JOINT COST MINIMIZATION APPROACH FOR THREE ECHELON SUPPLY CHAIN SYSTEM WITH MULTIPLE BUYERS UNDER INFLATION

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ABSTRACT
In this problem, we have discussed three echelon supply chain problem for inventory. Here we have been taken single supplier, single manufacturer and multiple buyers. Whole discussion has been done under inflation and time discounting over an infinite planning horizon by allowing partial backlogging for retailers. It is assumed that the units in inventory deteriorate over the time and follow a two-parameter weibull distribution and deteriorated units are not replaced. Results show that the total cost function is convex. With the convexity, a simple solution algorithm is presented to determine the optimal order quantity and optimal cycle time of the total cost function. The results are discussed with a numerical examples and particular cases of the model discussed in brief. A sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out.

KEYWORDS: Three echelon supply chain, weibull deterioration, inflation, partial backlogging, time discounting

MSC: 90B05

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1. INTRODUCTION

The terms multi-echelon and multi-level production/distribution network are also synonymous with such networks (or supply chains), when an item moves through more than one step before reaching the final customer. The area of supply chain management (SCM) has gained a lot of interest from researchers as well as practitioners in the industry. Essentially, the retailer (buyer) observes a deterministic demand and orders lots from the manufacturer

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(vendor). The vendor satisfies this downstream demand through manufacturing the requested product in lots, where each produced lot is shipped to the buyer in batches.

For a vertically integrated supply chain owned partially or jointly by the same company, such coordinated production– shipment policy provides valuable insights and optimal decisions that lead to global optimization. On the other hand, when individual entities are owned separately, such policy may not beneficial for all the parties equally as some may encounter an increase in their costs and hence become less eager to depart from their locally optimized policies. In such situations, sharing those benefits resulting from the coordinated approach becomes a major issue. Most of the work related to joint economic lot size production (JELP) has been conducted in the context of a two layer supply chain consisting of a single vendor and a single buyer. Goyal (1977) suggested a lot-for-lot policy with the assumption of infinite production rate for the manufacturer. Investigations reporting coordination in a three-level supply chain are few and far in the literature. Coordinating orders in a two-level (vendor–buyer) supply chain has been addressed in Hill (1997). Goyal and Gunasekaran (1995) observed an integrated production inventory marketing model to determine economic production quantity and economic order quantity for raw materials in a multi-echelon production system. Using an interval search approach, Goyal (2000) has developed a solution procedure for optimal production quantity in a single vendor single buyer production inventory system with unequal and equal sized shipments from the vendor to the buyer under capacity constraint of the transport equipment. Khouja (2003) was the first to consider a three stage supply chain with one or more firms at each stage. He discussed three level inventory coordination mechanisms among the members of the supply Chain. Lee (2005) added a new dimension to the single vendor single buyer problem by setting the number of raw material shipments received by the vendor per cycle to be a decision variable. Thus, the raw material ordering cost was considered explicitly in the model. Jaber and Goyal (2008) discussed coordination in a three-level supply chain with multiple suppliers, a vendor and multiple buyers. Sana (2011) proposed a three layer supply chain involving a supplier of raw materials, a manufacturer and a retailer for deterministic demand. Singh et al. (2012) studied a three echelon supply chain inventory model for deteriorating items with storage facility and lead time under inflation.

One of the assumptions in most of the inventory model has been a negligible level of inflation. But in recent times many countries have been confronted with fluctuating inflation rates that often have been far from negligence. The pioneer in this field was Buzacott (1975), who developed the first EOQ model taking inflation into account. This has become interesting factor for several researchers like Mishra (1979) proposed note on optimal inventory management under inflation. Bierman and Thomas (1977) considered an inventory decision under inflationary conditions. Ray and Chaudhuri (1997) provided an EOQ model with inflation time discounting. Jain et al. (2011) discussed an inflation implication on an inventory with expiration date, capital constraint and uncertain lead time in a multi-echelon supply chain. Yadav et al. (2013) developed a retailer's optimal policy under inflation in fuzzy environment with trade credit.

The control and maintenance of inventories for deteriorating items with shortages have received much attention of several researchers in the recent years because most of the physical goods deteriorate over time. In reality, some of the items are either damaged or decayed or affected by some other factors. Ghare and Schrader (1963) developed a model for an exponentially decaying inventory. An order level inventory model for items deteriorating at a constant rate was proposed by Dave and Patel (1981). Some of the significant recent work in this field has been done by Chung and Ting (1993), etc. Recent efforts of the deteriorating inventory research have been focused on considering the partial backlogging of the unsatisfied demand. The motivation is some reality issues since the case of complete backlogging is more likely only in a monopolistic market. In a non-monopolistic market, customers encountering shortages will respond differently. Some customers are willing to wait until the next replenishment, while others may be impatient and go elsewhere as waiting time increases. Therefore, partial backlogging is a necessary consideration for inventory management. Deb and Chaudhuri (1986) were the first to incorporate shortages into the inventory lot sizing problem with a linearly increasing time-varying demand. Abad (2001) derived a pricing and ordering policy for a variable rate of deterioration and partially backlogging. The partial backlogging was assumed to be exponential function of waiting time till the next replenishment. Dye et al. (2006) derived a deteriorating inventory model in which the partial backlogging rate linearly depends on the total number of customers in the waiting line. Chern et al. (2008) considered that the fraction of shortages backordered is a differentiable and decreasing function of time. Using the same partial backlogging ratio function as in Chern et al. (2008), Skouri et al. (2009) studied ramp type demand rate and Weibull distribution deterioration. Singh et al. (2010) developed an EOQ model with Pareto distribution for deterioration in a study. They have discussed Trapezoidal type demand and backlogging under trade credit policy.

