ON A SINGLE SERVER QUEUE WITH ARRIVALS IN BATCHES OF VARIABLE SIZE, GENERAL SERVICE IN THREE FLUCTUATING MODES, BALKING, RANDOM BREAKDOWNS AND A STAND-BY SERVER DURING BREAKDOWN PERIODS

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ABSTRACT
We study a single server queuing system with arrivals in batches of variable size and general service in three fluctuating modes with different mean service rates. It is assumed that just after completion of the previous service or just after completion of repairs or just after arrival of a fresh batch of customers at a time when the system is in an idle state, the service to next customer may start in mode j with probability p_j where $j = 1, 2, 3$ $\sum_{j=1}^{3} p_j = 1$. The system is assumed to be subject to random failures with the general repair time distribution. The system is equipped with a stand-by server which operates only during the breakdown periods of the system. We further assume that an arriving batch may balk and leave the system as soon as it arrives or may join the system. The balking rates have been assumed to be different during the working periods or the breakdown periods of the main server. Steady state results for the queue size distribution at a random epoch have been obtained explicitly. Probabilities of the idle state as well as the utilization factor of the system have been found explicitly. Many particular cases of interest are discussed.

KEYWORDS: batch arrivals, balking, general service in three fluctuating modes, random breakdowns, repair times, steady state

MSC: 90B22

RESUMEN
Estudiamos un sistema de colas con un solo servidor con arribos en paquetes de tamaño variable y serviciaje general en tres modos fluctuantes con diferentes tasas media de serviciaje. Se asume que justo después de completar el servicio previo o justamente después de completar la reparación o justo después de llegar un grupo nuevo de clientes en un momento en el que el sistema está en estado disponible, el servicio del próximo cliente puede comenzar en el modo j con probabilidad p_j donde $j = 1, 2, 3$ $\sum_{j=1}^{3} p_j = 1$.
Se asume que el sistema está sujeto a roturas aleatorias con una distribución general del tiempo de reparación. El sistema esta equipado con un servidor en espera que opera solamente durante los periodos de parada del sistema. Asumimos además que un paquete puede que arribe y deje el sistema tan pronto como llega o uede unirse al sistema. Las tasas de abandono se han sido asumidas diferentes durante los periodos de trabajo o de interrupción del servidor central. Se han obtenido resultados explícitos sobre el estado estable de la distribución del tamaño de las colas en una época aleatoria. Las probabilidades del estado libre así como el factor de utilización del sistema han sido hallados explícitamente Muchos casos particulares de interés son discutidos.

1. INTRODUCTION

A wide range of $M^{[X]}/G/1$ type queuing systems assume that the server provides one type of general service with the same mean rate of service to all customers. However, in many real life situations there could be variation in the mean service rate due to a variety of reasons such as fluctuation in internet speed in case of an online server or fluctuation in the server efficiency in case of a human server or fluctuations due to some extraneous factors including climatic conditions or intermittently available power supply etc. Recently, Baruah et al [4] have studied a

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queueing system in which the server provides general service in two fluctuating modes. In the present paper, we study a $M^{[X]}/G/1$ queueing system in which the server provides general service to customers in three fluctuating modes with different service rates. We further assume that the system is subject to random breakdowns and is equipped with a stand-by server which is employed only during breakdown periods. One finds enormous amount of work done on queueing systems with breakdowns. For some papers on random breakdowns and/or papers on queueing systems equipped with a stand-by server the reader may see Aissani et al [1], Fadhil et al [10, 11], Khalaf [14] and Madan [16,17], Madan et al [18] and Maraghi et al [19] Additionally, impatient customers are a very common phenomenon in a queueing system. Customers may arrive at the system but may leave without joining the system (balking) or customers may join the queue for some time but may leave the system without getting service (reneging). We assume that the system is subject to balking with different balking probabilities during the period when the server is in the working state (including the idle state) and during the period when the server is in the failed state (under repairs). For some papers on impatient customers the readers may see Ancker et al [2], Barrer [3], Baruah [4], Boxma [5], Choi et al [6], Choudhury et al [7], De Kok et al [8], Altman et al [9], Haight [12], Iravani et al [13], Lukasz et al [15] and Zhang et al [20].

