ON AN IMPROVED RATIO TYPE ESTIMATOR OF FINITE POPULATION MEAN IN SAMPLE SURVEYS

A K P C Swain
Former Professor of Statistics, Utkal University, Bhubaneswar-751004, India

ABSTRACT
In this paper an alternative ratio type exponential estimator is suggested and is compared with Bahl and Tuteja's ratio type exponential estimator and classical ratio estimator as regards bias and mean square error with large sample approximations both theoretically and with numerical illustration.

KEYWORDS: Simple random sampling, ratio type estimators, Bias, Mean square error.

MSC: 62D05

RESUMEN
En este trabajo se sugiere un estimador del tipo razón- exponencial alternativo y se compara con el de Bahl y Tuteja el de razón clásico, considerando sesgo y error cuadrático medio, bajo aproximaciones basadas en muestras grandes y haciendo ilustraciones teóricas y numéricas.

1. INTRODUCTION:

The use of auxiliary information dates back to year 1934 , when Neyman used it for stratification of the finite population. Cochran(1940) used auxiliary information in estimation procedure and proposed ratio method of estimation to provide more efficient estimator of the population mean or total compared to the simple mean per unit estimator under certain conditions when the auxiliary variable has positive correlation with the study variable in question.

Let $U = (U_1, U_2 ... U_N)$ be the finite population of size N. To each unit U_i $(i=1, 2..., N)$ in the population paired values (y_i, x_i) corresponding to study variable y and an auxiliary variable x, correlated with y are attached.

Now, define the population means of the study variable y and auxiliary variable x as

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i, \quad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Thus, the population ratio is defined as $R = \frac{\bar{Y}}{\bar{X}}$

Further, define the finite population variances of y and x and their covariance as $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$,

$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$, and $S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})$, respectively.

Thus, the squared coefficients of variation of y and x and their coefficient of covariation are respectively defined by C_{20}, C_{02} and C_{11} ,where

$$C_{rs} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{Y})^r (x_i - \bar{X})^s, r, s = 0,1,2,...$$

Define ρ as the correlation coefficient between y and x in the population.

A simple random sample 's' of size n is selected from U without replacement and the values (y_i, x_i), i=1, 2 … n are observed on the sampled units. Assume that y and x are positively correlated. The simple mean $\bar{y}$ is an unbiased estimator of the population mean $\bar{Y}$ and variance of $\bar{y}$ is given by

$$Var(\bar{y}) = \theta \bar{Y}^2 C_{20}$$

The classical ratio estimator of the population mean $\bar{Y}$,using auxiliary information on x is given by

$$\bar{y}_R = \frac{\bar{y}}{\bar{x}}\bar{X}, \tag{1.1}$$

where $\bar{y}$ and $\bar{x}$ are sample means of *y* and *x* respectively, defined by $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$.

The ratio estimator $\bar{y}_R$ envisages advance knowledge of $\bar{X}$. The bias(B) and mean square error (MSE)of $\bar{y}_R$ to $O(1/n)$ are given by(see Sukhatme and Sukhatme,1970)

$$B(\bar{y}_R) = \theta\bar{Y}(C_{02} - C_{11}) \tag{1.2}$$

$$MSE(\bar{y}_R) = \theta\bar{Y}^2(C_{20} + C_{02} - 2C_{11}) \tag{1.3}$$

where

$$\theta = \frac{N-n}{(N-1)n} \ , \ C_{rs} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{Y})^r (x_i - \bar{X})^s, r, s = 0,1,2$$

The ratio estimator $\bar{y}_R$ is a biased estimator and the bias decreases with increase in sample size.

In large samples $\bar{y}_R$ is more efficient than the simple mean per unit estimator $\bar{y}$ if $k = \rho\sqrt{\frac{C_{20}}{C_{02}}} > \frac{1}{2}$

or if $\rho > \frac{1}{2}$ when $C_{20} = C_{02}$.

