NON DIFFERENTIALBE PERTURBED NEWTON's METHOD FOR FUNCTIONS WITH VALUES IN A CONE

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ABSTRACT

This paper deals with variational inclusions of the form $0 \in f(x) + g(x) - K$ where f is smooth function from a reflexive Banach space X into a Banach space Y, g is a Lipschitz function from X into Y and K is a nonempty closed convex cone in the space Y. We show that the previous problem can be solved by an extension of the Zincenko's method which can be seen as a perturbed Newton's method. Numerical results are given at the end of the paper.

KEYWORDS: Variational inclusion, Set–valued map, pseudo-Lipschitz map, metric regularity, closed convex cone, normed convex process, Zincenko's iteration.

MSC: 90C59

RESUMEN

En este trabajo se tratan inclusiones variacionales de la forma $0 \in f(x) + g(x) - K$, donde f es una función suave de un espacio de Banach reflexivo X en un espacio de Banach Y, g es una función Lipschitz de X en Y, y K es un cono no vacío convexo y cerrado en el espacio Y. Se muestra que el problema anterior puede resolverse mediante una extensión del método de Zincenko, también visto como un método de Newton perturbado. Al final del trabajo se muestran algunos resultados numéricos.

1. INTRODUCTION

The variational inclusions were introduced by Robinson 1979, 1982[17, 18] as an abstract model for various problems encountered in fields such as mathematical programming, engineering, optimal control, economy, transport theory, etc.

Many variational inclusions appearing in the literature can be written on the form

$$0 \in f(x) + F(x). \tag{1.1}$$

where $f : X \to Y$, is a function , $F : X \rightrightarrows Y$ is a set-valued map and X, Y are Banach spaces.

In the smooth case (f is smooth), an interesting contribution to approximate a solution x^* of (1.1) is Dontchev's 1996a, 1996b[3, 4] work in which he introduced a sequence obtained with a partial linearization of the univoque part. He associated to (1.1) the Newton-type sequence

$$\begin{cases} x_0 \text{ is a given starting point} \\ 0 \in f(x_k) + f'(x_k)(x_{k+1} - x_k) + F(x_{k+1}). \end{cases} \tag{1.2}$$

and established the quadratic convergence when f' (the Fréchet derivative of f) is locally Lipchitz around the solution x^* under a pseudo-Lipschitz property for the set-valued map $(f+F)^{-1}$. For more on the Lipschitz property, also called Aubin property or Lipschitz like property, the reader could refer to Aubin 1984, [2], Aubin-Frankowska 1990, [1], Dontchev 2006, [5], Dontchev-Rockafellar 2009, [6], Mordukhovich 1993, [12], Mordukhovich 2006, [13], Rockafellar 1984, [21], Rockafellar-Wets 1998, [22]. We observe in Dontchev works, that the construction of the sequence (x_k) depends on a fixed point theorem given in Dontchev-Hager 1994, [7].

It seems natural to examinate the non differentiable case and we suppose that the univoque part of equation (1.1) can be written as the sum of a smooth function with a non differentiable perturbation and we study the following variational inclusion

$$0 \in f(x) + g(x) + F(x). \tag{1.3}$$

where $f : X \to Y$, is a smooth function , where $g : X \to Y$, is a Lipschitz function on a neighbourhood $\Omega \subset X$ of x^* solution of (1.3) and $F : X \rightrightarrows Y$.

Let us notice that inclusion (1.3) can be viewed as a perturbed problem associated to inclusion (1.1). Following Dontchev's method, the authors in Geoffroy-Piétrus 2004, [8] associated to (1.3) the sequence

$$\begin{cases} x_0 \text{ is a given starting point} \\ 0 \in f(x_k) + f'(x_k)(x_{k+1} - x_k) + g(x_k) + F(x_{k+1}) \end{cases} \tag{1.4}$$

which is the classical Zincensko 1963, [23] iteration method when the set-valued map F is the set $\{0\}$. In [8] as in [3], the authors obtained existence and convergence result for (1.4) under some metric regularity condition and using the fixed point theorem given in [7] .

