OPTIMAL INTEGRATED INVENTORY POLICY FOR STOCK-DEPENDENT DEMAND WHEN TRADE CREDIT IS LINKED TO ORDER QUANTITY

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ABSTRACT
The classical EOQ model is developed under the assumption that buyer must settle the payment due immediately when units are received in the inventory system. To attract the buyers, the vendor uses promotional tool viz permissible delay period when the buyer's order quantity is more than pre-specified quantity. In this paper, we analyze integrated inventory policy for vendor-buyer when demand is stock-dependent and trade credit is linked to order quantity. The joint total profit is maximized to determine buyer's order quantity and the number of shipments from the vendor to the buyer during one cycle. Numerical examples and sensitivity analysis are given to find critical inventory parameters. Managerial insights are also obtained.

KEYWORDS: Integrated Inventory Model, Stock-Dependent Demand, Trade-Credit Linked to Order Quantity

MSC: 90B05

RESUMEN
El clásico modelo EOQ se desarrolla bajo el supuesto de que el comprador deberá realizar el pago inmediatamente cuando las unidades se reciben en el sistema de inventario. Para atraer a los compradores, el vendedor utiliza el período de la promoción a saber period permisiblede demoracuando la cantidad de ordenes del comprador , está másque pre-especificada. En este trabajo se analiza la política de inventario integrado para el proveedor-comprador cuando la demanda depende de la existencia y el crédito commercial estávinculado a cantidad solicitada. El resultado ganancia total se maximize paradeterminar la cantidad de pedidos del comprador y el número de envos del vendedor al comprador durante un ciclo. Ejemplos numéricos y análisis de sensibilidad se dan paraencontrar los parámetros críticos de inventario. Conocimientos gerenciales se obtienen también.

1. INTRODUCTION

In business transactions, the offer of credit period to the buyer is considered to be sales promotional tool for the vendor. Goyal (1985) was first to determine an EOQ model with a constant demand rate under the condition of permissible delay in payments. Thereafter, several researchers examined the inventory models to obtain more insights into trade credit. One can refer review article on trade credit by Shah et al. (2012). The most of the cited references considered that credit is given for any order quantity.

In practice, vendor may offer delay period to settle the account to attract buyer to order more units. Khouja and Mehrez (1996) developed the vendor credit policies to determine optimum order quantity where credit terms are linked to order quantity. Chang *et al*. (2003, 2009) studied effect of deterioration of units in above model. The research articles by Chang (2004), Shin and Hwang (2003), Chung *et al*. (2005), Chung and Liao (2004, 2006), Shah and Shukla (2010, 2011) Shah et al. (2010), Shah (2010), deals with offer of trade credit linked to order quantity.

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The above stated models are discussed either from buyer's or vendor's point of view. Here, the dominant player takes the decision which is to be followed by the other player. Globalization of the business realizes the need of developing a win-win strategy for the buyer and vendor. Goyal (1976) developed a single-vendor single-buyer integrated inventory model. Banerjee (1986) assumed a lot-for-lot shipment policy for vendor in Goyal (1976). Goyal (1988) relaxed the lot-for-lot policy and established that the inventory cost reduces significantly if vendor's economic production quantity is an integral multiple of the buyer's purchase quantity. Many Researchers Lu (1995), Goyal (1995), Viswanathan (1998), Hill (1997,1999), Kelle *et al.* (2003), Yang and Wee (2003) established that more batching and frequent shipment policies are advantageous for the integrated inventory models.

Levin *et.al.* (1972) quoted that "large piles of goods attract more customers". This is termed as stock-dependent demand. Urban (2005) computed optimal order quantity when demand is stock-dependent. Roy and Chaudhari (2006) formulated inventory polices for finite planning horizon and incorporated shortages. Roy and Chaudhari (2012) discussed EPQ model for price-sensitive stock-dependent demand to study the effect of deterioration on the objective function.

