}{2}\right)\right\}C_x^2\right.$$

$$+\frac{\delta(\delta-1)}{2}\left\{h-\frac{K+\delta h}{\delta}\right\}^2C_x^2$$

$$=\theta\bar{Y}\left[-K^2+\frac{1}{2}\left\{K(g-h)+\delta gh\right\}^2+\frac{\delta-1}{2\delta}K^2\right]C_x^2 \qquad (2.8)$$

Bias and mean square error of t_g

Assuming $|e_1|<1$ for all possible samples, t_g can be expressed as

$$t_g=\bar{Y}(1+e_0)\left[\alpha\left(1-ge_1+\frac{g(g+1)}{2}e_1^2+...\right)\right.$$

$$\left.+(1-\alpha)\left(1+he_1+\frac{h(h-1)}{2}e_1^2+...\right)\right]^{\delta} \qquad (2.9)$$

On simplification and keeping up to second degree terms, we have

$$t_g=\bar{Y}\left[1+\delta\left\{h-\alpha(g+h)\right\}e_1+\delta\left\{\frac{h(h-1)}{2}+\left(\frac{g(g+1)}{2}-\frac{h(h-1)}{2}\right)e_1^2\right\}+\frac{\delta(\delta-1)}{2}\right.$$

$$\left.\left\{h-\alpha(g+h)\right\}^2e_1^2+e_0+\delta\left\{h-\alpha(g+h)\right\}e_0e_1\right] \qquad (2.10)$$

To $O\left(\frac{1}{n}\right)$

$$B(t_g)=\theta\bar{Y}\left[\delta\left\{h-\alpha(g+h\right\}C_{yx}+\delta\left\{\frac{h(h-1)}{2}+\alpha\left(\frac{g(g+1)}{2}-\frac{h(h-1)}{2}\right)\right\}C_x^2\right.$$

$$\left.+\frac{\delta(\delta-1)}{2}\left\{h-\alpha(g+h)\right\}^2C_x^2\right] \qquad (2.11)$$

$$MSE(t_g)=\theta\bar{Y}^2\left[C_y^2+\delta^2\left\{h-\alpha(g+h)\right\}^2C_x^2+2\delta\left\{h-\alpha(g+h)\right\}C_{yx}\right] \qquad (2.12)$$

Minimizing $MSE(t_g)$ with respect to α, we have

$$\alpha_{opt} = \frac{K+\delta h}{\delta(g+h)}$$

Substituting the optimum value of α in the approximate expressions for $MSE(t_g)$ we have to $O(1/n)$,

$$MSE(t_g)_{opt} = \theta\bar{Y}^2 C_y^2 \left(1-\rho^2\right)$$

$$B\left(t_g\right)_{opt} = \theta\bar{Y}\left[\delta\left\{h-\alpha_{opt}\left(g+h\right)\right\}C_{yx} + \delta\left\{\frac{h(h-1)}{2} + \alpha_{opt}\left(\frac{g(g+1)}{2} - \frac{h(h-1)}{2}\right)\right\}C_x^2\right.$$

$$\left. +\delta\frac{(\delta-1)}{2}\left\{h-\alpha_{opt}\left(g+h\right)\right\}^2 C_x^2\right]$$

$$= \theta\bar{Y}\left[\left\{h-\frac{K+\delta h}{\delta(g+h)}\left(g+h\right)\right\}C_{yx} + \delta\left\{\frac{h(h-1)}{2} + \frac{K+\delta h}{\delta(g+h)}\left(\frac{g(g+1)}{2} - \frac{h(h-1)}{2}\right)\right\}C_x^2\right.$$

$$\left. +\frac{\delta(\delta-1)}{2}\left\{h-\frac{K+\delta h}{\delta(g+h)}\left(g+h\right)\right\}^2 C_x^2\right]$$

$$= \theta\bar{Y}\left[\left\{h-\frac{K+\delta h}{\delta}\right\}C_{yx} + \delta\left\{\frac{h(h-1)}{2} + \frac{K+\delta h}{\delta(g+h)}\left(\frac{g(g+1)}{2} - \frac{h(h-1)}{2}\right)\right\}C_x^2\right.$$

$$\left. +\frac{\delta(\delta-1)}{2}\left\{h-\frac{K+\delta h}{\delta}\right\}^2 C_x^2\right]$$

$$= \theta\bar{Y}\left[\left(-K^2 + \delta\left\{\frac{h(h-1)}{2} + \frac{(K+\delta h)}{2}\left(g-h+1\right)\right\} + \frac{\delta-1}{2\delta}K^2\right)\right]C_x^2 \qquad (2.13)$$

3. COMPARISON OF BIASES AND MEAN SQUARE ERRORS OF T_G AND T*_G

$$B(t_g)_{opt} == \theta\bar{Y}\left[\left(-K^2 + \delta\left\{\frac{h(h-1)}{2} + \frac{(K+\delta h)}{2}\left(g-h+1\right)\right\} + \frac{\delta-1}{2\delta}K^2\right)\right]C_x^2$$

$$B(t_g^*)_{opt} = \theta\bar{Y}\left[-K^2 + \frac{1}{2}\left\{K\left(g-h\right)+\delta gh\right\}^2 + \frac{\delta-1}{2\delta}K^2\right]C_x^2$$

$$MSE(t_g) = MSE(t_g^*) = \theta\bar{Y}^2 C_y^2\left(1-\rho^2\right)$$

Thus, to $O\left(\frac{1}{n}\right)$, t_g and t_g^* are equally efficient.