Following Dontchev's works, we can find in the literature various contributions using metric regularity in the generalization to variational inclusions of the methods existing for solving classical equations. However the metric regularity is a very interesting tool for obtaining theoretical results (existence of sequences, rate of convergence, etc.) in the field of variational inclusions, it becomes a concept which is very difficult to be verified in practices that constitutes the main drawback. In addition, the methods resulting from the use of metric regularity assumptions (around the solution which is not known) furnish generally local convergence sequences.

The purpose here is to approximate a solution x^* of (1.3), in the case where $F : X \rightrightarrows Y$ is a non empty closed convex cone in the Banach space Y. For simplicity, F will be denoted by $-K$ in the rest of the paper. For this, we introduce a new algorithm and starting with a point x_0 and if x_k is computed, the new iterate x_{k+1} appears as a solution of a minimization problem. In addition, we show the convergence of this algorithm under classical conditions, observing that the metric regularity concept is not nedeed.
Let us remark that the method which is given in this paper has a semi-local convergence, the generated sequence converges to some x^* which is solution of (1.3).
In the next section, we recall some preliminaries on different concepts as normed convex processes and majorizing sequences . In Section 3, we introduce and describe the new algorithm for solving (1.3). Section 4 is devoted to our main theorem and its proof and finally in Section 5 we give some numerical results.

2. PRELIMINARIES

In this section, we collect some results that we will need to prove our main result.

2.1. Normed convex processes

Definition 2.1 *A mapping T from the real linear space X to the real linear space Y is a convex process if it satisfies*
a) $T(x) + T(z) \subset T(x+z)$ *for all* $x, z \in X$
b) $T(\lambda x) = \lambda T(x)$ *for every* $\lambda > 0$ *and every* $x \in X$
c) $0 \in T(0)$.

In fact, a convex process from X to Y is a mapping defined on X into subsets of Y, whose graph is a convex cone in $X \times Y$ containing the origin. If the graph is closed, then we refer to a closed convex process.
The idea of convex processes has been introduced by Rockafellar 1967,1970, [19, 20] and this concept has been clearly formalized and studied by Robinson in Robinson 1972,[16]. The following definitions and results come from Robinson's paper. For a convex process T, we define respectively the domain noted *dom*, the range and the inverse by : *dom* T is the set of points x for which $T(x) \neq \emptyset$, range T is $\bigcup\{T(x), x \in dom\ T\}$ and the inverse noted T^{-1} is a mapping from range T onto *dom* T with $T^{-1}(y) = \{x, y \in T(x)\}$.
Note that *dom* T and range T are both convex cones containing 0 and the inverse which always exists is itself a convex process. Finally if X and Y are normed spaces, we can define the norm of T by

$$\| T \| := \sup\{\inf\{\| y \|; y \in T(x)\}, \| x \| \leq 1, \quad x \in dom\ T, \}$$

and we shall call a convex process normed if its norm is finite.

Now we are going to give a theorem which is useful for the proof of our main theorem.

Theorem 2.1 [16] *Let X be a Banach space and Y a normed linear space. Let T and Δ be convex processes from X into Y; denote dom T by D and range T by R. Assume that T, T^{-1} and Δ are normed, and that $\| T^{-1} \| . \| \Delta \| < 1$. Suppose further that $K \subset$ dom Δ, $\Delta(D) \subset R$, D is closed, and $(T-\Delta)(x)$ is closed for each $x \in D$. Then the convex process $T - \Delta$ has the following properties :*

- *range $T \subset$ range $(T-\Delta)$,*
- $(T-\Delta)_R^{-1}$ *is a normed convex process, and* $\| (T-\Delta)_R^{-1} \| \leq \dfrac{\| T^{-1} \|}{(1 - \| T^{-1} \| . \| \Delta \|)}$.
and so closed.

2.2. Majorizing sequences

The following definitions, properties and examples come from Rheinboldt's paper Rheinboldt 1968,[14]. The so-called concept of majorizing sequences has been introduced by Kantorovich in Kantorovich 1949, [10]. Throughout this paper appears a new proof of semilocal convergence of Newton's method (for nonlinear equations) based on this concept.

