

PERFORMANCE ANALYSIS OF A REDUNDANT SYSTEM WITH WEIBULL FAILURE AND REPAIR LAWS

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ABSTRACT

The aim of the present paper is to analyse the performance measures of a two non-identical unit redundant system using semi-Markovian approach. The use of non-identical unit's redundant systems is preferred in industries but due to lack of proper maintenance and repair they results in decline of productivity. Hence, to enhance the reliability and productivity, beneficial initiative must be taken. For this purpose, a reliability model of a redundant system having one original and one duplicate unit is developed with an immediate repair facility. Repairman conducts the preventive maintenance of the unit after a pre-specific time to enhance the performance and efficiency of the system. All random variables follow Weibull distribution. Mean time to system failure, availability and profit function has been derived for the considered non-identical redundant system. To highlight the importance of the study graphs are drawn for these measures.

KEYWORDS: Performance Analysis, Two Non-identical Unit Redundant System, Weibull Failure and Repair Laws, Preventive Maintenance and Maximum Operation Time.

MSC: 90B22, 60K25

RESUMEN

Este paper tiene por objeto analizar medidas del comportamiento para un sistema redundante de dos sistemas de unidades no idénticas redundantes usando un enfoque semi-Markoviano. El uso de un sistema redundante de unidades no idénticas es preferido en las industrias pero debido a la falta de mantenimiento y reparación propia resulta en una declinación de la productividad. Entonces, para mejorar la fiabilidad y la productividad, una iniciativa beneficiosa tiene que ser tomada. Con este propósito, un modelo de fiabilidad de un sistema redundante que tiene una unidad original y un duplicado es desarrollado con una facilidad de reparación inmediata. El reparador lleva a cabo el mantenimiento preventivo de la unidad después de un tiempo pre-especificado para permitir la mejoría del comportamiento de la eficiencia del sistema. Todas las variables siguen una distribución de Weibull. El tiempo promedio de la falla del sistema, la función de disponibilidad y ganancia han sido derivadas para el sistema redundante considerado. Para destacar la importancia del estudio gráficos son establecidos para estas medidas.

1. INTRODUCTION

With the advancement of modern technology the configuration of industrial systems becomes more and more complex. The complexity of system reduces the quality and productivity of the system. To overcome this problem, system designers use cold standby redundancy as an effective technique for reliability enhancement of complex systems. The probability of failure of cold standby unit is zero. Air crafts, textile manufacturing systems, carbon recovery systems in fertilizer plants and satellite systems get high reliability using cold standby redundant systems. Gopalan and Nagarwalla (1985) analysed a standby system with one repairman and preventive maintenance. Goel and Sharma (1989) studied the effect of slow switch and two modes of failure on standby system. Repairable standby system with replaceable repair facility was analysed by Cao and Wu (1989). Gopalan and Bhanu (1995) used concept of on-line preventive maintenance and repair for a cold standby system. Chandrasekhar et al. (2004) carried out a study on two-unit cold standby system with Erlangian repair time. Zhang and Wang (2009) suggested a geometric model for a repairable cold standby system with priority in use and repair. Mahmoud and Moshref (2010) analyzed effect of preventive maintenance, hardware and human failure on cold standby systems. Moghaddass et al. (2011) analysed the reliability and availability of repairable system with repairman subject to shut-off rules. Wu and Wu (2011) developed a reliability model

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with two cold standby units, one repairman and a switch under poisson shocks. Kumar and Malik (2012) developed many stochastic models for a computer system using concept of preventive maintenance after maximum operation time and independent h/w and s/w failure.

Wang et al. (2006) carried out the cost benefit analysis of series systems with cold standby components. Ke et al. (2011) analysed a two-unit redundant system with detection delay and imperfect coverage. Ke et al. (2011) obtained the reliability measures of a repairable system with standby switching failures and reboot delay. Wang and Fang (2012) carried out the comparison of availability between two systems with warm standby units and different imperfect coverage.

All the studies referred above, discussed reliability models of cold standby systems having identical units under different set of assumptions. But, many times due to economic reasons, it is not always possible to keep an identical unit in standby. However, a duplicate unit can be kept in standby to improve the reliability and availability of the system. In most of the studies, all the researchers made the assumption that all the random variables related to failure time of the unit distributed exponentially and repair times are either arbitrary or constant distributed. But, the performance of most of the mechanical, industrial and electrical systems varies with respect to passes of time. So, their repair and failure is not necessarily constant distributed but may behave as any arbitrary distribution. There are many distributions such as Weibull, normal, and lognormal distributions that are useful in analysing failure processes of standby systems. These distributions have hazard rate functions that are not constant over time, thus providing a necessary alternative to the exponential failure law.