In this paper a three echelon supply chain with cooperative behavior is considered which consists of single supplier, single manufacturer and multiple buyers who are involved in procurement, producing and selling only one type of

finished product. Single supplier provides each type of component or raw material to the manufacturer and the manufacturer purchases each type of raw material from only single supplier according to the bill of materials of the finished product. The manufacturer produces finished products and distributes them to its buyers in separate and independent markets. The whole environment of business dealings has been assumed to be inflationary, which conforms to the practical market situation.
The whole combination is unique and very much practical. This setup has been explored numerically as well, an optimal solution has been found and the sensitivity of that solution has been also checked with respect to various system parameters. The final outcome shows that the model is not only economically feasible but stable also. A cost minimization model is derived along with an efficient solution algorithm that is based on the calculus approach.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are considered to develop the model.

2.1. Assumptions

1. This study considered cooperation between single supplier, single producer and multiple buyers.
2. Partial backlogging is allowed for buyers only. The partial backlogging is replenished in the next delivery.
3. Deterioration of the item follows a two-parameter weibull distribution and the deteriorated units are not replaced.
4. Multiple deliveries per order are considered. The planning horizon is infinite and cycles during the planning horizon are continuous. Since one cycle is considered, the items of the first delivery are made in the previous cycle.
5. Demand rate and production rate is deterministic and constant. Production rate is greater than demand rate.
6. Lead time is assumed to be negligible and single item is considered.
7. The cost components are divided into two classes. The costs that increase at the inflation rate occurring within the company are brought under class-I, whereas costs increasing at the inflation rate of the general economy comes under class-II. Two separate inflation rates; the internal (company) inflation rate i_1 and the external (general economy) inflation i_2, these two rates can be estimated by averaging the individual inflation rates of the costs in each class.

2.2. Notations

P Production rate (units/unit time)
N Number of buyers
d_i Demand rate per unit time for buyer i= 1, 2, 3, 4……N
B Fraction of the buyer's demand backordered
r The discount rate, representing the time value of money
i_1 The internal inflation rate, which is varied by company operation status
i_2 The external inflation rate, which is varied by the social economical situation
r_1 $r-i_1$ The discount rate minus the internal inflation rate
r_2 $r-i_2$ The discount rate minus the external inflation rate
g Scale parameter of deterioration rate of raw material
h Shape parameter of deterioration rate of raw material
α Scale parameter of deterioration rate of finished goods
β Shape parameter of deterioration rate of finished goods
u Scale parameter of deterioration rate of finished goods for buyer
v Shape parameter of deterioration rate of finished goods for buyer
Q_w Raw material's order quantity per order
Q_p Producer's finished goods production quantity per production
Q_b Buyer's received quantity per delivery from producer

k The number of delivery per order

T_1 The production period

T_2 The non-production period

T_3 Period that buyers are not out of stock

T_4 Period that buyers are out of stock

T_5 Time period between deliveries, $T_5 = T / k = T_3 + T_4$

T Length of cycle, $T = T_1 + T_2$

$I_w(t)$ Raw material's inventory level at any time t, $0 \le t \le T_1$

$I_{pj}(t)$ Producer's finished goods inventory level at any time $t, 0 \le t \le T_j$, $j = 1, 2$

$I_{bj}^i(t)$ i^{th} Buyer's finished goods inventory level at any time t, $0 \le t \le T_j$, $j = 3, 4$

C_{1w} Supplier's setup cost per order cycle

C_{1p} Producer's setup cost per production cycle

C_{2p} Producer's ordering cost per order cycle

C_{1b}^i i^{th} Buyer's ordering cost per order cycle

C_{mw} Raw material's per unit holding cost per unit time

C_{mp} Producer's finished goods per unit holding cost per unit time

C_{mb}^i i^{th} Buyer's finished goods per unit holding cost per unit time

C_{mb1}^i i^{th} Buyer's finished goods per unit backlog cost per unit time

C_{mb2}^i i^{th} Buyer's finished goods per unit shortage cost for lost sales

C_w Raw material' per unit cost

C_p Producer's finished goods per unit cost

C_b^i i^{th} Buyer's finished goods per unit cost

MI_p Producer's finished goods maximum inventory level

MI_b^i i^{th} Buyer's finished goods maximum inventory level

TUC_w Supplier's present worth total cost per unit time

TUC_p Producer's present worth total cost per unit time

TUC_b Buyer's present worth total cost per unit time

TUC The present worth total cost per unit time

3. MATHEMATICAL FORMULATION

The following scope applies to the study. A supplier procures raw materials from outside supplies and delivers the fixed quantities to the producer warehouse at a fixed time internal. The Producer withdraws raw materials from the warehouse and produces finished goods. The fixed quantities finished goods is delivered to the buyers at a fixed-time interval.

The derivations of cost are provided in this section. There are many exponents in the objective function and it is difficult to formulate an exact solution. To obtain an approximate solution we make the following assumptions. The present worth cost can be computed by applying the Taylors series expansion, for very small $(g, \alpha, \gamma, \mu \le 1)$, the second and higher order terms of g, α, γ and μ are neglected.

This study develops an integrated inventory model for deteriorating item in a multi echelon supply chain under inflation. A mathematical model with integrating single supplier single producer and multiple buyers is derived to obtain the optimal number of deliveries and order lot size, when the joint total cost of the supplier, the producer and the buyers is minimized.

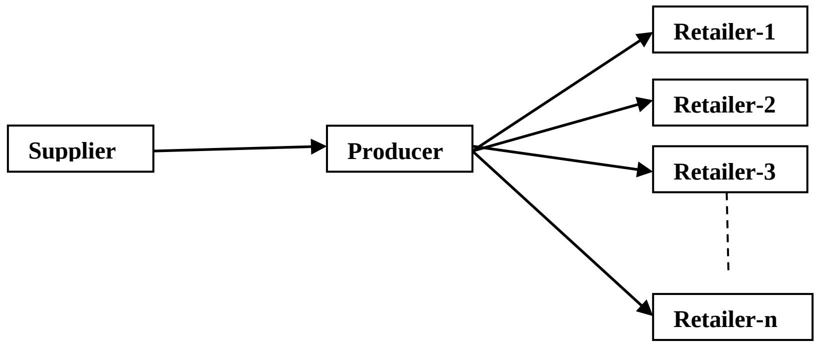

Figure1, A three echelon multi-retailers supply chain

3.1. Producer's raw materials or Suppliers Inventory System

The raw materials inventory system is shown in Fig 2(a). A supplier procures the raw materials and delivers the fixed quantities Q_w to the producer's warehouse at a fixed-time interval. The Producer withdraws raw materials from the warehouse. During the time period T_1, the inventory level decrease due to both producers demand and deterioration.