Definition 2.2 *Let (x_k) be a sequence in the metric space (X, ρ). Then a real nonnegative sequence (t_n) is said to majorize (x_k) if*

$$\rho(x_{k+1}, x_k) \leq t_{k+1} - t_k \qquad k = 0, 1, \ldots .$$

It is easy to observe that any majorizing sequence (t_k) of (x_k) is necessarily nondecreasing and for $m > k \geq 0$,

$$\rho(x_m, x_k) \leq \sum_{j=k}^{m-1} \rho(x_{j+1}, x_j) \leq \sum_{j=k}^{m-1} (t_{j+1} - t_j) = t_m - t_k.$$

Hence, if $\lim t_k = t^* < +\infty$ exists, then (x_k) is a Cauchy sequence in X; therefore, if X is complete, $\lim x_k = x^*$ also exists, and for $m \to +\infty$, the error estimate is immediate and $\rho(x^*, x_k) \leq t^* - t_k$ for $k = 0, 1, \ldots$.

For the majorizing principle to be useful, some estimations are necessary to obtain a majorizing sequence (t_k) for a given sequence (x_k). Thus, this principle requires appropriate assumptions, either about the generating mechanism of the sequence (x_k) or at least about the relation between the successive iterates of the sequence (x_k). For more results and specific studies the reader could refer to Rheinboldt's paper [14] and for our study we use this important lemma.

Lemma 2.1 [14] *Consider the sequence (t_k) defined by*

- *i)* $t_0 = 0, \quad t_1 = \alpha,$
- *ii)* $t_{k+1} - t_k = \dfrac{1}{1 - p_4 t_k}\Big[p_1(t_k - t_{k-1})^2 + (p_2 + p_3 t_{k-1})(t_k - t_{k-1})\Big]$, *for* $k = 1, 2, \ldots$ *with* $0 \leq p_4 \alpha < 1$, $p_i \geq 0$ *and* $i = 1, \ldots, 4$.

If for some parameter set (p_i^0, α^0) the solution (t_k^0) satisfies $t_k^0 \leq t_{k+1}^0$, $k \geq 0$, and $\lim t_k^0 = t^{0*} < 1/p_4$, then for any (p_i, α) with $0 \leq p_i \leq p_i^0$, $i = 1, \ldots, 4$, $0 \leq \alpha \leq \alpha^0$, thus the sequence (t_k) is nondecreasing and $\lim t_k = t^* \leq t^{0*}$. If $p_1 > 0$, $0 \leq p_2 < 1$, $\quad p_3 + p_4 = 2p_1$ and $0 < \alpha \leq (1 - p_2)^2/4p_1$, then the sequence (t_k) is strictly increasing and

$$\lim t_k = t^* = \frac{1}{2p_1}\Big[(1 - p_2) - \sqrt{(1 - p_2)^2 - 4p_1\alpha}\Big].$$

3. DESCRIPTION OF THE ALGORITHM

Let us consider a subset $X_0 \subset X$. For any fixed $w \in X_0$, we define a set valued mapping $T(w)$ from X to Y by

$$T(w)x := f'(w)x - K, \qquad x \in X.$$

It is easy to see that $T(w)$ is a normed convex process from X to Y. Its inverse is defined as follows, for any $y \in Y$ by

$$T^{-1}(w)y := \{z \in X, f'(w)z \in y + K\}$$

is also a normed convex process.
Given a starting point $x_0 \in X_0$ such that $T^{-1}(x_0)[-f(x_0) - g(x_0)] \neq \emptyset$, we define x_1 as the sum of x_0 and a projection of the origin in X on $T^{-1}(x_0)[-f(x_0) - g(x_0)]$. Then we repeat the procedure using x_1 as starting point. At the k^{th} step, we have x_k and we define x_{k+1} as the sum of x_k and a projection of the origin in X on $T^{-1}(x_k)[-f(x_k) - g(x_k)]$.
It is easy to see that an equivalent way to write the algorithm is, if x_k is already computed, the point x_{k+1} appear to be any solution of the minimization problem

$$minimize\{\parallel x - x_k \parallel / f(x_k) + g(x_k) + f'(x_k)(x - x_k) \in K\}. \tag{3.1}$$

The algorithm could be summerized in this way:

ALGORITHM Newton-Cone(f, g, K, x_0, ϵ).

Step 0. if $T^{-1}(x_0)[-f(x_0) - g(x_0)] = \emptyset$ terminate with failure.

Step 1. Do while $e > \epsilon$.

- Choose x as the solution of the problem
  $minimize\{\| x - x_0 \| \,/ f(x_0) + g(x_0) + f'(x_0)(x - x_0) \in K\}$.
- $e :=\| x - x_0 \|$; $x_0 = x$.