The most important probability distribution in reliability modelling is the Weibull distribution. The Weibull failure distribution may be used to model both increasing and decreasing failure rates. Suppose random variable V denotes the time to maximum operation time of an item/ device having Weibull distribution, and then its probability density function is denoted by $f_1(t) = \theta\eta t^{\eta-1} \exp(-\theta t^\eta)$ $t \geq 0$ and $\theta, \eta > 0$. It is

characterized by a hazard rate function of the form $h(t) = \theta\eta t^{\eta-1}$, $t \geq 0$ and $\theta, \eta > 0$ which is a power function. The function $\lambda(t)$ is increasing for $\eta > 0$, $\theta > 0$ and is decreasing for $\eta < 0$, $\theta < 0$. The reliability

function is given by $R(t) = \exp(-\theta t^\eta)$. Thus the failure free operating time of the system has a Weibull distribution with parameters θ and η . Here η is referred to as the shape parameter and θ is the scale parameter. Its effect on the distribution can be seen for several different values. For $\eta < 1$, the probability density function is similar in the shape to the exponential, and for large value of η ($\eta \geq 3$), the probability density function is some-what symmetrical, like the normal distribution. For $1 < \eta < 3$, the density is skewed. If we put $\eta = 1$ in pdf, Weibull distribution reduces to Exponential distribution and if $\eta = 2$, it reduces to Rayleigh distribution.

Kumar and Saini (2014) analysed cost-benefit of a single-unit system under preventive maintenance and Weibull distribution for random variables. Gupta et al. (2013) discussed a two dissimilar unit cold standby system model by taking Weibull failure and repair distribution. But, no work related the reliability and performance of two non-identical units has not been found in literature.

In the present paper, we develop a reliability model for a non-identical cold standby system for the evaluation of system reliability, mean time to system failure, steady state availability, busy period of server, expected number of repairs, expected number of visits by server and profit function of the system by considering all time random variables as Weibull distributed. The possible states of the proposed problem have been discussed under heading system description. A single repair facility has been provided to do repair and maintenance activities of original and duplicate unit. After a pre-specific time unit undergoes for preventive maintenance. All random variables are statistically independent. Switch devices and repairs are perfect. Semi-Markov process and regenerative point techniques are used to draw recurrence relations for various reliability characteristics. All time random variables are Weibull distributed. The probability density function of maximum operation time of

original and duplicate unit is denoted by $g(t) = \alpha\eta t^{\eta-1} \exp(-\alpha t^\eta)$. The pdf of failure times of the original and duplicate unit are denoted by $f(t) = \beta\eta t^{\eta-1} \exp(-\beta t^\eta)$ and $f_2(t) = h\eta t^{\eta-1} \exp(-h t^\eta)$ respectively. The preventive maintenance rate of the original and duplicate units is denoted by the probability density function $g_1(t) = \gamma\eta t^{\eta-1} \exp(-\gamma t^\eta)$. The random variables corresponding to repair rate of the original and duplicate units have the probability density function $f_1(t) = k\eta t^{\eta-1} \exp(-k t^\eta)$ and

$f_3(t) = l\eta t^{\eta-1} \exp(-lt^\eta)$ respectively with $t \geq 0$ and $\theta, \eta, \alpha, \beta, h, k, l > 0$. The probability /cumulative density functions of direct transition time from regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state S_k, S_r once in $(0, t]$ have been denoted by $q_{ij,kr}(t)/Q_{ij,kr}(t)$. To improve the importance of the study, graphical and numerical results are drawn for a particular case for mean time to system failure, availability and profit function.

Nomenclature	
O	Operative unit
DCs	Duplicate cold standby unit
Do	Duplicative unit is operative
~ / *	Symbol for Laplace -Steiltjes Transform (LST) / Laplace Transfor(LT)
⊗/⊙	Symbol for Laplace-Stieltjes convolution/Laplace convolution
Fur/FUR	Denotes the failed original unit under repair/continuously under repair
DFur/DFUR	Denotes the failed duplicate unit under repair/continuously under repair
DPm/DPM	Denotes that duplicate unit under preventive maintenance/ continuously under preventive maintenance
Pm/PM	Denotes that original unit under preventive maintenance/ continuously under preventive maintenance
WPm/WPM	Denotes that original unit waiting for preventive maintenance/ continuously waiting for preventive maintenance
DWPm/DWPM	Denotes that duplicate unit waiting for preventive maintenance/ continuously waiting for preventive maintenance
Fwr/FWR	Original unit after failure waiting for repair/continuously waiting for repair
DFwr / DFWR	Duplicate unit after failure waiting for repair/continuously waiting for repair
MTSF	Mean Time to System Failure

2. MODEL DESCRIPTION

In this section, a stochastic model has been developed for two non-identical unit's systems. The system may be any of the following states describes as follows:

State 0: Original unit is operative, duplicate unit in cold standby and system is in upstate. The service facility at S_0 remain idle.