The supplier's raw materials inventory system at any time t can be represented by the following differential equation:

$$\frac{dI_w(t)}{dt} = -P - ght^{h-1}I_w(t) \qquad 0 \le t \le T_1 \qquad \text{………. (1)}$$

Using the boundary condition, $I_w(T_1) = 0$. The solution of (1) is given by

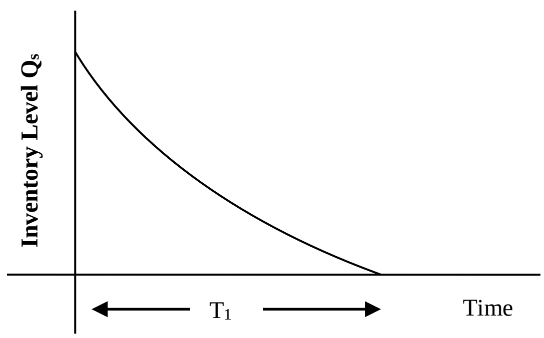

Figure. 2 (a)

$$I_w(t) = Pe^{-gt^h}\int_t^{T_1} e^{gu^h}\,du \qquad 0 \le t \le T_1 \qquad \text{……… (2)}$$

Since $I_w(0) = Q_w$, the maximum inventory level of the raw materials, i.e. the order quantity per order from supplier is

$$Q_w = P\int_0^{T_1} e^{gu^h}\,du = P\int_0^{T_1}\sum_{n=0}^{\infty}\frac{(gu^h)^m}{\text{n!}}du \approx P\left(T_1 + \frac{gT_1^{h+1}}{h+1}\right) \qquad \text{……… . (3)}$$

1. Present Value setup cost: Since setup in each cycle is done at the start of each cycle, the present value setup cost $S_w = C_{1w}$ … . (4)

2. Present value inventory holding cost: Inventory occurs during period T_1, the present-value inventory cost during the period is

$$HD_w = \sum_{m=1}^{2} C_{mw} \int_0^{T_1} I_w(t) e^{-r_m t} dt \qquad \text{Where} \qquad r_m = r - i_m \qquad m = 1, 2$$

$$HD_w \approx \sum_{m=1}^{2} C_{mw} P \left\{ \frac{T_1^2}{2} - \frac{r_m T_1^3}{6} + \frac{g h T_1^{h+2}}{(h+1)(h+2)} \right\} \qquad \text{………} \quad (5)$$

3. Present-value item cost: The item cost includes the loss due to the deterioration as well as the cost of the item sold. Because the order is done at $t = 0$, the present value item cost is

$$IT_w = C_w Q_w \approx C_w P \left(T_1 + \frac{g T_1^{h+1}}{h+1} \right) \qquad \text{……….. (6)}$$

The present value total cost during the cycle is the sum of the setup cost S_w, the inventory holding cost HD_w and the item cost IT_w. For raw materials, the present value total cost per unit time is

$$TUC_w = \frac{S_w + HD_w + IT_w}{T} \qquad \text{………….. (7)}$$

3.2. Producer's finished goods inventory system.

The producer inventory system in Fig 2(b) can be divided into two independent phases depicted by T_1 and T_2. This methodology reduces the complexity in our problems derivation on and analysis. Each phase has its own time unit, t which starts from the beginning of the phase T_i. During time period T_1 There is an inventory buildup and hence deterioration becomes effective. At $t = T_1$, the production stops and the inventory level increase to its maximum level MI_p. There is no production during time period T_2, the inventory level decrease due to demand and deterioration. The Inventory level becomes zero at $t = T_2$.

The inventory system depicted in Figure 2 (b) is represented by the following differential equations;

$$\frac{dI_{p_1}(t)}{dt} = P - \sum_{i=1}^{N} d_i - \alpha \beta t^{\beta-1} I_{p_1}(t) \qquad 0 \le t \le T_1 \qquad \text{…………. (8)}$$

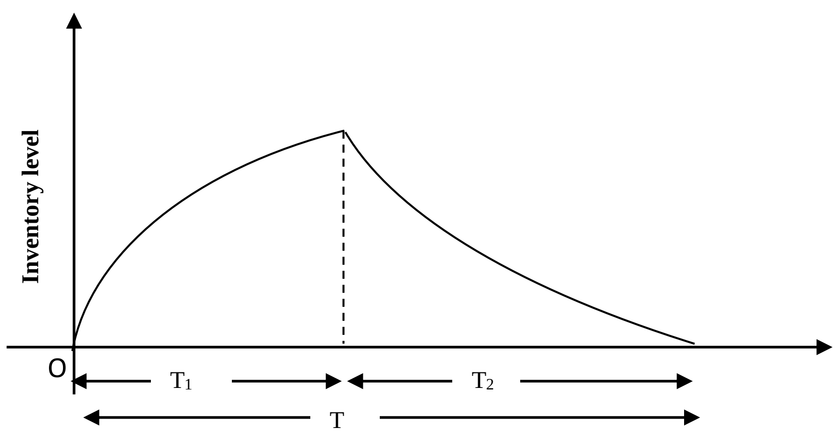

Figure 2 (b)

$$\frac{dI_{p_2}(t)}{dt} = -\sum_{i=1}^{N} d_i - \alpha \beta t^{\beta-1} I_{p_2}(t) \qquad 0 \le t \le T_2 \qquad \text{… ………… (9)}$$

The first-order differential equations are solved using the boundary conditions $I_{p_1}(0) = (0)$ and $I_{p_2}(T_2) = (0)$, then

$$I_{p_1}(t) = \left(P - \sum_{i=1}^{N} d_i \right) e^{-\alpha t^{\beta}} \int_0^t e^{\alpha u^{\beta}} du \qquad 0 \le t \le T_1 \qquad \text{…………. (10)}$$

and

$$I_{p_2}(t)=\sum_{i=1}^{N} d_i e^{-\alpha t^\beta}\int_t^{T_2} e^{\alpha u^\beta}du \qquad 0\le t\le T_2 \qquad \ldots \quad \ldots\ldots \quad \ldots (11)$$