In this paper, we analyze an integrated single-vendor single buyer inventory model when the demand rate is stock-dependent, the production rate is finite and proportional to the demand rate and trade credit is permitted only if buyer orders more units than the pre-specified order quantity by the buyer. The joint total profit per unit time is maximized with respect to order quantity and number of shipments from vendor to the buyer. A computational procedure is outlined to find the best optimal solution. The numerical examples and sensitivity analysis are given to validate the developed model.

2. NOTATION AND ASSUMPTIONS

2.1 Notation

$R\ I\ t$ Stock-dependent demand rate; $\alpha + \beta I(t)$ where $\alpha > 0$ is constant scale demand and $0 < \beta < 1$ is stock-dependent parameter.

A_v Vendor's setup cost per set-up

A_b Buyer's ordering cost per order

C_p Production cost per unit

C_b Buyer's purchase cost per unit

s The unit retail price to customers, where $s > C_b > C_p$

I_v Vendor's inventory holding cost rate per unit per annum, excluding interest charges

I_b Buyer's inventory holding cost rate per unit per annum, excluding interest charges

I_{vp} Vendor's opportunity cost /\$/ unit time

I_{bp} Buyer's opportunity cost /\$/ unit time

I_{be} Buyer's interest earned /\$/ unit time

ρ Capacity utilization which is ratio of demand to the production rate; $\rho < 1$ and known

M Allowable credit period for the buyer offered by the vendor

Q Buyer's order quantity per order (a decision variable)

Q_d Pre specified order quantity to qualify for offer of trade credit

T Cycle time (a decision variable)

T_d The time length when Q_d - units are depleted to zero

n Number of shipments from vendor to the buyer (a decision variable)

TVP Vendor's total profit per unit time

TBP Buyer's total profit per unit time

π (Sum of TVP and TBP) joint total profit per unit time

2.2 ASSUMPTIONS

The following assumptions are made in deriving the proposed model.

1. The supply chain under consideration comprises of single-vendor and single-buyer for a single product.
2. Lead-time is zero. Shortages are not allowed.
3. The buyer qualifies for trade credit offer if order is equal or larger than the pre-specified quantity Q_d by the vendor. Otherwise, the buyer must use cash on delivery strategy.
4. During the credit period, the buyer earns interest at the rate I_{be} per unit on the generated revenue. At the end of the credit period the buyer settles the payments due against the purchase made and incurs an opportunity cost at a rate of I_{bp} for unsold items in stock.

3. MATHEMATICAL MODEL

In this section, we develop an integrated inventory model when demand is stock-dependent and trade credit is only offered if buyer's order quantity is equal or greater than a pre-specified quantity.

3.1 VENDOR'S TOTAL PROFIT PER UNIT TIME

The total profit per unit time for the vendor comprises of sales revenue, set-up cost, holding cost and opportunity cost as follows:

(1) Sales revenue: The total sales revenue per unit time is $(C_b - C_p)\frac{Q}{T}$. (See Appendix A for computation of Q)

(2) Set-up cost: nQ-units are manufactured in one production run by the vendor. Therefore, the set-up cost per unit time is $\frac{A_v}{nT}$.

(3) Holding cost: Using Joglekar (1988), the vendor's average inventory per unit time is
$$\frac{C_p(I_v + I_{vp})\left[(n-1)(1-\rho)+\rho\right]}{\beta^2 T}\alpha\left(e^{\beta T} - \beta T - 1\right).$$

(4) Opportunity cost: If Q_d or more units are ordered by the buyer, the credit period of M – units is permissible to settle the account. In this scenario, vendor endures a capital and payment received. Equivalently, when $T \geq T_d$, the delay in payment is permissible and corresponding opportunity cost per unit time is $\frac{C_b I_{vp} QM}{T}$. On the other hand, when $T < T_d$ the vendor receives payments on deliver and so no opportunity cost will occur.