State 1: Original unit is under preventive maintenance after completion of maximum operation time, duplicate unit is operative and system is in upstate. The service facility at S_1 is busy in preventive maintenance of the original unit.

State 2: Original unit is under repair after failure, duplicate unit is operative and system is in upstate. The service facility at S_2 is busy in repair activity of the failed original unit.

State 3: Original unit is operative, duplicate unit is repair after failure and system is in upstate. The service facility at S_3 is busy in repair activity of the failed duplicate unit.

State 4: Original unit is operative, duplicate unit is under preventive maintenance after completion of maximum operation time and system is in upstate. The service facility at S_4 is busy in preventive maintenance of the duplicate unit.

State 5: Original unit is failed and waiting for repair, duplicate unit is continuously under preventive maintenance after completion of maximum operation time from previous state and system is in downstate. The service facility at S_5 is busy in preventive maintenance of the duplicate unit.

State 6: Original unit is failed and continuously under repair from past state, duplicate unit is failed and waiting for repair and system is in downstate. The service facility at S_6 is busy in repair of the original unit.

State 7: Original unit is failed and continuously under repair from past state, duplicate unit is under waiting for preventive maintenance and system is in downstate. The service facility at S_7 is busy in repair of the original unit.

State 8: Original unit is waiting for preventive maintenance after completion of maximum operation time, duplicate unit is under preventive maintenance continuously after completion of maximum operation time from previous state and system is in downstate. The service facility at S_8 is busy in preventive maintenance of the duplicate unit.

State 9: Original unit is continuously under preventive maintenance after completion of maximum operation time from previous state, duplicate unit is waiting for preventive maintenance after completion of maximum operation time and system is in downstate. The service facility at S_9 is busy in preventive maintenance of the original unit.

State 10: Original unit is continuously under preventive maintenance after completion of maximum operation time from previous state, duplicate failed unit is waiting for repair and system is in downstate. The service facility at S_{10} is busy in preventive maintenance of the original unit.

State 11: Original unit is waiting for repair, duplicate failed unit is continuously under repair from previous state and system is in downstate. The service facility at S_{11} is busy in repair of the failed duplicate unit.

State 12: Original unit is waiting for preventive maintenance after completion of maximum operation time, duplicate failed unit is continuously under repair from previous state and system is in downstate. The service facility at S_{12} is busy in repair of the failed duplicate unit.

Out of these, states $S_0 S_1 S_2 S_3$ and S_4 are the operative and regenerative states while all other are non-regenerative and failed states.

3. RELIABILITY INDICIES

3.1 Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)dt \quad \text{as} \quad (1)$$

$$\begin{aligned} p_{01} &= \frac{\alpha}{\alpha + \beta}, p_{02} = \frac{\beta}{\alpha + \beta}, p_{10} = \frac{\gamma}{\alpha + h + \gamma}, p_{1,10} = \frac{h}{\alpha + \gamma + h} = p_{13,10}, p_{19} = \frac{\alpha}{\alpha + \gamma + h} = p_{14,9}, \\ p_{20} &= \frac{k}{\alpha + k + h}, p_{26} = \frac{h}{\alpha + h + k} = p_{23,6}, p_{27} = \frac{\alpha}{\alpha + h + k} = p_{24,7}, p_{30} = \frac{l}{l + \alpha + \beta}, \\ p_{3,12} &= \frac{\alpha}{\alpha + \beta + l} = p_{31,12}, p_{3,11} = \frac{\beta}{\alpha + \beta + l} = p_{32,11}, p_{40} = \frac{\gamma}{\alpha + \beta + \gamma}, p_{45} = \frac{\beta}{\alpha + \beta + \gamma} = p_{42,5}, p_{48} = \\ &\frac{\alpha}{\alpha + \beta + \gamma} = p_{41,8}, p_{52} = p_{63} = p_{74} = p_{81} = p_{94} = p_{10,3} = p_{11,2} = p_{12,1} = 1 \end{aligned} \quad (2)$$