Since $I_{p_2}(0)=MI_p$, the producer's maximum inventory level is

$$MI_p=\sum_{i=1}^{N} d_i\int_0^{T_2} e^{\alpha u^\beta}du=\sum_{i=1}^{N} d_i\int_0^{T_2}\left(\sum_{n=0}^{\infty}\frac{\left(\alpha u^\beta\right)^n}{n!}\right)du\approx\sum_{i=1}^{N} d_i\left(T_2+\frac{\alpha T_2^{\beta+1}}{\beta+1}\right) \qquad \ldots\ldots.. (12)$$

The production quantity is $Q_p=PT_1$ ………. (13)

By the boundary condition $I_{p_1}(T_1)=I_{p_2}(0)$, one can derive the following equation;

$$\left(P-\sum_{i=1}^{N} d_i\right)e^{-\alpha T_1^\beta}\int_0^{T_1} e^{\alpha u^\beta}du=\sum_{i=1}^{N} d_i\int_0^{T_2} e^{\alpha u^\beta}du$$

For a very small value of α, second and higher order terms of α are neglected. The above Eq. can be reduced as

$$\left(P-\sum_{i=1}^{N} d_i\right)\left(T_1-\frac{\alpha\beta T_1^{\beta+1}}{\beta+1}\right)=\sum_{i=1}^{N} d_i\left(T_2+\frac{\alpha\beta T_2^{\beta+1}}{\beta+1}\right) \qquad \ldots\ldots\ldots\ldots\ldots (14)$$

Since T_1 in Eq. (14) is a high power equation, it is difficult to solve analytically for the value of T_1 when $\alpha T_1 << 1$, the method used in Mishra (1975) and Wee (2007), where $\alpha\beta T_1^{\beta+1}/\beta+1$ is neglected, results in the following approximate value for T_1

$$T_1\approx\frac{\sum_{i=1}^{N} d_i}{P-\sum_{i=1}^{N} d_i}\left(T_2+\frac{\alpha\beta T_2^{\beta+1}}{\beta+1}\right) \qquad \ldots\ldots\ldots\ldots.. (15)$$

1. Present-value ordering cost: Since replenishment in each cycle is done at the start of each cycle, the present-value ordering cost for raw material is

$$OR_p=C_{2p} \qquad \ldots\ldots\ldots\ldots. (16)$$

2. Present-value setup cost: At the start of the cycle, the cycle has an initial production setup cost. The present value setup cost is

$$S_p=C_{1p} \qquad \ldots\ldots\ldots \quad (17)$$

3. Present-value inventory cost: Inventory is carried during T_1 and T_2 time periods. If this system does not consider the buyers, all of the holding costs belong to the producer. If this system considers the buyers, the holding costs of the items that are delivered to the buyers belong to the buyers. It should be subtracted from the producer. The present-value holding cost is

$$HD_p=\sum_{m=1}^{2} C_{mp}\left[\int_0^{T_1} I_{p_1}(t)e^{-r_m t}dt+\int_0^{T_2} I_{p_2}(t)e^{-r_m(T_1+t)}dt-\left(\sum_{i=1}^{N}\int_0^{T_3} I_{b_3}^i(t)e^{-r_m t}dt\sum_{j=0}^{k_i-1} e^{-jr_mT_5}\right)\right]$$

where $r_m=r-i_m \quad m=1,2$

$$HD_p=\sum_{m=1}^{2} C_{mp}\left[\left(P-\sum_{i=1}^{N} d_i\right)\left\{\frac{T_1^2}{2}-\frac{r_mT_1^3}{3}-\frac{\alpha\beta T_1^{\beta+2}}{(\beta+1).(\beta+2)}\right\}\right.$$

$$\left.+\sum_{i=1}^{N} d_i\left\{\frac{(1-r_mT_1).T_2^2}{2}-\frac{r_mT_2^3}{6}+\frac{\alpha\beta T_2^{(\beta+2)}}{(\beta+1).(\beta+2)}\right\}-\sum_{i=1}^{N} d_i\left(\frac{T_3^2}{2}-\frac{r_mT_3^3}{6}+\frac{uvT_3^{(v+2)}}{(v+1).(v+2)}\right).\left(\frac{\left(1-e^{-r_mT}\right)}{\left(1-e^{-r_mT_5}\right)}\right)\right] \qquad \ldots\ldots\ldots (18)$$

4. Present-value item cost: The item cost includes the loss due to the deterioration as well as the cost of the item sold. Because set up is done at $t=0$, the present worth item cost is

$$IT_p=C_p.Q_p=C_p.PT_1 \qquad \ldots\ldots\ldots\ldots.. (19)$$

Therefore, the present-value total cost during the cycle is the sum of the setup cost S_p, ordering cost OR_p, the holding cost HD_p and the item cost IT_p. The present value total cost per cycle is

$$TUC_p = \frac{S_p + OR_p + HD_p + IT_p}{T} \quad \text{……….. (20)}$$

3.3. Buyer's finished goods inventory system

The change in buyer's Inventory level is depicted in fig 2(c). Since P >> d, we assumed that the initial delivery is made at $t = 0$ in the buyer's inventory system. Part of stock is delivered towards backorders, leaving a balance of MI_r units in the initial inventory. During time period T_3, the inventory level decrease due to both demand deterioration. At $t = T_3$ the inventory level is zero. During the time period T_4, part of the shortage is backlogged and part of it results in lost sales. Only the backlogged items are replaced in the next replenishment. There are k deliveries in $T = T_1 + T_2$ time period. In T_4 segment of Figure 2 (c), the broken line indicates the complete shortage and the dark line indicates shortage due to partial backlogging.