Hence, the total profit per unit time for the vendor is

$$TVP(n) = \begin{cases} TVP_1(n), T < T_d \\ TVP_2(n), T \geq T_d \end{cases} \quad (1)$$

where

$$TVP_1(n) = \frac{(C_b - C_p)Q}{T} - \frac{A_v}{nT} - \frac{C_p(I_v + I_{Vp})\left[(n-1)(1-\rho)+\rho\right]\alpha\left(e^{\beta T} - \beta T - 1\right)}{\beta^2 T} \quad (2)$$

$$TVP_2\ \ n\ = \frac{C_b - C_p\ \ Q}{T} - \frac{A_v}{nT} - \frac{C_p(I_v + I_{Vp})\left[\ n-1\ \ 1-\rho\ +\rho\right]\alpha\ \ e^{\beta T} - \beta T - 1}{\beta^2 T}$$
$$- \frac{C_b\, I_{vp}\, QM}{T}$$
(3)

3.2 Buyer's total profit per unit time

The total profit per unit time for the buyer comprises of sales revenue, ordering cost, holding cost, opportunity cost and interest earned. These costs are computed as follows:

(1) Sales revenue: The total sales revenue per unit time is $s - C_b\ \ \frac{Q}{T}$. (See Appendix A for computation of Q)

(2) Ordering cost: The ordering cost per unit time is $\frac{A_b}{nT}$.

(3) Holding cost: The buyer's holding cost (excluding interest charges) per unit time is $\frac{C_b\, I_b\, \alpha\ \ e^{\beta T} - \beta T - 1}{\beta^2\, T}$.

(4) Opportunity cost: Based on the lengths of T, M and T_d, the following four cases arises (i) $0 < T < T_d$ (ii) $T_d \le T \le M$ (iii) $T_d \le M \le T$ (Fig. 1) and (iv) $M \le T_d \le T$
The cases (iii) and (iv) are similar.

Opportunity cost per unit time

$$= \begin{cases} \frac{C_b\, I_{bp}\, Q}{T} & ,0 < T < T_d \\ 0 & ,T_d \le T \le M \\ \frac{C_b\, I_{bp}}{\beta^2 T}\alpha\ \ e^{\beta\ T-M}\ \ - \beta(T-M)-1 & ,T_d \le M \le T\ or\ M \le T_d \le T. \end{cases}$$

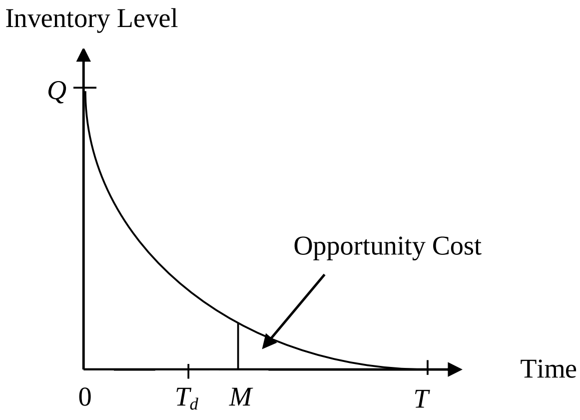

Figure.1 Opportunity cost for $T_d \le M \le T$ or $M \le T_d \le T$

Interest earned: As discussed in opportunity cost interest earned per unit time in all the four cases is as follows.

Interest earned per unit time

$$=\begin{cases} 0, \quad 0 < T < T_d \quad \text{because payment is to be made on delivery} \\ \dfrac{sI_{be}}{T}\left(\int_0^T R\ \ I\ \ t\ \ \ tdt + Q\ \ M - T\ \ \right), T_d \le T \le M \quad \text{figure 2} \\ \dfrac{sI_{be}}{T}\left(\int_0^M R\ \ I\ \ t\ \ \ tdt\right), T_d \le M \le T \ \ or\ M \le T_d \le T \quad \text{figure 3} \end{cases}.$$