Step 2. Return x.

Remark 3.1 *If $K = \{0\}$ the procedure is exactly the same given in 1963 by Zincenko in [23].*

Remark 3.2 *If instead of the cone K, we consider a general set-valued mapping F depending on x, we cannot conclude with our method. If we want to consider x_{k+1} as the solution of the problem given only by the constraint of (3.1) replacing K by F, we have some analogy with the method introduced by Pietrus-Geoffroy 2004, [9]. But the problem studied in their paper doesn't ensures the unicity of the iterate x_{k+1} when starting with x_k, thus the numerical treatment is not easy. Moreover, the convergence result obtained is only local under a metric regularity assumption.*

Remark 3.3 *The continuity of the linear operator $f'(x_k)$ (Frechet derivative) and the fact that K is closed and convex , imply that the feasible set of (3.1) is a closed convex set for all $k \in \mathbb{N}^*$. Then the existence of a feasible point $\tilde{x}$ implies that any solution of (3.1) must lie in the intersection of the feasible set of (3.1) with the closed ball of center x_k and radius $\| \tilde{x} - x_k \|$. Since X is reflexive and the function $\| x - x_k \|$ is weakly lower semicontinuous, a solution of (3.1) exists (see Levi 1966,[11]). Then it is clear that if (3.1) is feasible then it is solvable and its convexity implies that any local solution will be global.*

4. CONVERGENCE ANALYSIS OF THE ALGORITHM

In this section we give the main result of our paper.

Theorem 4.1 *Let X, X_0, Y, K, f g and T be as previously defined.*
Suppose that there exists a point $x_0 \in X_0$, such that $T(x_0)$ carries X onto Y, and there are real numbers B, L, l and η satisfying the following properties:

- *(a) $\|T^{-1}(x_0)\| \leq B$.*
- *(b) For each $x, y \in X_0$, we have*
$$\|f'(x) - f'(y)\| \leq L\|x - y\|,$$
$$\|g(x) - g(y)\| \leq l\|x - y\|.$$
- *(c) $\|x_1 - x_0\| \leq \eta \leq \min\Big(\frac{1}{BL}, \frac{(1-Bl)^2}{2BL}\Big)$, where x_1 is obtained from x_0 by the algorithm (such a point exists by the remarks in the previous section).*

- *(d) Let*

$$t^* = \frac{1}{BL}\Big((1-Bl) - \sqrt{(1-Bl)^2 - 2BL\eta}\Big).$$

If $B(x_0,t^*) \subset X_0$, then the algorithm (3.1) generates a sequence (x_k) such that for all $k \geq 0$, x_k remains in $B(x_0,t^*)$ and converges to some x^* such that $f(x^*)+g(x^*) \in K$.

Proof: We can observe that if for any $k \in I\!N$ we set

$$A_k = \Big(f(x_{k+1})+g(x_{k+1})-f(x^*)-g(x^*)\Big) - \Big(f(x_{k+1})-f(x_k)-f'(x_k)(x_{k+1}-x_k)+g(x_{k+1})-g(x_k)\Big)$$

we get $A_k \in K - f(x^*) - g(x^*)$. If the sequence (x_k) converges to x^*, we obtain by continuity that $(A_k) \to 0$.

Since $K - f(x^*) - g(x^*)$ is closed, we obtain $f(x^*)+g(x^*) \in K$ and then x^* solves our problem.

So, the rest of the proof will be focused in proving that $x_k \to x^*$ which will be done by showing that there exists a convergent sequence (t_k) which majorizes (x_k), which means that (x_k) is a Cauchy sequence and converges to some $x^* \in X_0$.

We follow by induction showing that the sequence (x_k) satisfies (3.1), remains in $B(x_0,t^*)$ and there exists a nondecreasing sequence (t_k) satisfying

$$\|x_{k+1}-x_k\| \leq t_{k+1}-t_k, \qquad \forall k \in I\!N. \tag{4.1}$$

The third assumption gives the existence of a point x_1 which solves (3.1) for $k=0$. Moreover by setting $t_1 := \eta$ and $t_0 := 0$ we obtain $\|x_1 - x_0\| \leq t_1 - t_0 = \eta$ and our result is checked for $k=0$.