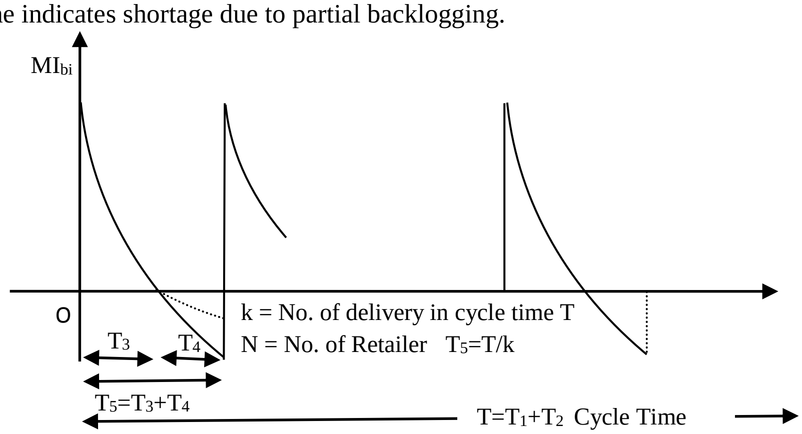

Figure 2 (c)

In Figure 2 (c), the buyer's inventory system can be represented by the following equation;

$$\frac{dI_{b_3}^i(t)}{dt} = -d_i - uvt^{v-1} I_{b_3}^i(t) \qquad 0 \le t \le T_3 \;\; i = 1,2,3,........N \quad \text{………… (21)}$$

$$\frac{dI_{b_4}^i(t)}{dt} = -Bd_i \qquad 0 \le t \le T_4 \quad \text{………… (22)}$$

The first order differential equation is solved using the boundary conditions, $I_{b_3}^i(T_3) = 0$, and $I_{b_4}^i(0) = 0$, one has

$$I_{b_3}^i(t) = d_i e^{-ut^v} \int_t^{T_3} e^{uh^v} dh \qquad 0 \le t \le T_3 \quad \text{………… (23)}$$

and $$I_{b_4}^i(t) = -Btd_i \qquad 0 \le t \le T_4 \quad \text{…… …….. (24)}$$

Since $I_{b_3}^i(T_3) = MI_b^i$ i.e. the buyer's maximum inventory level is

$$MI_b^i = d_i \int_0^{T_3} e^{uh^v} dh = d_i \int_0^{T_3} \left(\sum_{n=0}^{\infty} \frac{(uh^v)^n}{n!} \right) dh \approx d_i \left(T_3 + \frac{uT_3^{(v+1)}}{(v+1)} \right) \quad \text{....…. (25)}$$

The quantity per delivery to the i[th] buyer is

$$Q_b^i = MI_b^i + Bd_i T_4 \approx d_i \left(T_3 + \frac{uT_3^{(v+1)}}{(v+1)} \right) + Bd_i T_4 \quad \text{……….. (26)}$$

1. Present-value ordering cost: The present value of the total replenishment cost during T is given by

$$OR_b = k\sum_{i=1}^{N} C_{1b}^{i} \qquad \text{………… (27)}$$

2. Present-value inventory holding cost: Inventory is carried during time period T_3. The present value holding cost is

$$HD_b = k\sum_{i=1}^{N}\left(\sum_{m=1}^{2} C_{mb}^{i}\int_0^{T_3} I_{b_3}^{i}(t)e^{-r_m t}dt\right) = k\sum_{i=1}^{N}\left(\sum_{m=1}^{2} C_{mb}^{i}\int_0^{T_3}\left(d_i e^{-ut^v}\int_t^{T_3} e^{uh^v}dh\right)e^{-r_m t}dt\right)$$

$$HD_b \approx k\sum_{i=1}^{N}\left(\sum_{m=1}^{2} C_{mb}^{i} d_i\left\{\frac{T_3^2}{2} - \frac{r_m T_3^3}{6} + \frac{uvT_3^{(v+2)}}{(v+1).(v+2)}\right\}\right) \qquad \text{………… (28)}$$

3. **Present-value shortage cost:** Shortage occurs during time period T_4. The present value shortage/backlog cost is

$$BA = k\sum_{i=1}^{N}\sum_{m=1}^{2} C_{mb1}^{i}\int_0^{T_4}\left\{-I_{b_4}^{i}(t)\right\}.e^{-r_m(T_3+t)}dt$$

$$BA = k\sum_{i=1}^{N}\sum_{m=1}^{2} C_{mb1}^{i}.B.d_i\left\{\frac{(1-r_m.T_3).T_4^2}{2} - \frac{r_m.T_4^3}{3}\right\} \qquad \text{……… … (29)}$$

4.Present-value lost sale: Lost sale occurs during time period T_4. During this time period, the complete shortage is $d_i T_4$ and the partial backlog is $Bd_i T_4$. The difference between them indicates the lost sales. The present value lost sale cost is

$$LS = k\sum_{i=1}^{N}\sum_{m=1}^{2} C_{mb2}^{i}\int_0^{T_4}(d_i - Bd_i).e^{-r_m(T_3+t)}dt = k\sum_{i=1}^{N}\sum_{m=1}^{2} C_{mb2}^{i}.(1-B).d_i\int_0^{T_4}\left(\sum_{n=0}^{\infty}\frac{\{-r_m(T_3+t)\}^n}{n!}\right)dt$$

$$LS \approx k\sum_{i=1}^{N}\sum_{m=1}^{2} C_{mb2}^{i}.(1-B).d_i.\left\{(1-r_m T_3)T_4 - \frac{r_m T_4^2}{2}\right\} \qquad \text{……….. (30)}$$

5. Present-value item cost: The item cost includes loss due to deterioration as well as the cost of the item sold. Because order is at $t=0$ and $t = T_3 + T_4$. The present value item cost is

$$IT_b = k\sum_{i=1}^{N}\left[C_b^i.MI_b^i + C_b^i.B.\mathrm{T}_4.d_i.e^{-r_2(T_3+T_4)}\right]$$

$$IT_b = k\sum_{i=1}^{N}\left[C_b^i.d_i\left(T_3 + \frac{uT_3^{(v+1)}}{v+1}\right) + C_b^i.B.\mathrm{T}_4.d_i.\{1 - r_2(T_3+T_4)\}\right] \qquad \text{……….. (31)}$$