It can be easily verified that $p_{01} + p_{02} = p_{10} + p_{19} + p_{1,10} = p_{10} + p_{14,9} + p_{10} = p_{20} + p_{27} + p_{26} = p_{20} + p_{23,6} + p_{24,7} = p_{30} + p_{3,12} + p_{3,11} = p_{30} + p_{31,12} + p_{32,11} = p_{40} + p_{45} + p_{48} = p_{40} + p_{41,8} + p_{42,5} = p_{52} = p_{63} = p_{74} = p_{81} = p_{94} = p_{10,3} = p_{11,2} = p_{12,1} = 1$ (3)

The mean sojourn times (μ_i) is the state S_i are

$$\mu_0 = \frac{\Gamma(1+1/\eta)}{(\alpha + \beta)^{1/\eta}}, \mu'_1 = \Gamma(1/\eta + 1) \left[\frac{1}{(\alpha + h + \gamma)^{1/\eta}} + \frac{1}{(\alpha + \gamma + h)(\gamma)^{1/\eta}} (\alpha + h) \right], \mu_3 = \frac{\Gamma(1+1/\eta)}{(\alpha + \beta + l)^{1/\eta}},$$

$$\begin{aligned}
\mu_2' &= \Gamma(1/\eta + 1) \left[\frac{1}{(\alpha + h + k)^{\frac{1}{\eta}}} + \frac{1}{(\alpha + k + h)(k)^{\frac{1}{\eta}}} (\alpha + h) \right], & \mu_2 &= \frac{\Gamma(1 + 1/\eta)}{(\alpha + k + h)^{1/\eta}} \\
\mu_3' &= \Gamma(1/\eta + 1) \left[\frac{1}{(\alpha + \beta + l)^{\frac{1}{\eta}}} + \frac{1}{(\alpha + \beta + l)(l)^{\frac{1}{\eta}}} (\alpha + \beta) \right], & \mu_4 &= \frac{\Gamma(1 + 1/\eta)}{(\alpha + \beta + \gamma)^{1/\eta}}, \\
\mu_4' &= \Gamma(1/\eta + 1) \left[\frac{1}{(\alpha + \beta + \gamma)^{\frac{1}{\eta}}} + \frac{1}{(\alpha + \gamma + \beta)(\gamma)^{\frac{1}{\eta}}} (\alpha + \beta) \right] & \mu_1 &= \frac{\Gamma(1 + 1/\eta)}{(\alpha + \gamma + h)^{1/\eta}}
\end{aligned} \tag{4}$$

3.2 Reliability and Mean Time to System Failure (MTSF)

By probabilistic arguments, the recurrence relations for mean time to system failure are obtained by considering $\varphi_i(t)$ the cumulative density function of first passage time from the regenerative state S_i to a failed state and all failed states are observed as absorbing state. The recurrence relations for $\varphi_i(t)$ are as follows:

$$\varphi_i(t) = \sum_j Q_{i,j}(t) \otimes \varphi_j(t) + \sum_k Q_{i,k}(t) \tag{5}$$

Taking LST of above relation (5) and solving for $\tilde{\varphi}_0(s)$. We have

$$R^*(s) = \frac{1 - \tilde{\varphi}_0(s)}{s} \tag{6}$$

The system's reliability can be obtained by taking Laplace inverse transform of (6). The MTSF of the system is given by

$$\text{MTSF} = \lim_{s \rightarrow 0} \frac{1 - \tilde{\varphi}_0(s)}{s} = \frac{N}{D} \tag{7}$$

where $N = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2$ and $D = 1 - p_{01}p_{10} - p_{02}p_{20}$

3.3 Steady State Availability

Let $A_i(t)$ denotes the probability of up-state of a system at time point 't' entered regenerative state S_i at $t = 0$. The recursive relations are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) \odot A_j(t) \tag{8}$$

where $S_i, S_j \in E$ and can transit through n transitions. The upstate probability of system to stay

at a particular state is denoted by $M_i(t)$. Here $M_0(t) = e^{-(\alpha + \beta)t^\eta}$,

$$M_1(t) = e^{-(\alpha + h + \gamma)t^\eta}, M_2(t) = e^{-(\alpha + k + h)t^\eta}, M_3(t) = e^{-(\alpha + \beta + l)t^\eta}, M_4(t) = e^{-(\alpha + \beta + \gamma)t^\eta} \tag{9}$$