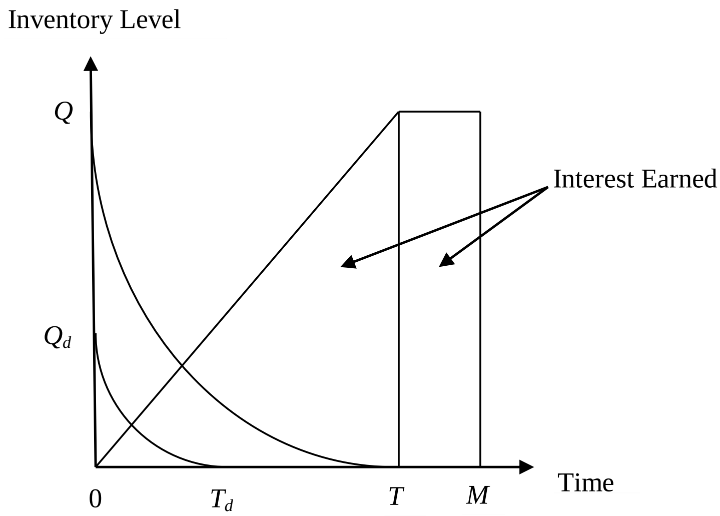

Figure.2 Interest earned by buyer when $T_d \le T \le M$

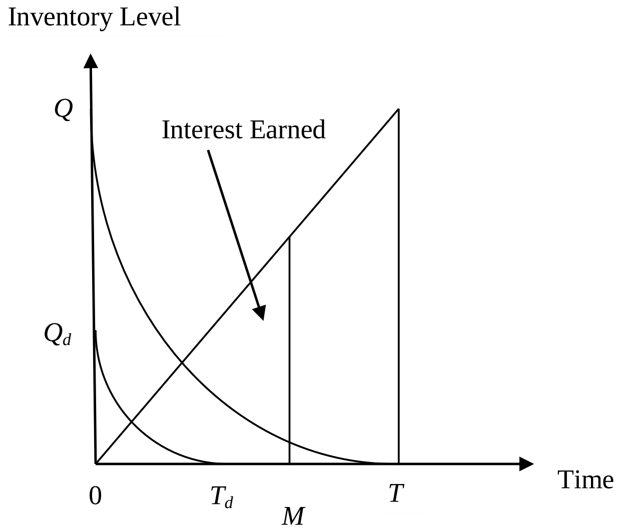

Figure.3 Interest earned by buyer when $T_d \le M \le T$

Hence, the buyer's total profit per unit time is

$$TBP(T) = \begin{cases} TBP_1(T), 0 < T < T_d \\ TBP_2(T), T_d \le T \le M \\ TBP_3(T), T_d \le M \le T \\ TBP_4(T), M \le T_d \le T \end{cases} \tag{4}$$

;where

$$TBP_1(T) = \frac{(s-C_b)Q}{T} - \frac{A_b}{T} - \frac{C_b I_b \alpha}{\beta^2 T}\left(e^{\beta T} - \beta T - 1\right) - \frac{C_b I_{bp} Q}{T} \tag{5}$$

$$TBP_2(T) = \frac{(s-C_b)Q}{T} - \frac{A_b}{T} - \frac{C_b I_b \alpha}{\beta^2 T}\left(e^{\beta T} - \beta T - 1\right) + \frac{sI_{be}}{T}\left(\int_0^T R(I(t))dt + Q(M-T)\right) \tag{6}$$

$$TBP_3(T) = TBP_4(T) = \frac{(s-C_b)Q}{T} - \frac{A_b}{T} - \frac{C_b I_b \alpha}{\beta^2 T}\left(e^{\beta T} - \beta T - 1\right)$$

$$-\frac{C_b I_{bp}\alpha}{\beta^2 T}\left(e^{\beta(T-M)} - \beta(T-M) - 1\right) + \frac{sI_{be}\alpha}{\beta^2 T}\left(e^{\beta T} - (1+\beta M)e^{\beta(T-M)}\right) \tag{7}$$