The present worth total cost during the delivery is the sum of the ordering cost OR_b, the holding cost (HD_b), the backlog cost (BA), the lost sale cost (LS) and the item cost (IT_b). The present value total cost per delivery is

$$TUC_b^{'} = \frac{OR_b + HD_b + BA + LS + IT_b}{T} \qquad \text{……….. (32)}$$

There are k deliveries per cycle per buyer. The fixed time interval between the deliveries is $T_5 = T/k$. Therefore, the present worth total cost per cycle at $t=0$ is

$$TUC_b = TUC_b^{'}\sum_{j=0}^{k-1} e^{-jr_m T_5}$$

$$TUC_b = \left(\frac{OR_b + HD_b + BA + LS + IT_b}{T}\right).\left(\frac{1-e^{-r_m T}}{1-e^{-r_m T_5}}\right) \qquad \text{…….. (33)}$$

This study develops an integrated production inventory model for deteriorating items with weibull distribution under partial backlogging, inflation and time value of money with multiple buyers. For very small α, r_m and value

($\alpha, r_m, \mu \le 1$), an approximate model with multiple buyer and a single producer is developed to derive the optimal production policy and lot size. The present value total cost per unit time of the producer and the buyers is sum of TUC_W, TUC_P and TUC_b. Since $T_3 = T_5 - T_4$, $T_5 = T / k_i$ and $T = T_1 + T_2$, the problem can be stated as an optimization problem and it can formulate as

Minimize $TUC(k, T_2, T_4) = TUC_s + TUC_p + TUC_b$ ……….. (34)

Subject to $0 \le T_2, 0 \le T_4 \le T / k$ ……….. (35)

4. SOLUTION PROCEDURE

The optimization technique is used to minimize (34) to derive T_2 and T_4 as follow:

Step1. Since the number of delivery per order k, is an integer value, start by choosing an integer value of $k \ge 1$.

Step2. Take the derivative of $TUC(k, T_2, T_4)$ with respect to T_2 and T_4, and equate the result to zero. The necessary condition of optimality is $\partial TUC(\mathrm{k}, T_2, T_4)/\partial T_2 = 0$ and $\partial TUC(\mathrm{k}, T_2, T_4)/\partial T_4 = 0$ These simultaneous equations can be solved for T_2 and T_4

Step3. Find those values of T_2 and T_4 from step 2 for that

$$\partial^2 TUC(\mathrm{k}, T_2, T_4)/\partial T_2^{\,2} . \partial^2 TUC(\mathrm{k}, T_2, T_4)/\partial T_4^{\,2} - \left(\partial^2 TUC(\mathrm{k}, T_2, T_4)/\partial T_2 \partial T_4\right)^2 > 0$$

and $\partial^2 TUC(\mathrm{k}, T_2, T_4)/\partial T_2^2$ is positive

Step4. Using these values of T_2 and T_4 in eq. (34) and find the minimum value of TUC

Step5. Repeat steps 2 and 3 for all possible values of k until the minimum $TUC(k^*, T_2^*, T_4^*)$ is found. The values of $TUC(k^*, T_2^*, T_4^*)$ constitute the optimal solutions that satisfy the condition mentioned in step 3.

Step6. Derive the T_1^*, T_3^*, T_5^*, T^*, Q_s^*, Q_p^*, Q_b^*, $TUC_w{}^*$, $TUC_p{}^*$ and $TUC_b{}^*$

5. NUMERICAL EXAMPLE

Optimal production and replenishment policy to minimize the present worth total system cost may be obtained by using the methodology proposed in the proceeding sections. The following numerical example is illustrated the model.

Consider a three-echelon supply chain with three retailers $(N = 2)$, a producer and a supplier for single item, the value of parameters adopted in this study are production rate P=1000; demand rate d=200; setup cost C_{1w}=\$100;C_{1p}=\$50; ordering cost C_{2p}=\$90; C_{1b}=\$80; holding cost C_{mw}=\$0.6; C_{mp}=\$0.8; C_{mb}=\$1; backlog cost C_{mb1}=\$10; lost sale cost C_{mb2}=\$5; item cost C_w=\$8; C_p=\$10; C_b=\$12; deterioration rate g=0.08, h=2, α=0.05, β=2, u=0.08, v=3; inflation rate r=0.2, i_1=0.08, i_2=0.14; fraction backorder B=0.8.

The computational results are shown in Table 1. The raw material, producer and retailer's costs are presented in Table 2.

The major conclusions and the special condition are drawn from numerical are as follow;

1. In this example, TUC* is \$20867.5 while the optimal values of k*, T_1^*, T_2^*, T_3^* and T_4^* are 17, 1.29, 8.70, 0.45 and 0.13 respectively. For the raw materials Q_w is 1347.2 units. As for the finished goods, Q_p is 1290 units and Q_b is 221.9 units. The time period between deliveries is 0.58 units.

2. If the optimal solution is calculated independently from the raw material's view, k* is 1 and TUC* is \$27521.9. An increase of \$6654.4 per unit time. If the optimal solution is derived independently from the producer's view, k* is 2 and TUC* is \$25970. An increase of \$5103 per unit time. The reason is the entire buyer's cost will increase. If the optimal solution is derived independently from the retailer's view, k* is 15 and TUC* is \$20872. Since the entire producer's cost will increase, an increase of \$5 per unit time.