Taking Laplace transformation of above equation (8) and solving for $A_0^*(s)$. The steady state availability is given by

$$A_0(\infty) = \lim_{s \rightarrow 0} s A_0^*(s) = \frac{N_2}{D_2} \tag{10}$$

where

$$\begin{aligned}
N_2 = & (M_0(t)((1 - p_{41.8}p_{14.9} - p_{31.12}p_{13.10})(1 - p_{42.5}p_{24.7} - p_{32.11}p_{23.6}) - (p_{42.5}p_{14.9} + p_{32.11}p_{13.10}) \\
& (p_{41.8}p_{24.7} + p_{31.12}p_{23.6})) + ((M_1(t) + M_3(t)p_{13.10} + M_4(t)p_{14.9})(p_{01}(1 - p_{42.5}p_{24.7} - p_{32.11}p_{23.6}) \\
& + p_{02}(p_{41.8}p_{24.7} + p_{31.12}p_{23.6}))) + ((M_2(t) + M_3(t)p_{23.6} + M_4(t)p_{24.7})(p_{01}(p_{42.5}p_{14.9} + p_{32.11}p_{13.10}) + \\
& p_{02}(1 - p_{41.8}p_{14.9} - p_{31.12}p_{13.10})))
\end{aligned}$$

$$\begin{aligned}
D_2 = & \mu_0((1 - p_{41.8}p_{14.9} - p_{31.12}p_{13.10})(1 - p_{42.5}p_{24.7} - p_{32.11}p_{23.6}) - (p_{42.5}p_{14.9} + p_{32.11}p_{13.10}) \\
& (p_{41.8}p_{24.7} + p_{31.12}p_{23.6})) + (\mu_1 + \mu_3 p_{13.10} + \mu_4 p_{14.9})(p_{01}(1 - p_{42.5}p_{24.7} - p_{32.11}p_{23.6}) + \\
& p_{02}(p_{41.8}p_{24.7} + p_{31.12}p_{23.6})) + (\mu_2 + \mu_3 p_{23.6} + \mu_4 p_{24.7})(p_{01}(p_{42.5}p_{14.9} + p_{32.11}p_{13.10}) + \\
& p_{02}(1 - p_{41.8}p_{14.9} - p_{31.12}p_{13.10}))
\end{aligned}$$

3.4 Busy Period Analysis for Server

Let $B_i^R(t)$, $B_i^{Pm}(t)$ be the probability that the server is busy in repairing and preventive maintenance of the unit at an instant 't' given that the system entered state i at t = 0. The recursive relations for $B_i^R(t)$ are as follows:

$$B_i^R(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) \odot B_j^R(t) \quad (11)$$

$$B_i^{Pm}(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) \odot B_j^{Pm}(t)$$

where $S_i, S_j \in E$ and can transit through n transitions. The upstate probability of server's

business at a particular state is denoted by $W_i(t)$. Here $W_2(t) = e^{-(\alpha+k+h)t^n}$, $W_3(t) = e^{-(\alpha+\beta+l)t^n}$,

$W_1(t) = e^{-(\alpha+k+\gamma)t^n}$, $W_4(t) = e^{-(\alpha+\beta+\gamma)t^n}$. By taking Laplace transformation of (11) and solving for

$$B_0^{*R}(s). \text{ The busy period of the server due to repair is given by } B_0^R = \lim_{s \rightarrow 0} sB_0^{*R}(s) = \frac{N_3^R}{D_2},$$

$$B_0^{Pm} = \lim_{s \rightarrow 0} sB_0^{*Pm}(s) = \frac{N_4^{Pm}}{D_2}$$

where $N_3^R = (W_3^*(0)p_{13.10})(p_{01}(1 - p_{42.5}p_{24.7} - p_{32.11}p_{23.6}) + p_{02}(p_{41.8}p_{24.7} + p_{31.12}p_{23.6})) +$
 $(W_2^*(0) + W_3^*(0)p_{23.6})(p_{01}(p_{42.5}p_{14.9} + p_{32.11}p_{13.10}) + p_{02}(1 - p_{41.8}p_{14.9} - p_{31.12}p_{13.10}))$

$N_4^{Pm} = (W_1^*(0) + W_4^*(0))(p_{01}(1 - p_{42.5}p_{24.7} - p_{32.11}p_{23.6}) + p_{02}(p_{41.8}p_{24.7} + p_{31.12}p_{23.6})) +$
 $(W_4^*(0)p_{24.7})(p_{01}(p_{42.5}p_{14.9} + p_{32.11}p_{13.10}) + p_{02}(1 - p_{41.8}p_{14.9} - p_{31.12}p_{13.10}))$

and D_2 is already mentioned in previous section.