Bahl and Tuteja(1991) have suggested a ratio type exponential estimator given by

$$\bar{y}_{BTR} = \bar{y}\exp(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}) \tag{1.4}$$

To $O(1/n)$,the bias and mean square error of $\bar{y}_{BTR}$ are given by

$$B(\bar{y}_{BTR}) = \theta\bar{Y}(\frac{3}{8}C_{02} - \frac{1}{2}C_{11}) \tag{1.5}$$

$$MSE(\bar{y}_{BTR}) = \theta\bar{Y}^2(C_{20} + \frac{1}{4}C_{02} - C_{11}) \tag{1.6}$$

$\bar{y}_{BTR}$ is more efficient than $\bar{y}_R$ and $\bar{y}$ if $\frac{1}{4} < k < \frac{3}{4}$, (Bahl and Tuteja,1991):

Upadhyaya,Singh,Chatterjee and Yadav (2011) have shown that to $O(1/n^2)$,Bahl and Tuteja's(1991) ratio type exponential estimator is more efficient than the classical ratio estimator $\bar{y}_R$ when y and x follow bivariate normal distribution , coefficients of variation of y and x are equal and

$$0 < \rho < \frac{3}{4}.$$

In the following we suggest a ratio type estimator with square root transformation and compare it with the Bahl-Tuteja's (1991) ratio type exponential estimator and classical ratio estimator both theoretically and with the help of some natural and artificial populations.

2. A RATIO TYPE ESTIMATOR USING SQUARE ROOT TRANSFORMATION

Define the ratio type estimator using square root transformation as

$$\bar{y}_{SQR} = \bar{y}(\frac{\bar{X}}{\bar{x}})^{1/2} \tag{2.1}$$

2.1. Bias and Mean square error of $\bar{y}_{SQR}$

Write $\bar{y} = \bar{Y}(1+e_0)$ and $\bar{x} = \bar{X}(1+e_1)$ with $E(e_0) = E(e_1) = 0$, $V(e_0) = \theta C_{20}$, $V(e_1) = \theta C_{02}$ and $Cov(e_o, e_1) = \theta C_{11}$.

Expanding $\bar{y}_{SQR}$ in power series under the assumption that $|e_1| < 1$ for all possible samples and retaining terms up to fourth degree in e_0 and e_1 ,we have

$$\bar{y}_{SQR} = \bar{Y} + \bar{Y}(e_0 - \frac{1}{2}e_1 + \frac{3}{8}e_1^2 - \frac{15}{48}e_1^3 + \frac{105}{384}e_1^4 - \frac{1}{2}e_0e_1 + \frac{3}{8}e_0e_1^2 - \frac{15}{48}e_0e_1^3) \tag{2.2}$$

$$(\bar{y}_{SQR} - \bar{Y})^2 = \bar{Y}^2[\ (e_0^2 + \frac{1}{4}e_1^2 - e_0e_1)\ + e_0^2e_1^2\ +$$

$$+\frac{5}{4}e_0e_1^2 - e_0^2e_1 - \frac{33}{24}e_0e_1^3 - \frac{3}{8}e_1^3 + \frac{87}{192}e_1^4\ ] \tag{2.3}$$

Thus,

$$B(\bar{y}_{SQR}) = \bar{Y}[\ E(e_0) - \frac{1}{2}E(e_1) + \frac{3}{8}E(e_1^2) - \frac{15}{48}E(e_1^3) + \frac{105}{384}E(e_1^4)$$

$$-\frac{1}{2}E(e_0e_1) + \frac{3}{8}E(e_0e_1^2) - \frac{15}{48}E(e_0e_1^3)\ ] \tag{2.4}$$

$$MSE(\bar{y}_{SQR}) = E(\bar{y}_{SQR} - \bar{Y})^2 = \bar{Y}^2\left[E(e_0^2) + \frac{1}{4}E(e_1^2) - E(e_0e_1)\right]$$

$$+\bar{Y}^2\left[E(e_0^2e_1^2) + \frac{5}{4}e_0e_1^2 - E(e_0^2e_1) - \frac{33}{24}E(e_0e_1^3) - \frac{3}{8}E(e_1^3) + \frac{87}{192}E(e_1^4)\right] \tag{2.5}$$