3.3 Joint total profit per unit time

In integrated system, the vendor and buyer decide to take joint decision which maximize the profit of the supply chain. The joint total profit per unit time for the integrated system is

$$\pi(n,T) = \begin{cases} \pi_1(n,T) = TVP_1(n) + TBP_1(T), 0 < T < T_d \\ \pi_2(n,T) = TVP_2(n) + TBP_2(T), T_d \le T \le M \\ \pi_3(n,T) = TVP_2(n) + TBP_3(T), T_d \le M \le T \\ \pi_4(n,T) = TVP_2(n) + TBP_3(T), M \le T_d \le T. \end{cases} \tag{8}$$

where

$$\pi_1(n,T) = (s - C_p - C_b I_{bp})\left(\frac{Q}{T} - \frac{\bar{A}}{T} - \frac{1}{T}(\phi+\psi)\alpha\left(e^{\beta T} - \beta T - 1\right)\right) \tag{9}$$

$$\pi_2(n,T) = \left(s - C_p - (C_b I_{vp} - sI_{be})M\right)\frac{Q}{T} - sI_{be}Q - \frac{\bar{A}}{T}$$

$$-\frac{1}{T}(\phi+\psi)\alpha\left(e^{\beta T} - \beta T - 1\right) + \frac{sI_{be}}{T}\int_0^T R(I(t))dt \tag{10}$$

$$\pi_3(n,T) = (s - C_p - C_b I_{vp} M)\left(\frac{Q}{T} - \frac{\bar{A}}{T} - \frac{1}{T}(\phi+\psi)\alpha\left(e^{\beta T} - \beta T - 1\right)\right)$$

$$-\frac{C_b I_{bp}\alpha}{\beta^2 T}\left(e^{\beta(T-M)} - \beta(T-M) - 1\right) + \frac{sI_{be}\alpha}{\beta^2 T}\left(e^{\beta T} - (1+\beta M)e^{\beta(T-M)}\right) \tag{11}$$

$$\bar{A} = A_b + \frac{A_v}{n}$$

$$\phi = \frac{C_p\ I_v + I_{vp}\ \left[\ n-1\ \ 1-\rho\ +\rho\right]}{\beta^2}$$

$$\psi = \frac{C_b I_b}{\beta^2}$$

4. COMPUTATIONAL PROCEDURE

For fixed *T*, we note that $\pi\ \ n,T$ is a concave function of *n* because $\frac{\partial^2 \pi\ \ n,T}{\partial n^2} = -\frac{2A_v}{n^3 T} < 0$. Therefore to find optimum number of shipments n^* we will have a local optimal solution. The optimum value of cycle time can be obtained by setting $\frac{\partial \pi}{\partial T} = 0$ for fixed *n*.

Algorithm:

Step 1: Set parametric values.

Step 2: Compute T_d using $\frac{1}{\beta} ln\left(1+\frac{\beta Q_d}{\alpha}\right)$ for given value of Q_d.

Step 3: Set *n* =1.

Step 4: Knowing T_d and *M*, compute *T* by solving $\frac{\partial \pi_j}{\partial T} = 0$ for $j = 1,2,3.$

Step 5: Find corresponding profit π_j for $j = 1,2,3.$

Step 6: Increment *n* by 1.

Step 7: Repeat step 4 and 6 until $\pi\ \ n-1,T\ \ n-1\ \ \le \pi\ \ n,T\ \ n\ \ \ge \pi\ \ n+1,T\ \ n+1$.

Once the optimal solution n^*,T^* is obtained, the optimal order quantity can be obtained.