3.5 Expected Number of Repairs, Replacements & Visits by Server

Let $E_i^R(t)$, $E_i^{Pm}(t)$ & $N_i(t)$ be the expected number of repairs, preventive maintenances and visits by server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $E_i^R(t)$ are given as

$$E_i^R(t) = \sum_j Q_{i,j}^{(n)}(t) \otimes [\delta_j + E_j^R(t)] \quad (12)$$

$$E_i^{Pm}(t) = \sum_j Q_{i,j}^{(n)}(t) \otimes [\delta_j + E_j^{Pm}(t)]$$

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t) \otimes [\delta_j + N_j(t)]$$

Where S_j is any regenerative state to which the given regenerative state i transits and $\delta_j=1$, if j is the regenerative state where the server does job afresh, otherwise $\delta_j = 0$. Taking Laplace Stieltjes transformation of relation (12) and solving for $E_0^{**R}(s)$. The expected numbers of repairs per unit time are given by

$$E_0^R(\infty) = \lim_{s \rightarrow 0} sE_0^{**R}(s) = \frac{N_5^R}{D_2}, \quad E_0^{Pm}(\infty) = \lim_{s \rightarrow 0} sE_0^{**Pm}(s) = \frac{N_6^{Pm}}{D_2}, \quad N_0(\infty) = \lim_{s \rightarrow 0} s\tilde{N}_0(s) = \frac{N_7}{D_2}$$

where

$$N_5^R = (p_{32.11} + p_{31.12} + p_{30})p_{13.10}(p_{01}(1 - p_{42.5}p_{24.7} - p_{32.11}p_{23.6}) + p_{02}(p_{41.8}p_{24.7} + p_{31.12}p_{23.6})) + (p_{20} + p_{23.6} + p_{24.7} + (p_{32.11} + p_{31.12} + p_{30})p_{23.6})(p_{01}(p_{42.5}p_{14.9} + p_{32.11}p_{13.10}) + p_{02}(1 - p_{41.8}p_{14.9} - p_{31.12}p_{13.10}))$$

$$N_6^{Pm} = ((p_{10} + p_{14.9} + p_{13.10}) + (p_{40} + p_{42.5} + p_{41.8})p_{14.9})(p_{01}(1 - p_{42.5}p_{24.7} - p_{32.11}p_{23.6}) + p_{02}(p_{41.8}p_{24.7} + p_{31.12}p_{23.6})) + ((p_{40} + p_{42.5} + p_{41.8})p_{24.7})(p_{01}(p_{42.5}p_{14.9} + p_{32.11}p_{13.10}) + p_{02}(1 - p_{41.8}p_{14.9} - p_{31.12}p_{13.10}))$$

$$N_7 = (p_{01} + p_{02})(((1 - p_{41.8}p_{14.9} - p_{31.12}p_{13.10})(1 - p_{42.5}p_{24.7} - p_{32.11}p_{23.6})) - ((p_{42.5}p_{14.9} + p_{32.11}p_{13.10})(p_{41.8}p_{24.7} + p_{31.12}p_{23.6})))$$

and D_2 is already mentioned in previous section.

4. PROFIT ANALYSIS

The net profit in steady state incurred by the system model can be obtained as follows:

$$P = K_0A_0 - K_1B_0^{Pm} - K_2B_0^R - K_3E_0^{Pm} - K_4E_0^R - K_5N_0 \quad (13)$$

K_0 = Revenue generated by system per unit up-time

K_i = Expenditure per unit time on different repair activities

5. CASE STUDIES WITH DISCUSSIONS

(i) When shape parameter $\eta = 0.5$ then maximum operation /failure of original unit/failure of duplicate unit/preventive maintenance/ repair of original/repair of duplicate unit time distributions reduce to:

$$g(t) = \frac{\alpha}{2\sqrt{t}} e^{-\alpha\sqrt{t}}, \quad f(t) = \frac{\beta}{2\sqrt{t}} e^{-\beta\sqrt{t}}, \quad g_1(t) = \frac{\gamma_1}{2\sqrt{t}} e^{-\gamma_1\sqrt{t}}, \quad f_1(t) = \frac{k}{2\sqrt{t}} e^{-k\sqrt{t}} \quad (14)$$

$$f_2(t) = \frac{h}{2\sqrt{t}} e^{-h\sqrt{t}}, \quad f_3(t) = \frac{l}{2\sqrt{t}} e^{-l\sqrt{t}}; \quad \text{where } t \geq 0 \text{ and } \eta, \alpha, \beta, \gamma_1, h, k, l > 0$$

(ii) When shape parameter $\eta = 1.0$ then failure/preventive maintenance/arrival time of the server/replacement/transition rate/ repair time distributions reduce to exponentials, then

$$g(t) = \alpha e^{-\alpha t}, \quad f(t) = \beta e^{-\beta t}, \quad g_1(t) = \gamma_1 e^{-\gamma_1 t}, \quad f_1(t) = k e^{-kt} \quad (15)$$

$$f_2(t) = h e^{-ht}, \quad f_3(t) = l e^{-lt}; \quad \text{where } t \geq 0 \text{ and } \eta, \alpha, \beta, \gamma_1, h, k, l > 0$$