The biases of t_g and t_g^* are functions of g,h and δ and as such vary with different values g,h and δ .The best choices of these free parameters are those for which the first order biases vanish.For some specified values for g,h and δ the biases of t_g and t_g^* are tabulated in Table 1.

Table 1 Comparison of Biases without the common multiplier $\theta\bar{Y}$

Sl. No.	δ	g	h	α_{opt}	$B(t_g)$	$B(t_g^*)$
1	1	1	1	$\frac{K+1}{2}$	$(\frac{K+1}{2}-K^2)C_x^2$	$-(K^2-\frac{1}{2})C_x^2$
2	1	0	1	$K+1$	$-K^2C_x^2$	$-K(K+\frac{1}{2})C_x^2$
3.	1	1	0	K	$K(1-K)C_x^2$	$-K(K-\frac{1}{2})C_x^2$
4.	-1	1	1	$\frac{1-K}{2}$	$\frac{(K-1)}{2}C_x^2$	$-\frac{1}{2}C_x^2$
5.	-1	0	1	$1-K$	0	$-\frac{K}{2}C_x^2$
6.	-1	1	0	$-K$	$-KC_x^2$	$\frac{K}{2}C_x^2$

4. NUMERICAL ILLUSTRATIONS

Considering 5 natural populations described in Table 2 **we compare the approximate absolute biases of power** transformation ratio type estimator with exponential ratio type estimator in Table 3.

Table 2 Description of populations

Pop^n No.	Description	N	y	x	ρ_{yx}	C_x	C_y	K
1.	Sampford (1962)	17	Acreage under oats in 1957	Acreage of crops and grass in 1947	0.4	0.22	0.45	0.818
2.	Singh and Chaudhary (1986)	16	Area under wheat 1979-80	Total cultivated area during 1978-79	0.96	0.74	0.69	0.895
3.	Konijn (1973)	16	Food expenditure	Total expenditure	0.95	0.08	0.11	1.3062
4.	Murthy (1967)	16	Output for factories (000Rs)	Fixed capital (000Rs.)	0.84	0.15	0.09	0.504
5.	Swain (2003)	19	No. of milk cows in 1957	No. of milk cows census 1956	0.72	1.14	1.12	0.7073

Comments:

(i) t_1 is less biased than t_1^* for population 2, 3.

(ii) t_2 is less biased than t_2^* for population 1, 2, 3, 4 and 5.

(iii) t_3 is less biased than t_3^* for population 1, 2, 3

(iv) t_4 is less biased than t_4^* for population 1, 2, 3, 4 and 5

(v) t_5 is less biased than t_5^* for population 1, 2, 3, 4 and 5

(vi) t_6 is less biased than t_6^* for population 1

Table 3 Absolute Biases of Estimators without constant multiplier $\theta\overline{Y}$ **.**

Population / Bias	1	2	3	4	5
$B(\overline{y}_R)$	0.008809	0.057498	0.00196	0.01116	0.38039
$B(t_1)$	0.01209	0.0805	0.00346	0.0113	0.45928
$B(t_1^*)$	0.00770	0.165	0.00764	0.0056	0.00026
$B(t_2)$	0.03190	0.438	0.01084	0.00569	0.65015
$B(t_2^*)$	0.05170	0.683	0.01502	0.01136	1.10975
$B(t_3)$	0.00768	0.052	0.0025	0.00565	0.26905
$B(t_3^*)$	0.01210	0.193	0.0067	0.00002	0.19055
$B(t_4)$	0.00392	0.02865	0.001	0.00556	0.19003
$B(t_4^*)$	0.02371	0.2737	0.0031	0.01123	0.6497
$B(t_5)$	0.00048	0.000091	0.0001	0.00002	0.4227
$B(t_5^*)$	0.01931	0.24496	0.004	0.00564	0.4596
$B(t_6)$	0.04007	0.49019	0.00843	0.01136	0.91921
$B(t_6^*)$	0.05169	0.24514	0.00425	0.00569	0.45960

5. CONCLUSION

t_g and t_g^* are equally efficient in large samples. But biases of these classes vary for different values of g, h and δ. The choices of g,h and δ are arbitrary and should be so chosen to make first order bias zero. However, overall there is little to choose between t_g and t_g^* in large samples, if we can assume that the bias is negligible for large sample size. The special cases of the generalized classes of ratio-type estimators discussed in this paper can be distinguished and compared on the basis of biases of these estimators depending on the values of the free parameters involved in the their construction. Further generalization of the proposed classes may be effected by applying linear transformation to the auxiliary variable x and thus we define the resulting generalized classes as

$$t_G = \overline{y}\left[\alpha\left(\frac{A\overline{X}+B}{A\overline{x}+B}\right)^g + (1-\alpha)\left(\frac{A\overline{x}+B}{A\overline{X}+B}\right)^h\right]^\delta$$

$$t_G^* = \overline{y}\left[\alpha e^{g\frac{A(\overline{X}-\overline{x})}{A\overline{X}+B}} + (1-\alpha)e^{h\frac{A(\overline{x}-\overline{X})}{A\overline{X}+B}}\right]^\delta$$

where α, g, h, δ, A and B are real constants to be suitably chosen.

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