5. NUMERICAL EXAMPLES AND INTERPRETATIONS

Example 1 Consider,
$\alpha = 10000$ units, $\beta = 10\%$, $\rho = 0.7$, $C_b = \$10$ / unit, $C_p = \$5$ / unit, $A_v = \$400$ / setup, $A_b = \$50$ / order, $I_v = 10\%$ /unit/annum, $I_b = 10\%$/unit/annum, $I_{bp} = 8\%$/unit/annum, $I_{be} = 5\%$ /\$/annum, $I_{vp} = 2\%$/unit/annum, $s = \$25$/unit and $M = 30$days.

Table 1 Optimal solutions for different Q_d

Q_d	Q^*	n^*	T^* **(days)**	**Profit($)**		
				Buyer	**Vendor**	**Joint**
1000	1369	5	49.65	150150	48809	198959
2000	1369	5	49.65	150150	48809	198959
3000	3000	2	124.46	150051	49074	199125
4000	4000	2	124.46	150051	49073	199124
5000	1369	5	49.65	150150	48809	198959
6000	1369	5	49.65	150150	48809	198959

The optimal shipments and ordering units with buyer, vendor and joint profit for different values of Q_d are exhibited in Table 1.
From Table 1, it is seen that the vendor's total profit and joint total profit of the system increase with increase in Q_d and then further increase in pre-specified units lower their profits whereas for the buyer, it is opposite trend. It is seen that the buyer's optimal order quantity Q^* is equal to Q_d when and less than Q_d when $Q_d \geq 5000$. Thus, vendor is advised to set threshold which is effective. If the threshold set by the vendor is too high, the buyer will be reluctant to order a quantity greater than the threshold to take advantage of delayed payments.

Example 2 Consider the data given in Example 1. We study the effect of delayed payments for $Q_d = 3000$ units.

Table 2 Optimal solutions for different M $Q_d = 3000$

M (days)	Q^*	n^*	T^* (days)	Case	Profit($) Buyer	Vendor	Joint
20	3000	2	126.36	$M \leq T_d \leq T$	149796	49136	198932
30	3000	2	124.46	$M \leq T_d \leq T$	150051	49074	199125
40	3000	2	121.63	$M \leq T_d \leq T$	150319	49008	199327
50	3000	2	117.81	$M \leq T_d \leq T$	150603	48938	199541
60	3000	2	112.89	$M \leq T_d \leq T$	150904	48863	199767

From Table 2, it is observed that longer credit period increases buyer's total profit and joint profit of the supply chain. The longer credit period reduces vendor's total profit because payment will be received late for the purchases made.

Example 3. In this example, we carry out sensitivity analysis to find the critical inventory parameters. The changes in the optima cycle time, purchase quantity and joint profit is studied by varying inventory parameters as $-20\%, -10\%, 10\%$ and 20%. The results are exhibited in Figure 4.