(iii) When shape parameter $\eta = 2.0$ then failure/preventive maintenance/arrival time of the server/replacement/transition rate/ repair time distributions reduce to Rayleigh having the pdf-

$$g(t) = 2\alpha e^{-\alpha t^2}, f(t) = 2\beta e^{-\beta t^2}, g_1(t) = 2\gamma_1 e^{-\gamma_1 t^2}, f_1(t) = 2ke^{-kt^2}$$

$$f_2(t) = 2he^{-ht^2}, f_3(t) = 2le^{-lt^2}; \text{ where } t \geq 0 \text{ and } \eta, \alpha, \beta, \gamma_1, h, k, l > 0$$
(16)

6. NUMERICAL RESULTS

Table 1: MTSF vs. Failure Rate (β) for various values of shape parameter

B	$\alpha=2, \eta=0.5,$ $\gamma=5, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=0.5,$ $\gamma=7, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=0.5$ $\gamma=5, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=0.5,$ $\gamma=7, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=0.5,$ $\gamma=5, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=0.5$ $\gamma=7, k=1.5$ $h=0.009$ $l=1.4$
0.01	4.9918	11.7910	5.9647	13.8080	8.9395	20.8745
0.02	4.7950	10.7622	5.7599	12.6725	8.6486	19.1946
0.03	4.6123	9.8940	5.5696	11.7138	8.3781	17.7760
0.04	4.4423	9.1515	5.3921	10.8934	8.1260	16.5621
0.05	4.2836	8.5094	5.2264	10.1835	7.8904	15.5116
0.06	4.1351	7.9487	5.0712	9.5632	7.6698	14.5936
0.07	3.9960	7.4548	4.9255	9.0165	7.4627	13.7844
0.08	3.8654	7.0165	4.7886	8.5310	7.2680	13.0659
0.09	3.7426	6.6251	4.6596	8.0970	7.0846	12.4235
0.10	3.6268	6.2733	4.5379	7.7067	6.9116	11.8458

Table 2: Availability vs. Failure Rate (β) for various values of shape parameter

β	$\alpha=2, \eta=0.5,$ $\gamma=5, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=0.5,$ $\gamma=7, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=1,$ $\gamma=5, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=1,$ $\gamma=7, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=2,$ $\gamma=5, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=2,$ $\gamma=7, k=1.5,$ $h=0.009,$ $l=1.4$
0.01	0.9394	0.9724	0.8941	0.9367	0.8715	0.9109
0.02	0.9347	0.9674	0.8916	0.934	0.8701	0.9092
0.03	0.9299	0.9625	0.8892	0.9312	0.8687	0.9076
0.04	0.9252	0.9576	0.8869	0.9285	0.8673	0.906
0.05	0.9205	0.9527	0.8845	0.9259	0.8659	0.9044
0.06	0.9158	0.9478	0.8821	0.9232	0.8645	0.9028
0.07	0.9112	0.943	0.8798	0.9206	0.8632	0.9012
0.08	0.9065	0.9382	0.8775	0.918	0.8619	0.8996
0.09	0.9019	0.9333	0.8752	0.9154	0.8605	0.8981
0.1	0.8973	0.9286	0.873	0.9129	0.8592	0.8966

Table 3: Profit vs. Failure Rate (β) for various values of shape parameter

β	$\alpha=2, \eta=0.5,$ $\gamma=5, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=0.5,$ $\gamma=7, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=1,$ $\gamma=5, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=1,$ $\gamma=7, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=2,$ $\gamma=5, k=1.5,$ $h=0.009,$ $l=1.4$	$\alpha=2, \eta=2,$ $\gamma=7, k=1.5,$ $h=0.009,$ $l=1.4$
0.01	4357.7	4522.4	4178.2	4384.1	4134.2	4329
0.02	4331.4	4494.8	4165.1	4369.2	4126.6	4320.1
0.03	4305.1	4467.4	4152	4354.4	4119.1	4311.3

0.04	4279	4440.1	4139.1	4339.8	4111.6	4302.7
0.05	4252.9	4413	4126.3	4325.3	4104.3	4294.1
0.06	4227.1	4385.9	4113.6	4311	4097	4285.6
0.07	4201.3	4359	4101	4296.8	4089.8	4277.2
0.08	4175.7	4332.2	4088.5	4282.7	4082.6	4268.9
0.09	4150.2	4305.6	4076.2	4268.8	4075	4260.7
0.1	4124.9	4279	4064	4255	4068.5	4252.6