2. THE MATHEMATICAL MODEL

Customers arrive at the system in batches of variable size in accordance with a compound Poisson process. Let $\lambda c_i dt(i=1,2,3, \ ...)$ be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t,t+dt]$, where $0 \le c_i \le 1$, $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the mean arrival rate of batches. The arriving batches wait in the queue in the order of their arrival. It is further assumed that customers with each batch are pre-ordered for the purpose of service.

There is one server providing service in three fluctuating modes. The customers are served one by one on a first come first served basis. We assume that just after completion of the previous service or just after completion of repairs , the service to next customer may start in mode j with probability p_j where $j=1,2,3$ $\sum_{j=1}^{3} p_j = 1$. The same rule applies when a fresh service starts on arrival of a batch at a time when the server is idle. We further assume that mode j service times S_j follow general distribution G_j and density function $g_j(s)$ with mean service rate μ_j, where $j=1,2,3$. Let $\mu_j(x)dx$, $j=1,2,3$ be the conditional probability density of completion of service under mode j of service during the interval $(x,x+dx]$, given that the elapsed time is x, so that

$$\mu_j(x) = \frac{g_j(x)}{1-G_j(x)}, \ j=1,2,3 \tag{2.1}$$

and, therefore,

$$g_j(s) = \mu_j(s) e^{-\int_0^s \mu_j(x)dx} \tag{2.2}$$

Let αdt be the probability that the system will breakdown. We assume that the system may breakdown only while the main server is providing service.

Let βdt be the probability of completion of repairs and repair times follow a general arbitrary distribution with

$$\beta(x) = \frac{B(x)}{1-B(x)} \tag{2.3}$$

and thus

$$b(u)=\beta(u)\exp\left(-\int_0^u \beta(x)dx\right) \tag{2.4}$$

The service time S_b of a customer served by the standby server when the main server is under repairs is exponentially distributed with mean service time $\frac{1}{\nu}$, $\nu>0$.

Let $(1-a_1)$ $(0\le a_1\le 1)$ be the probability that an arriving batch balks (does not join the queue) during the period when the server is in the working state (including the idle state) and let $(1-a_2)$ $(0\le a_2\le 1)$ be the probability that an arriving batch balks (does not join the queue) when the server is in failed state (under repairs).

Various stochastic processes involved in the system are assumed to be independent of each other

3. DEFINITIONS AND NOTATIONS

$P_n^{(j)}(x,t)$: Probability that at time t, the server is providing service in mode $j=1,\ 2,\ 3$ and there are $n(\ge 0)$ customers in the queue excluding one customer in service with elapsed service time x. Consequently, $P_n^{(j)}(t)=\int_0^\infty P_n^{(j)}(x,t)dx$ denotes the probability that at time t, the server is providing service in mode $j=1,\ 2,\ 3$ and there are $n(\ge 0)$ customers in the queue excluding one customer in service irrespective of the value of x. Further, let $P_n(t)=\sum_{j=1}^{3} P_n^{(j)}(t)$ be the probability that at time t, there are *n* customers in the queue excluding one customer in service irrespective of the mode of service, $j=1,\ 2,\ 3$.

$R_n(x,t)$: Probability that at time t there are *n* ($n\ge 0$) customers in the queue excluding one customer being served by the stand-by server and the system is under repairs since time *x*

$M(t)$: Probability that at time t, the server is idle and there is no customer in the system

4. STEADY STATE EQUATIONS GOVERNING THE SYSTEM

Let

$$\lim_{t\to\infty} P_n^{(j)}(x,t)=P_n^{(j)}(x),\ \lim_{t\to\infty} P_n^{(j)}(t)=P_n^{(j)},\ \lim_{t\to\infty} P_n(t)=\sum_{j=1}^{3}\lim_{t\to\infty} P_n^{(j)}=P_n\quad j=1,\ 2,\ 3,$$

$$\lim_{t\to\infty} M(t)=M$$

denote the steady state probabilities corresponding to $P_n^{(j)}(x,t)$, $P_n^{(j)}(t)$ $j=1,\ 2,\ 3$, $P_n(t)$ and $E(t)$ respectively.