Let us suppose now that we have obtained $x_1,\dots,x_k$ from the algorithm given by (3.1) and $t_0,t_1,\dots,t_k$ such that

$$\|x_j - x_{j-1}\| \leq t_j - t_{j-1}, \qquad \forall j = 1,\dots k.$$

Thus, this implies that the quantities $x_0,x_1,\dots,x_k$ belong to the set $B(x_0,t^*)$.

The first assumption of the theorem allows us to obtain this important following information

$$\|T^{-1}(x_0)\|.\|f'(x_j)-f'(x_0)\| \leq BL\|x_j - x_0\| \leq BLt^* < 1.$$

We also have

$$T(x_j)x = f'(x_j)x - K = \Big(f'(x_0) + (f'(x_j)-f'(x_0)\Big)x - K = (T(x_0)-\Delta_j)(x)$$

with $\Delta_j = -f'(x_j) + f'(x_0)$.
In this case, the application of Theorem 2.1 allows us to obtain that $T(x_j)$ carries X onto Y, $T^{-1}(x_j)$ is normed and

$$\|T^{-1}(x_j)\| \leq \frac{\|T^{-1}(x_0)\|}{1-\|T^{-1}(x_0)\|.\|f'(x_j)-f'(x_0)\|} \leq \frac{B}{1-BL\|x_j-x_0\|}, \quad \forall j=1,\dots,k.$$

The fact that $T(x_j)$ carries X onto Y implies that (3.1) is feasible, hence solvable for $j = k$ and we obtain the existence of x_{k+1} which solve (3.1).

Now let us consider the problem which consists to find a x which is solution of the problem

$$f(x_k) + g(x_k) + f'(x_k)(x - x_k) \in f(x_{k-1}) + f'(x_{k-1})(x_k - x_{k-1}) + g(x_{k-1}) + K. \qquad (4.2)$$

We can observe that since x_k solve (3.1) then the right-hand side of (4.2) in contained in cone K and we can conclude that any x satisfying (4.2) is necessarily feasible for (3.1).
We can rewrite (4.2) in the following way

$$-f(x_k) - g(x_k) + f(x_{k-1}) + g(x_{k-1}) + f'(x_{k-1})(x_k - x_{k-1}) \in f'(x_k)(x - x_k) - K,$$

and we observe that the left-hand side of the previous inclusions is exactly $T(x_k)(x - x_k)$.
Thus we can obtain x as the solution of the following inclusion

$$x - x_k \in T^{-1}(x_k)\Big(- f(x_k) - g(x_k) + f(x_{k-1}) + g(x_{k-1}) + f'(x_{k-1})(x_k - x_{k-1})\Big). \qquad (4.3)$$

Since the right-hand of (4.3) contains an element of least norm , we can find some $\tilde{x}$ satisfying (4.3) (and also (4.2))which satisfies:

$$\|\tilde{x} - x_k\| \leq \|T^{-1}(x_k)\|\Big(\| - f(x_k) + f(x_{k-1}) + f'(x_{k-1})(x_k - x_{k-1})\| + \|g(x_k) - g(x_{k-1})\|\Big).$$

The Lipschitz assumptions on f' and g and the estimation obtained for $\|T^{-1}(x_j)\|$ $(j = 1, \dots, k)$ yields

$$\|\tilde{x} - x_k\| \leq \frac{B}{1 - BL\|x_k - x_0\|}\Big(\frac{L}{2}\|x_k - x_{k-1}\|^2 + l\|x - x_k\|\Big).$$

Using the induction assumption, we obtain

$$\|\tilde{x} - x_k\| \leq \frac{B}{1 - BLt_k}\Big(\frac{L}{2}(t_k - t_{k-1})^2 + l(t_k - t_{k-1})\Big).$$

The right-hand side of the last inequality invite us to consider the sequence defined by

$$t_{k+1} - t_k = \frac{B}{1 - BLt_k}\Big(\frac{L}{2}(t_k - t_{k-1})^2 + l(t_k - t_{k-1})\Big),$$

with $t_0 = 0$ and $t_1 = \eta$.
We can apply the Lemma 2.1 setting $p_1 = \frac{BL}{2}$; $p_2 = Bl$; $p_3 = 0$; $p_4 = BL$; $\alpha = \eta$ where B, L, l,and η are previously defined and then conclude that the sequence (t_k) is strictly increasing and converges to

$$t^* = \frac{1}{BL}\Big(1 - Bl - \sqrt{(1 - Bl)^2 - 2BL\eta}\Big).$$

With the help of this sequence we obtain

$$\|x_{k+1} - x_k\| \leq \|\tilde{x} - x_k\| \leq t_{k+1} - t_k.$$

In other words the sequence (x_k) is majorized by the sequence (t_k) and the proof is acheived.