Table 1 . Numerical results for illustrated example

K	T_1	T_2	T_3	T_4	Q_w	Q_p	Q_b	TUC_w	TUC_p	TUC_b	TUC
1	1.29	8.70	2.95	7.04	1347.2	1290	4038.6	1179.9	6562.7	19514.7	27521.9
2	1.29	8.70	2.45	2.54	1347.2	1290	2081.0	1179.9	**6480.1**	18006.2	25970.1
3	1.29	8.70	2.00	1.33	1347.2	1290	1353.6	1179.9	6551.2	15551.0	23557.8
4	1.29	8.70	1.67	0.82	1347.2	1290	992.6	1179.9	6620.8	14257.0	22381.9
5	1.29	8.70	1.42	0.57	1347.2	1290	782.9	1179.9	6680.7	13582.1	21775.6
6	1.29	8.70	1.23	0.43	1347.2	1290	647.9	1179.9	6728.6	13215.7	21437.7
7	1.29	8.70	1.08	0.34	1347.2	1290	551.6	1179.9	6768.6	12949.4	21236.4
8	1.29	8.70	0.96	0.28	1347.2	1290	480.3	1179.9	6801.8	12765.7	21110.2
9	1.29	8.70	0.86	0.24	1347.2	1290	425.8	1179.9	6831.1	12630.2	21027.7
10	1.29	8.70	0.78	0.21	1347.2	1290	382.9	1179.9	6854.1	12554.1	20972.4
11	1.29	8.70	0.71	0.19	1347.2	1290	347.6	1179.9	6875.5	12496.3	20934.7
12	1.29	8.70	0.65	0.18	1347.2	1290	319.7	1179.9	6894.3	12528.3	20908.8
13	1.29	8.70	0.60	0.16	1347.2	1290	292.9	1179.9	6909.7	12395.7	20891.2
14	1.29	8.70	0.55	0.15	1347.2	1290	269.3	1179.9	6927.2	12267.5	20879.6
15	1.29	8.70	0.51	0.14	1347.2	1290	249.8	1179.9	6940.3	**12186.5**	20872.4
16	1.29	8.70	0.48	0.13	1347.2	1290	234.4	1179.9	6948.8	12188.3	20868.7
17	**1.29**	**8.70**	**0.45**	**0.13**	**1347.2**	**1290**	**221.9**	**1179.9**	**6958.4**	**12301.0**	**20867.5**
18	1.29	8.70	0.42	0.13	1347.2	1290	209.8	1179.9	6968.9	12340.6	20868.4
19	1.29	8.70	0.40	0.12	1347.2	1290	198.6	1179.9	6974.2	12314.1	20870.9
20	1.29	8.70	0.37	0.13	1347.2	1290	189.7	1179.9	6986.1	12454.1	20874.9
…	……….	………..	………..	………	………..	……..	…………	…………	………….	…………….	……………
…	……….	………..	…………	………	………….	…….	…………	…………..	…………	…………..	……………
30	1.29	8.70	0.23	0.07	1347.2	1290	114.2	1179.9	7031.5	14636.2	20954.1

3. A graphical representation and numerical analysis are presented to show the convexity of TUC. Based on above discussion and graphical representation of Figure 3 (a) and 3 (b), one can say that TUC is a convex function. When $k^* = 17$ the sufficient conditions are $\partial^2 TUC/\partial T_2^2 = 1112.46$, $\partial^2 TUC/\partial T_4^2 = 2758.78$, $\partial^2 TUC/\partial T_2 \partial T_4 = 0$

and $\partial^2 TUC(\mathrm{k}, T_2, T_4)/\partial T_2^2 \,.\, \partial^2 TUC(\mathrm{k}, T_2, T_4)/\partial T_4^2 - \left(\partial^2 TUC(\mathrm{k}, T_2, T_4)/\partial T_2 \partial T_4\right)^2 = 3.06 \times 10^6 > 0$

4. When k increase T_1, T_2 will same and T_3, T_4 and T_5 will decrease. The reason is multiple deliveries will increase the number of production to avoid the excess inventory.

5. When there is complete backordering i.e. B=1, at k=17, TUC is $21024. There is no lost sale cost in this situation. An increase of $157 per unit time.

Table 2. The supplier, producer and buyer's cost for **k = 17**

Buyer's Costs		Producer's Costs		Supplier's Costs	
OR_b	2560.3	OR_p	60	S_w	100
HD_b	2722.8	HD_p	5658.4	HD_w	838.9
IT_b	8245.9	IT_p	1290.3	IT_w	1077.8
BA	1749.9				
LS	3372.1				

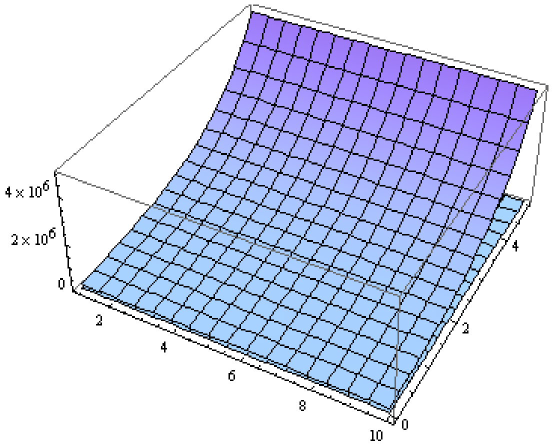

Figure 3 (a)

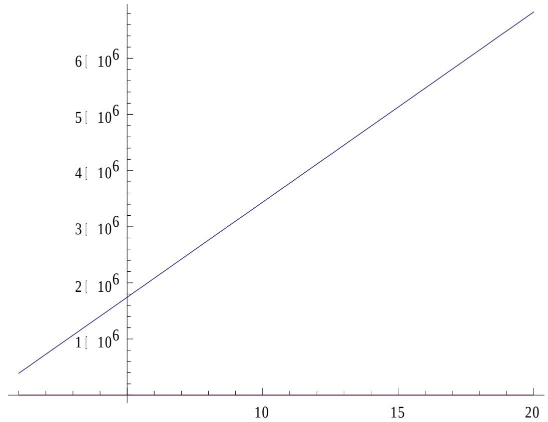

Figure 3 (b)

In figure 3(a), Variation of TUC w. r. t. T_1 (from 0 to 10) and T_3 (from 0 to 5) and in figure 3(b) Variation TUC w. r. t. the value of k (No. of delivery)

6. When deterioration for raw material is not considered (i.e. g=0, h=0) then at k=15, the TUC is $20799. A decrease of $73 per unit time.
7. When deterioration for finished goods of producer is not considered (i.e. α=0, β=0) then at k=16, the TUC is $12426. A decrease of $8442 per unit time.
8. When deterioration for finished goods of buyer is not considered (i.e. u=0, v=0) then at k=16, the TUC is $21529. An increase of $661 per unit time.