Then connecting states of the system at time $t+dt$ with those at time t and then taking limit as $t\to\infty$, we obtain the following set of steady state equations governing the system:

$$\frac{d}{dx}P_n^{(1)}(x)+(\lambda+\mu_1(x)+\alpha)P_n^{(1)}(x)=\lambda(1-a_1)P_n^{(1)}(x)+a_1\lambda\sum_{i=1}^{n}c_iP_{n-i}^{(1)}(x), \quad n>1 \tag{4.1}$$

$$\frac{d}{dx}P_0^{(1)}(x)+(a_1\lambda+\mu_1(x)+\alpha)P_n^{(1)}(x)=0 \tag{4.2}$$

$$\frac{d}{dx}P_n^{(2)}(x)+(a_1\lambda+\mu_2(x)+\alpha)P_n^{(2)}(x)=\lambda(1-a_1)P_n^{(1)}(x)+a_1\lambda\sum_{i=1}^{n}c_iP_{n-i}^{(2)}(x), \; n>1 \tag{4.3}$$

$$\frac{d}{dx}P_0^{(2)}(x)+(a_1\lambda+\mu_2(x)+\alpha)P_n^{(2)}(x)=0 \tag{4.4}$$

$$\frac{d}{dx}P_n^{(3)}(x)+(a_1\lambda+\mu_3(x)+\alpha)P_n^{(3)}(x)=\lambda(1-a_1)P_n^{(3)}(x)+a_1\lambda\sum_{i=1}^{n}c_iP_{n-i}^{(3)}(x), \; n>1 \tag{4.5}$$

$$\frac{d}{dx}P_0^{(3)}(x)+(a_1\lambda+\mu_3(x))+\alpha P_n^{(3)}(x)=0 \tag{4.6}$$

$$\begin{aligned}\frac{d}{dx}R_n(x)+(\lambda+\beta(x)+\nu)R_n(x)&=\lambda(1-a_2)R_n(x)+a_2\sum_{i=1}^{\infty}\lambda c_iR_{n-i}(x)\\ &\quad+\nu R_{n+1}(x), \qquad n\geq 1\end{aligned} \tag{4.7}$$

$$\frac{d}{dx}R_0(x)+(\lambda+\beta(x)+\nu)R_0(x)=\lambda(1-a_2)R_n(x)+\nu R_1(x) \tag{4.8}$$

$$\lambda a_1 M=\int_0^\infty P_0^{(1)}(x)\mu_1(x)dx+\int_0^\infty P_0^{(2)}(x)\mu_2(x)dx+\int_0^\infty P_0^{(3)}(x)\mu_3(x)dx \tag{4.9}$$

The equations (4.1) – (4.8) are to be solved subject to the following boundary conditions:

$$P_n^{(1)}(0)=p_1\left[\begin{aligned}&\int_0^\infty P_{n+1}^{(1)}(x)\mu_1(x)dx+\int_0^\infty P_{n+1}(x)\mu_2(x)dx\\ &\qquad+\int_0^\infty P_{n+1}^{(3)}(x)\mu_3(x)dx+\lambda a_1c_{n+1}M+\int_0^\infty R_n(x)\beta(x)dx\end{aligned}\right], \quad n\geq 0$$

(4.10)

$$P_n^{(2)}(0)=p_2\left[\begin{aligned}&\int_0^\infty P_{n+1}^{(1)}(x)\mu_1(x)dx+\int_0^\infty P_{n+1}^{(2)}(x)\mu_2(x)dx\\ &\qquad+\int_0^\infty P_{n+1}^{(3)}(x)\mu_3(x)dx+\lambda a_1c_{n+1}M+\int_0^\infty R_n(x)\beta(x)dx\end{aligned}\right], \quad n\geq 0$$