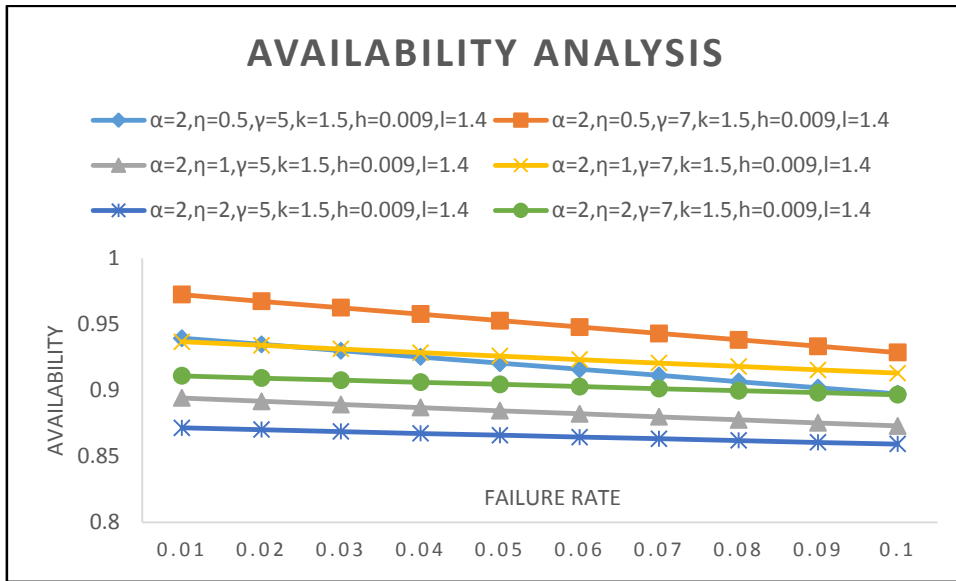


Figure. 1: Steady State Availability vs. Failure Rate

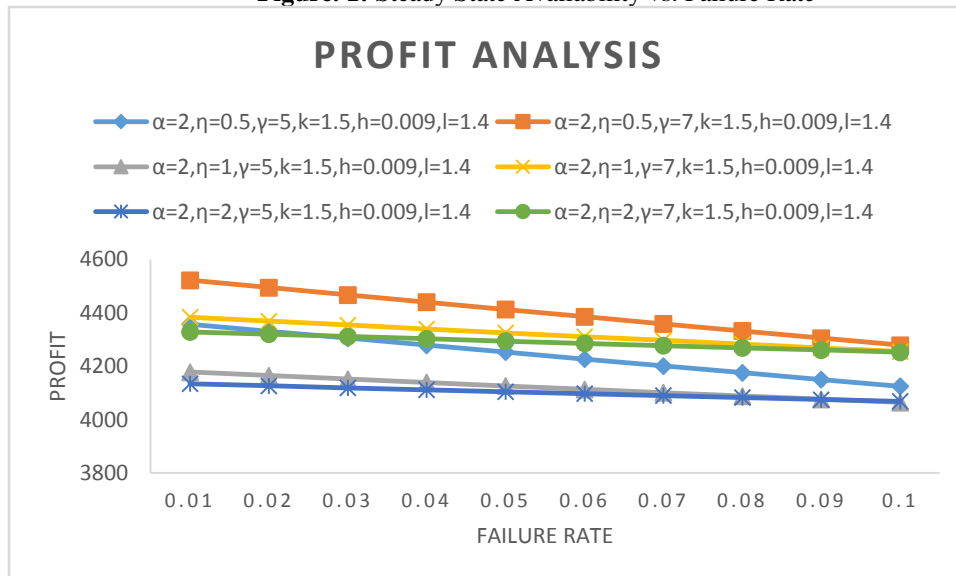


Figure. 2: Profit vs. Failure Rate

7. CONCLUSION

For a particular case, having values $\alpha = 2, \eta = .5, \gamma = 5, h = .009, k = 1.5, l = 1.4$ behaviour of various reliability measures such as MTSF, availability and net expected steady state profit of the system discussed here for a two-unit cold standby system under Weibull failure and repair laws. The values of K_i for profit function are assumed as $K_0 = 5000, K_1 = 500, K_2 = 400, K_3 = 350, K_4 = 300, K_5 = 325$. From the numerical results depicted above in Table's 1-3 shows that the MTSF, availability and profit of the system declines with the increase of failure rate (β) and shape parameter (η) while values of these parameters increase with increment of the repair rate and preventive maintenance rate (γ). Finally, we conclude that by increasing the repair rate of the original and duplicate unit system can be made more profitable.

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