6. SENSITIVITY ANALYSIS

The change in the values of parameters may happen due to uncertainties in any decision-making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in decision making. Using the numerical example given in the previous section, the sensitivity analysis of various parameters has been done. We derive the optimal solution for fixed values of the parameters P, d, C_{1w}, C_{1p}, C_{2p}, C_{1b}, C_{mw}, C_{mp}, C_{mb}, C_{mb1}, C_{mb2}, C_w, C_p, C_b, g, h, α, β, u, v, r, i_1, i_2 and B (1000, 400, 100, 50, 90, 80, 0.6, 0.8, 1, 10, 5, 8, 10, 12, 0.08, 2, 0.05, 2, 0.05, 3, 0.2, 0.08, 0.14 and 0.8).The optimal values of k^0 and TUC^0 are derived; when one of the parameters increases or decreases by 5% and 10% while other parameters remain unchanged. The results are presented in Tables below. The percentage of cost increase index is defined as

$$PCI = \frac{TUC^0 - TUC^*}{TUC^*} \times 100$$

Table 3. Effect of change of parameters on T_1^0 and T_3^0

	P	d	g	h	α	β	u	v	r	i_1	i_2	B
-10% changed												
T_1^0	1.52	1.06	1.31	0.30	1.13	0.30	1.29	0.30	1.30	1.29	1.29	1.29
T_3^0	0.45	0.45	0.45	0.26	0.45	0.26	0.45	0.26	0.46	0.45	0.44	0.41
-5% changed												
T_1^0	1.41	1.18	1.30	0.30	1.19	0.30	1.29	0.30	1.30	1.29	1.29	1.29
T_3^0	0.45	0.45	0.45	0.26	0.45	0.26	0.45	0.26	0.45	0.45	0.44	0.43
+5% changed												
T_1^0	1.18	1.40	1.29	0.30	1.36	0.30	1.29	0.30	1.29	1.29	1.30	1.29
T_3	0.45	0.45	0.45	0.26	0.45	0.26	0.45	0.26	0.44	0.45	0.45	0.46
+10% changed												
T_1^0	1.08	1.50	1.28	0.30	1.44	0.30	1.29	0.30	1.28	1.30	1.30	1.29
T_3^0	0.45	0.45		0.26	0.45	0.26	0.45	0.26	0.44	0.45	0.45	0.48

The main conclusions are from sensitivity analysis:

1. From the table 4, change of the value of demand rate (d) affects the value of TUC and which is presented by PCI. When demand rate (d) increases by 10%, the value of PCI increases by over 8.36%. The values of PCI are more sensitive to the shape parameters (h, v and β).
2. The values of PCI are least sensitive to the scale parameter (g and u).
3. The values of PCI are little sensitive to the parameters i_1 (internal inflation rate), i_2 (external inflation rate) and B (fraction of demand backordered).
4. The parameters production rate (P), demand rate (d), scale parameters (g, α and u), shape parameters (β), inflation rate (i_1, i_2), and fraction of demand backordered (B) influence the value of PCI in the same direction and the parameters shape parameters (h, v) and discount rate (r) influence the value of PCI in the opposite direction.
5. The value of production time (T_1) is little sensitive to the parameters P, d, g, α, u, r, and highly sensitive to the parameters h, β, v but no sensitive to the parameters i_1, i_2 and B.
6. The value of T_3 is little sensitive to the parameters r, i_2 and B, highly sensitive to the h, β and B but no sensitive to the parameters h, d, g, α, u, i_1.

Table 4. Different values of TUC^0 and PCI^0 with respect to the percentage change of the value of different

-10% changed												
	P	d	g	h	α	β	u	v	r	i_1	i_2	B
TUC^0	20551.8	19067.4	20861.3	21827.8	20470.8	19733.0	20865.7	21828.5	21155.2	20814.9	20756.6	20760.5
PCI	-1.51	-8.62	-0.02	+4.20	-1.90	-5.43	-0.009	+4.60	+1.38	-0.25	-0.53	-0.51
-5% changed												
TUC^0	20716.9	19974.9	20864.4	21827.5	20633.1	20667.7	20866.6	21827.8	21011.7	20841.2	20812.2	20818.0
PCI	-0.72	-4.40	-0.01	+4.20	-1.12	-0.95	0.00	+4.60	+0.69	-0.25	-0.26	-0.23
+5% changed												
TUC^0	21004.5	21746.5	20870.5	21827.4	21018.1	23265.4	20868.4	21826.8	20722.6	20893.8	20922.6	20909.9
PCI	+0.65	+4.21	+0.01	+4.20	+0.72	+11.49	0.00	+4.60	-0.69	+0.12	+0.26	+0.20
+10% changed												
TUC^0	21128.7	22613.0	20873.5	21827.3	21236.1	25049.0	20869.4	21826.5	20577.1	20920.0	20977.4	20946.2
PCI	+1.25	+8.36	+0.02	+4.20	+1.76	+20.04	0.009	+4.60	-1.38	+0.25	+0.52	+0.37

parameters like P, d, g, h, **α**, **β**, u, v, r, i_1, i_2, and B for k=17

7. CONCLUSIONS

In this study, we have derived a three-layer supply chain involving supplier, producer and multiple retailers with replenishment lot size. Different rates of deterioration for supplier, producer and buyer have taken because raw material, finished goods for producer and finished goods for buyers are under different management. Partial backlogging has taken for buyers. The study has been conducted under the discounted cash flow (DCF) approach as it permits a proper recognition of the financial implication of the opportunity cost in inventory analysis. Multiple buyers and multiple deliveries are the most important policies to reduce inventory as well as many other costs. Joint decision also reduces optimal joint cost when compared with an independent cost by supplier or producer or retailers. To make it admissible to all parties, the integrated policy should offer some kind of profit sharing. The profit sharing policy can be in the form of advanced payment, per item cost discount. The study is particularly useful for the inventory systems where all parties form a strategic alliance with mutually beneficial objective. Numerical example and sensitivity discussion has been performed in this study and we have showed that a certain value of k optimize the total system cost. This research can be extended to consider the permissible delay in payments, imperfect production, variable production and demand rate.

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