$$P_n^{(3)}(0) = p_3 \left[\begin{array}{l} \int_0^\infty P_{n+1}^{(1)}(x)\,\mu_1(x)dx + \int_0^\infty P_{n+1}^{(2)}(x)\,\mu_2(x)dx \\ \qquad + \int_0^\infty P_{n+1}^{(3)}(x)\,\mu_3(x)dx + \lambda\, a_1 c_{n+1}\, M + \int_0^\infty R_n(x)\beta(x)dx \end{array} \right], \quad n \geq 0 \tag{4.11}$$

$$R_n(0) = \alpha \int_0^\infty P_n^{(1)}(x)dx + \alpha \int_0^\infty P_n^{(2)}(x)dx + \alpha \int_0^\infty P_n^{(3)}(x)dx \tag{4.12}$$

$$= \alpha(P_n^{(1)} + P_n^{(2)} + P_n^{(3)}), \qquad n \geq 0 \tag{4.13}$$

5. STEADY STATE QUEUE SIZE DISTRIBUTION AT A RANDOM EPOCH

We define the following probability Generating Functions:

$$P^{(j)}(x,z) = \sum_{n=1}^{\infty} z^n P_n^{(j)}(x), \quad j = 1,\ 2,\ 3,$$

$$P^{(j)}(z) = \sum_{n=1}^{\infty} z^n P_n^{(j)}, \quad j = 1,\ 2,\ 3,$$

$$P(z) = \sum_{j=1}^{3} \ P^{(j)}(z),$$

$$C(z) = \sum_{i=1}^{\infty} z^i c_i,$$

$$R(z) = \sum_{n=0}^{\infty} z^n R_n(x) \tag{5.1}$$

We apply supplementary variable technique procedure to the above equations of section 4, and get the following main results

$$P^{(1)}(z) = \left(\frac{-\,p_1\,(a_1(\lambda - \lambda C(z)))M}{D(z)} \right) \left(\frac{1 - G_1^*[a_1(\lambda - \lambda C(z)) + \alpha]}{a_1(\lambda - \lambda C(z)) + \alpha} \right) \tag{5.2}$$

$$P^{(2)}(z) = \left(\frac{-\,p_2\,(a_1(\lambda - \lambda C(z)))M}{D(z)} \right) \left(\frac{1 - G_2^*[a_1(\lambda - \lambda C(z))) + \alpha]}{a_1(\lambda - \lambda C(z))) + \alpha} \right) \tag{5.3}$$

$$P^{(3)}(z)=\left(\frac{-p_3\,(a_1(\lambda-\lambda C(z)))M}{D(z)}\right)\left(\frac{1-G_3^*[a_1(\lambda-\lambda C(z)))+\alpha]}{a_1(\lambda-\lambda C(z)))+\alpha}\right)$$

(5.4)

$$R(z)=\left(\frac{N(Z)}{D(z)}\right)\left(\frac{1-B^*[a_2(\lambda-\lambda C(z)))+\nu-\frac{\nu}{z}]}{a_2(\lambda-\lambda C(z)))+\nu-\frac{\nu}{z}}\right) \tag{5.5}$$

Where $\bar{G}_j[a_1(\lambda-\lambda C(z))+\alpha]=\int_0^\infty e^{-[a_1(\lambda-\lambda C(z))+\alpha]x}.\,dG_j(x)$ is the Laplace-Stieltjes transform of the service time $G_j(x)$ in the *j* th mode of service, $j=1,\,2,\,3$ and

$\bar{B}\ \left[a_2(\lambda-\lambda C(z))+v-\frac{v}{z}\right]=\int_0^\infty e^{-\left[a_2(\lambda-\lambda C(z))+v-\frac{v}{z}\right]x}.\,dB(x)$ is the Laplace-Stieltjes transform of the repair time $B(x)$ and

$$\begin{aligned}N(z)=&-\alpha[a_1(\lambda-\lambda C(z)))+\alpha]\,p_1\left(\frac{1-G_1^*[a_1(\lambda-\lambda C(z)))+\alpha]+\alpha]}{a_1[a_1(\lambda-\lambda C(z)))+\alpha]+\alpha}\right)M\\&-\alpha\,[a_1(\lambda-\lambda C(z)))+\alpha]p_2\left(\frac{1-G_1^*[a_1(\lambda-\lambda C(z)))+\alpha]+\alpha]}{[a_1(\lambda-\lambda C(z)))+\alpha]+\alpha}\right)M\\&-\alpha[a_1(\lambda-\lambda C(z)))+\alpha]\,p_3\left(\frac{1-G_1^*[a_1(\lambda-\lambda C(z)))+\alpha]}{[a_1(\lambda-\lambda C(z)))+\alpha]+\alpha}\right)M\end{aligned} \tag{5.6}$$