5. NUMERICAL RESULTS

In this part, we consider two examples in finite dimension where X is taken to be $I\!R^2$, $Y = I\!R^3$ and $K = I\!R^2_- \times \{0\}$. The numerical experiments have been performed with a processor "Intel ® core i7" using the Maple Sofware that facilitated the treatment of the minimization problem.

Example 5.1. Let us consider the system:

$$\begin{cases} x_1^2 + x_2^2 - |x_1 - 0.5| - 1 \le 0 \\ x_1^2 + (x_2 - 1)^2 - |x_1 - 0.5| - 1 \le 0, \\ (x_1 - 1)^2 + (x_2 - 1)^2 - 1 = 0 \end{cases} \tag{9}$$

We can remark that the point $x^* = (\frac{1}{2}, 1 - \frac{1}{2}\sqrt{3})$ is one of the solution of the system.

For applying our algorithm, we take :

$$f(x) := \big(x_1^2 + x_2^2 - 1, x_1^2 + (x_2 - 1)^2 - 1, (x_1 - 1)^2 + (x_2 - 1)^2 - 1\big),$$

$$g(x) := \big(-|x_1 - 0.5|, -|x_1 - 0.5|, 0\big),$$

and $x_0 = (0.55, 0.1)$ for the guess point.

In the table M_k is the value of the minimization problem at the step k.

step k	$\|x_k - x^*\|_\infty$	M_k
0	5×10^{-2}	
1	2.5×10^{-2}	10^{-3}
4	3.12×10^{-3}	1.3×10^{-5}
7	1.95×10^{-4}	5.08×10^{-8}
10	4.88×10^{-5}	3.18×10^{-9}
13	6.1×10^{-6}	4.97×10^{-11}
16	15.3×10^{-6}	3.1×10^{-12}
17	7.63×10^{-7}	7.76×10^{-13}
21	4.77×10^{-8}	3.03×10^{-15}

Table 1: *Solutions of (9) starting from* $x_0 = (0.55, 0.1)$.

Example 5.2. Let us consider, the system which has been given by Robinson in Robinson 1972,[15].

$$\begin{cases} x_1^2 + x_2^2 - 1 \le 0 \\ x_1^2 + (x_2 - 1)^2 - 1 \le 0, \\ (x_1 - 1)^2 + (x_2 - 1)^2 - 1 = 0 \end{cases} \tag{10}$$

The solution set is a circular arc centered at $(1, 1)$ and with endpoints at $(\frac{1}{2}, 1 - \frac{1}{2}\sqrt{3})$ and $(0, 1)$. Since the data are smooth, it is convenient to take

$$f(x) := \big(x_1^2 + x_2^2 - 1, x_1^2 + (x_2 - 1)^2 - 1, (x_1 - 1)^2 + (x_2 - 1)^2 - 1\big).$$

and

$$g(x) := \big(0, 0, 0\big).$$

If the guess point is $(0.55, 0.1)$, we obtain the following table.

step k	$\|x_k - x^*\|_\infty$	M_k
0	5×10^{-2}	
1	2.03×10^{-2}	3.5×10^{-3}
2	2.37×10^{-6}	4.11×10^{-6}
3	3.25×10^{-2}	5.63×10^{-12}
4	6.11×10^{-24}	1.05×10^{-23}

Table 2*: Solutions of (10) starting from* $x_0 = (0.55, 0.1)$.

For this example, the convergence is very fast with a good accuracy and is conform to the results founded by Robinson in 1974.

We remark that in the case of smooth data ($g = 0$) our method is the same than the Newton's method introduced by Robinson in [15], thus the convergence is quadratic.
In the nonsmooth case, it is not possible to apply the Newton's method introduced by Robinson and our method seems to have a linear convergence. We hope to obtain superlinear convergence in the semi-smooth case in a forth'coming paper.

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