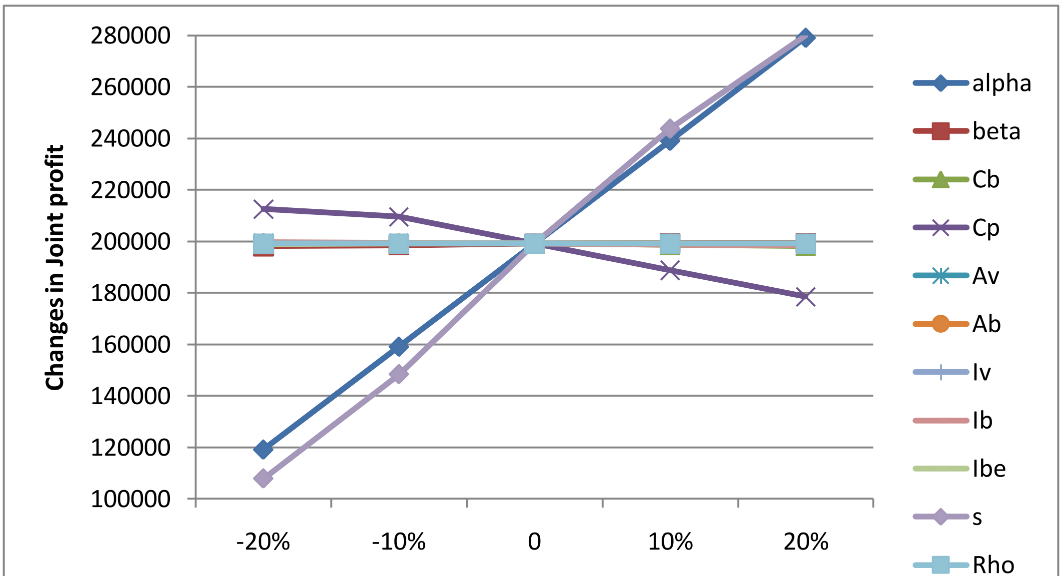

Figure. 4 Percentage change in inventory parameters

It is observed that joint profit has significant positive impact of scale demand and retail price set by the buyer. It is evident that both the player should take advantage of demand increase and setting agreeable selling price. Production cost of supplier reduced joint total profit. It is advised to the supplier to use advanced technology which reduces this production cost. Other inventory parameters have very small contribution in increasing profit of the supply chain.

Example 4. In table 4, we compare independent *Vs* joint decision, for pre-specified quantity $Q_d = 3000$ units at which buyer qualities for getting delay period facility.

Table 4 Optimal Solution of independent and scenario

Scenario			Buyer	Vendor	Joint
Independent	Total Shipments	4			
	Ordering Quantity	1548			
	Cycle Time (days)	56			
	Total Annual Profit ($)		150205	48828	199033
Integrated	Total Shipments	2			
	Ordering Quantity	3000			
	Cycle Time (days)	124			
	Total Annual Profit ($)		150050	49074	199124
	Readjusted Total Annual Profit ($)		150274	48850	199124

where

$$\text{Buyer's profit} = \pi\ n,T \ \times \frac{TBP\ \ P,T}{\left[TBP\ \ P,T\ \ + TVP\ \ n\ \right]}$$

$$= 199124 \times \frac{150205}{(150205 + 48828)} = \$\ 150274$$

$$\text{Supplier's profit} = \pi\ n,T \ \times \frac{TVP\ \ n}{\left[TBP\ \ P,T\ \ + TVP\ \ n\ \right]}$$

$$= 199124 \times \frac{48828}{(150205 + 48828)} = \$\ 48850$$

Table 4 shows that the total annual profit under joint decision \$199125 (= \$150051+ \$49074) which is greater than the total profit under independent decision \$199033 (= \$150205+\$48828). It establishes that joint decision is advantageous to both the players. The last row of table 4 is about readjustment of the profits (Goyal (1976)) to encourage players for joint decision.

5. CONCLUSION

An integrated inventory policy comprising of single-vendor single-buyer is studied when demand is stock-dependent and credit terms are linked to order quantity. The computational procedure is outlined to optimize joint total profit per unit time with respect to number of shipments from the vendor to the buyer and cycle time when vendor's stock depletes to zero.

Based on the results, it is observed that joint profit for the supply chain increases in joint decision compared to independent decision but reduces that of the buyer. To attract the buyer for the joint decision vendor should set proper threshold to offer credit period.

In future, one can study optimum threshold for the vendor to study inventory polices. Our model will be worth if variants like deterioration, floor constraints *etc.* are incorporated.

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APPENDIX A

The rate of change of inventory at any instant of time can be discussed by differential equation

$$\frac{dI(t)}{dt} = -\left(\alpha + \beta I(t)\right), 0 \le t \le T$$

with $I(0) = Q$ and $I(T) = 0$. Using $I(T) = 0$, the solution of the differential equation is

$I(t) = \frac{\alpha}{\beta}\left[e^{\beta(T-t)} - 1\right], \ 0 \le t \le T.$ The units to be purchased $Q = I(0) = \frac{\alpha}{\beta}\left[e^{\beta T} - 1\right]$.