$$\begin{aligned}D(z)=&\;z-p_1G_1^*[a_1(\lambda-\lambda C(z)))+\alpha]+p_2G_2^*[a_1(\lambda-\lambda C(z)))+\alpha]+p_3G_3^*[a_1(\lambda-\lambda C(z)))+\alpha]\\&-p_1\alpha\,B^*[a_2(\lambda-\lambda C(z)))+\nu-\frac{\nu}{z}]\left(\frac{1-G_1^*[a_1(\lambda-\lambda C(z)))+\alpha]}{a_1(\lambda-\lambda C(z)))+\alpha}\right)\\&-p_2\alpha\,B^*[a_2(\lambda-\lambda C(z)))+\nu-\frac{\nu}{z}]\left(\frac{1-G_2^*[a_1(\lambda-\lambda C(z)))+\alpha]}{a_1(\lambda-\lambda C(z)))+\alpha}\right)\\&-p_3\alpha\,B^*[a_2(\lambda-\lambda C(z)))+\nu-\frac{\nu}{z}]\left(\frac{1-G_3^*[a_1(\lambda-\lambda C(z)))+\alpha]}{a_1(\lambda-\lambda C(z)))+\alpha}\right)\end{aligned} \tag{5.7}$$

Now, we proceed to determine the only unknown constant *E* which appears in each of the equations (5.2) to (5.5). We define $P(z)$ to be the steady state probability generating function of the queue length irrespective of the state of the system, so that adding equations (5.2) to (5.5) we obtain

$$P(z)=P^{(1)}(z)+P^{(2)}(z)+P^{(3)}(z)+R(z)=\frac{N(z)}{D(z)}.$$

We shall use the following normalizing condition at z=1.

$$M+P^{(1)}(1)+P^{(2)}(2)+P^{(3)}(1)+R(1)=1 \tag{5.8}$$

We note that each of the probability generating functions $P^{(1)}(z),\ P^{(2)}(z),\ P^{(2)}(z)$ and $R(z)$ are of the zero/zero form. Therefore, using L'Hopital's rule we get

$$P^{(1)}(1) = \underset{z\to 1}{Lim}\ P^{(1)}(z)$$

$$= \frac{p_1\lambda a_1 E(I)\left(\frac{1-G_1^*(\alpha)}{\alpha}\right)M}{1-(\lambda a_1 E(I)+\alpha(\lambda a_2 E(I)-\nu)E(R))\left(p_1\left(\frac{1-G_1^*(\alpha)}{\alpha}\right)+p_2\left(\frac{1-G_2^*(\alpha)}{\alpha}\right)+p_3\left(\frac{1-G_3^*(\alpha)}{\alpha}\right)\right)} \tag{5.9}$$

$$P^{(2)}(1) = \underset{z\to 1}{Lim}\ P^{(2)}(z)$$

$$= \frac{p_2\lambda a_1 E(I)\left(\frac{1-G_2^*(\alpha)}{\alpha}\right)M}{1-(\lambda a_1 E(I)+\alpha(\lambda a_2 E(I)-\nu)E(R))\left(p_1\left(\frac{1-G_1^*(\alpha)}{\alpha}\right)+p_2\left(\frac{1-G_2^*(\alpha)}{\alpha}\right)+p_3\left(\frac{1-G_3^*(\alpha)}{\alpha}\right)\right)} \tag{5.10}$$

$$P^{(3)}(1) = \underset{z\to 1}{Lim}\ P^{(3)}(z) =$$

$$\frac{p_3\lambda a_1 E(I)\left(\frac{1-G_3^*(\alpha)}{\alpha}\right)M}{1-(\lambda a_1 E(I)+\alpha(\lambda a_2 E(I)-\nu)E(R))\left(p_1\left(\frac{1-G_1^*(\alpha)}{\alpha}\right)+p_2\left(\frac{1-G_2^*(\alpha)}{\alpha}\right)+p_3\left(\frac{1-G_3^*(\alpha)}{\alpha}\right)\right)} \tag{5.11}$$

$$R(1) = \underset{z\to 1}{Lim}\ R(z)$$

$$= \frac{\alpha\lambda a_1 E(I)\left(p_1\left(\frac{1-G_2^*(\alpha)}{\alpha}\right)+p_1\left(\frac{1-G_2^*(\alpha)}{\alpha}\right)+p_1\left(\frac{1-G_2^*(\alpha)}{\alpha}\right)\right)M}{1-(\lambda a_1 E(I)+\alpha(\lambda a_2 E(I)-\nu)E(R))\left(p_1\left(\frac{1-G_1^*(\alpha)}{\alpha}\right)+p_2\left(\frac{1-G_2^*(\alpha)}{\alpha}\right)+p_3\left(\frac{1-G_3^*(\alpha)}{\alpha}\right)\right)} \tag{5.12}$$

Using (5.40) to (5.43) in the normalizing condition (5.39 and simplifying we get

$$M = \frac{1-(\lambda a_1 E(I)+\alpha(\lambda a_2 E(I)-\nu)E(R))\left(p_1\left(\frac{1-G_1^*(\alpha)}{\alpha}\right)+p_2\left(\frac{1-G_2^*(\alpha)}{\alpha}\right)+p_3\left(\frac{1-G_3^*(\alpha)}{\alpha}\right)\right)}{1+\alpha(\lambda a_1 E(I)+\alpha(\lambda a_2 E(I)-\nu)E(R))\left(p_1\left(\frac{1-G_1^*(\alpha)}{\alpha}\right)+p_2\left(\frac{1-G_2^*(\alpha)}{\alpha}\right)+p_3\left(\frac{1-G_3^*(\alpha)}{\alpha}\right)\right)}. \tag{5.13}$$

Thus all the probability generating functions in the main results (5.2) to (5.5) are completely and explicitly determined. Further, the utilization factor $\rho = 1 - M$ of the system is obtained as follows:

$$\rho = \frac{(1+\alpha)\,\lambda\, a_1 E(I)\left(p_1\left(\frac{1-G_1^*(\alpha)}{\alpha}\right) + p_2\left(\frac{1-G_2^*(\alpha)}{\alpha}\right) + p_3\left(\frac{1-G_3^*(\alpha)}{\alpha}\right)\right)}{1+\alpha(\lambda\, a_1 E(I) + \alpha(\lambda\, a_2 E(I) - \nu)E(R))\left(p_1\left(\frac{1-G_1^*(\alpha)}{\alpha}\right) + p_2\left(\frac{1-G_2^*(\alpha)}{\alpha}\right) + p_3\left(\frac{1-G_3^*(\alpha)}{\alpha}\right)\right)} \tag{5.14}$$

6. SOME PARTICULAR CASES

6.1. No balking, three modes of general service times and random breakdowns with general repair times and exponential service time by the stand-by server

The results of this case can be obtained by letting $a_1 = 1 = a_2$ in the main results found above in equations (5.2) to (5.12)..

6.2. No balking, three modes of general service times and random breakdowns with general repair times and no stand-by server

The results of this case can be obtained by letting $a_1 = 1 = a_2$ and $\nu = 0$ in the main results found above in equations (5.2) to (5.12)

6.3. Balking, two modes of general service times and random breakdowns with general repair times and exponential service time by the stand-by server

The results of this case can be obtained by letting $p_3 = 0$ in the main results found above in equations (5.2) to (5.12)

6.4. Balking, one mode of general service times and random breakdowns with general repair times and exponential service time by the stand-by server

The results of this case can be obtained by letting $p_1 = 1,\ p_2 = 0 = p_3$ in the main results found above in equations (5.2) to (5.12)

6.5. No balking, one mode of general service times and no breakdowns

The results of this case can be obtained by letting $a_1 = 1 = a_2,\ p_1 = 1,\ p_2 = 0 = p_3, \alpha = 0$, $G_j^*[\alpha] = G_j^*[0] = 1$ and $\underset{\alpha \to 0}{Lim}\left(\frac{1-G_j(\alpha)}{\alpha}\right) = E(S_i),\ j = 1,2,3$ and using $p_1 + p_2 + p_3 = 1$ in the main results found above in (5.2) to (5.12). Thus we obtain

$$P^{(1)}(z) = \frac{(1-G_1^*[\lambda - \lambda C(z)])M}{G_1^*[\lambda - \lambda C(z)] - z} \tag{6.1}$$

$$P^{(2)}(z) = 0 \tag{6.2}$$

$$P^{(3)}(z) = 0 \tag{6.3}$$

$$R(z) = 0 \tag{6.4}$$

Where

$$M = 1 - \lambda E(I)(E(S_1)) \tag{6.5}$$

$$\rho = \lambda E(I)(E(S_1)) \tag{6.6}$$

We note that the results (6.1), (6.5) and (6.6) are known results (Gaver [ ]

6.6. Balking, three modes of exponential service times, random breakdowns with exponential repair times and exponential service time by the stand-by server

In this case we let $G_j^*[m] = \dfrac{\mu_j}{\mu_j + m}$, $j = 1, 2, 3$ and $B^*[k] = \dfrac{\beta}{\beta + k}$ in the main results (5.2) to (5.12) and get

$$P^{(1)}(z) = \left(\frac{-p_1(a_1(\lambda - \lambda C(z)))M}{D(z)} \right) \left(\frac{1}{\mu_1 + a_1(\lambda - \lambda C(z)) + \alpha} \right) \tag{6.7}$$

$$P^{(2)}(z) = \left(\frac{-p_2(a_1(\lambda - \lambda C(z))M}{D(z)} \right) \left(\frac{1}{\mu_2 + a_1(\lambda - \lambda C(z)) + \alpha} \right) \tag{6.8}$$

$$P^{(3)}(z) = \left(\frac{-p_3(a_1(\lambda - \lambda C(z))M}{D(z)} \right) \left(\frac{1}{\mu_3 + a_1(\lambda - \lambda C(z)) + \alpha} \right) \tag{6.9}$$

$$R(z) = \left(\frac{N(Z)}{D(z)} \right) \left(\frac{1}{\beta + a_2\left(\lambda - \lambda C(z) + \nu - \dfrac{\nu}{\nu z} \right)} \right) \tag{6.10}$$

Where

$$N(z) = -\alpha\ (a_1(\lambda - \lambda C(z))\left(\frac{p_1}{\mu_1 + a_1(\lambda - \lambda C(z)) + \alpha}\right)M$$

$$-\alpha\ (a_1(\lambda - \lambda C(z)))\left(\frac{p_2}{\mu_2 + a_1(\lambda - \lambda C(z)) + \alpha}\right)M$$

$$-\alpha\ (a_1(\lambda - \lambda C(z)))\left(\frac{p_3}{a_1(\lambda - \lambda C(z))}\right)M \tag{6.11}$$

$$D(z) = z - \left(\frac{p_1\mu_1}{\mu_1 + a_1(\lambda - \lambda C(z))} + \frac{p_2\mu_2}{\mu_2 + a_1(\lambda - \lambda C(z)) + \alpha} + \frac{p_3\mu_3}{\mu_3 + a_1(\lambda - \lambda C(z)) + \varepsilon}\right)$$

$$-\ p_1\alpha\left(\frac{\beta}{\beta + a_2\,(\lambda - \lambda z) + \nu - \frac{\nu}{z}}\right)\left(\frac{1}{\mu_1 + a_1(\lambda - \lambda C(z)) + \alpha}\right)$$

$$-\ p_2\alpha\left(\frac{\beta}{\beta + a_2\,(\lambda - \lambda C(z)) + \nu - \frac{\nu}{z}}\right)\left(\frac{1}{\mu_2 + a_1(\lambda - \lambda C(z)) + \alpha}\right)$$

$$-\ p_3\alpha\left(\frac{\beta}{\beta + a_2\,(\lambda - \lambda C(z)) + \nu - \frac{\nu}{z}}\right)\left(\frac{1}{\mu_3 + a_1(\lambda - \lambda C(z)) + \alpha}\right) \tag{6.12}$$

$$P^{(1)}(1) = \lim_{z\to 1} P^{(1)}(z)$$

$$= \frac{p_1\lambda\, a_1 E(I)\left(\frac{1}{\mu_1 + \alpha}\right)M}{1 - (\lambda\, a_1 E(I) + \alpha(\lambda\, a_2 E(I) - \nu)E(R))\left(\frac{p_1}{\mu_1 + \alpha} + \frac{p_2}{\mu_2 + \alpha} + \frac{p_3}{\mu_3 + \alpha}\right)} \tag{6.13}$$

$$P^{(2)}(1) = \lim_{z\to 1} P^{(2)}(z)$$

$$= \frac{p_2\lambda\, a_1 E(I)\left(\frac{1}{\mu_2 + \alpha}\right)M}{1 - (\lambda\, a_1 E(I) + \alpha(\lambda\, a_2 E(I) - \nu)E(R))\left(\frac{p_1}{\mu_1 + \alpha} + \frac{p_2}{\mu_2 + \alpha} + \frac{p_3}{\mu_3 + \alpha}\right)} \tag{6.14}$$

$P^{(3)}(1) = \underset{z\to 1}{Lim}\ P^{(3)}(z)$

$$= \frac{p_3\lambda a_1 E(I)\left(\dfrac{1}{\mu_3+\alpha}\right)M}{1-(\lambda a_1 E(I)+\alpha(\lambda a_2 E(I)-\nu)E(R))\left(\dfrac{p_1}{\mu_1+\alpha}+\dfrac{p_2}{\mu_2+\alpha}+\dfrac{p_3}{\mu_3+\alpha}\right)} \quad (6.15)$$

$R(1) = \underset{z\to 1}{Lim}\ R(z)$

$$= \frac{\alpha\lambda a_1 E(I)\left(\left(\dfrac{p_1}{\mu_1+\alpha}\right)+\left(\dfrac{p_2}{\mu_2+\alpha}\right)+\left(\dfrac{p_3}{\mu_3+\alpha}\right)\right)M}{1-(\lambda a_1 E(I)+\alpha(\lambda a_2 E(I)-\nu)E(R))\left(\left(\dfrac{p_1}{\mu_1+\alpha}\right)+p_2\left(\dfrac{p_2}{\mu_2+\alpha}\right)+\left(\dfrac{p_3}{\mu_3+\alpha}\right)\right)} \quad (6.16)$$

$$M = \frac{1-(\lambda a_1 E(I)+\alpha(\lambda a_2 E(I)-\nu))E(R))\left(\left(\dfrac{p_1}{\mu_1+\alpha}\right)+\left(\dfrac{p_2}{\mu_2+\alpha}\right)+\left(\dfrac{p_3}{\mu_3+\alpha}\right)\right)}{1+\alpha(\lambda a_1 E(I)+\alpha(\lambda a_2 E(I)-\nu))E(R))\left(\left(\dfrac{p_1}{\mu_1+\alpha}\right)+\left(\dfrac{p_2}{\mu_2+\alpha}\right)+\left(\dfrac{p_3}{\mu_3+\alpha}\right)\right)}. \quad (6.17)$$

$$\rho = \frac{(1+\alpha)\lambda a_1 E(I)\left(\left(\dfrac{p_1}{\mu_1+\alpha}\right)+\left(\dfrac{p_2}{\mu_2+\alpha}\right)+\left(\dfrac{p_3}{\mu_3+\alpha}\right)\right)}{1+\alpha(\lambda a_1 E(I)+\alpha(\lambda a_2 E(I)-\nu)E(R))\left(\left(\dfrac{p_1}{\mu_1+\alpha}\right)+\left(\dfrac{p_2}{\mu_2+\alpha}\right)+\left(\dfrac{p_3}{\mu_3+\alpha}\right)\right)} \quad (6.18)$$

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