Now,

$$E(e_1^4) = \frac{N-n}{n^3}\left[\frac{N^2+N-6nN+6n^2}{(N-1)(N-2)(N-3)}C_{04} + \frac{3N(N-n-1)(n-1)}{(N-1)(N-2)(N-3)}C_{02}^2\right]$$

$$E(e_0e_1^3) = \frac{N-n}{n^3}\left[\frac{N^2+N-6nN+6n^2}{(N-1)(N-2)(N-3)}C_{13} + \frac{3N(N-n-1)(n-1)}{(N-1)(N-2)(N-3)}C_{11}C_{02}\right]$$

$$E(e_0^2e_1^2) = \frac{N-n}{n^3}\left[\frac{N^2+N-6nN+6n^2}{(N-1)(N-2)(N-3)}C_{22} + \frac{N(N-n-1)(n-1)}{(N-1)(N-2)(N-3)}(C_{20}C_{02} + 2C_{11}^2\right]$$

$$E(e_1^3) = \frac{(N-n)(N-2n)}{(N-1)(N-2)n^2}C_{03}$$

$$E(e_0e_1^2) = \frac{(N-n)(N-2n)}{(N-1)(N-2)n^2}C_{12} \qquad E(e_0^2e_1) = \frac{(N-n)(N-2n)}{(N-1)(N-2)n^2}C_{21}$$

where

$$C_{rs} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{Y})^r(x_i - \bar{X})^s / \bar{Y}^r\bar{X}^s \quad , \ (r, s = 1,2,3,4).$$

(See Sukhatme ,Sukhatme and Asok (1984))

Hence,

$$B(\bar{y}_{SQR}) = \bar{Y}\frac{N-n}{N-1}\frac{1}{n}\left[(\frac{3}{8}C_{02}-\frac{1}{2}C_{11})\right]$$
$$+\bar{Y}\frac{N-n}{N-1}\frac{1}{n}\left[\frac{A}{n}(\frac{3}{8}C_{12}-\frac{15}{48}C_{03})+\frac{B}{n^2}(\frac{105}{384}C_{04}-\frac{15}{48}C_{13})\right.$$
$$\left.+\frac{3D}{n^2}(\frac{105}{384}C_{02}^2-\frac{15}{48}C_{11}C_{02})\ \right] \tag{2.6}$$

$$MSE(\bar{y}_{SQR}) = \bar{Y}^2\frac{N-n}{N-1}\frac{1}{n}\left[(C_{20}+\frac{1}{4}C_{02}-C_{11})+\frac{1}{n}A(\frac{5}{4}C_{12}-C_{21}-\frac{3}{8}C_{03})\right] \tag{2.7}$$

$$+\bar{Y}^2\frac{N-n}{N-1}\frac{1}{n}\left[\frac{1}{n^2}B(\frac{87}{192}C_{04}+C_{22}-\frac{33}{24}C_{13})+\frac{1}{n^2}D(C_{20}C_{02}+2C_{11}^2-\frac{33}{8}C_{11}C_{02}+\frac{87}{64}C_{02}^2)\right]$$

where

$$A=\frac{N-2n}{N-2} \qquad B=\frac{N^2+N-6nN+6n^2}{(N-2)(N-3)} \qquad D=\frac{N(N-n-1)(n-1)}{(N-2)(N-3)}$$

2.2. Bias and Mean square error of $\bar{y}_{BTR}$

Expanding $\bar{y}_{BTR}$ in power series with the same assumptions used in expanding $\bar{y}_{SQR}$ and retaining terms up to and including degree four in e_0 and e_1 ,we have

$$\bar{y}_{BTR}-\bar{Y}=\bar{Y}(e_0-\frac{e_1}{2}+\frac{3}{8}e_1^2-\frac{13}{48}e_1^3+\frac{73}{384}e_1^4-\frac{e_0e_1}{2}+\frac{3}{8}e_0e_1^2-\frac{13}{48}e_0e_1^3)$$
$$(\bar{y}_{BTR}-\bar{Y})^2=\bar{Y}^2(e_0^2+\frac{1}{4}e_1^2-e_0e_1+\frac{5}{4}e_0e_1^2-e_0^2e_1-\frac{3}{8}e_1^3+\frac{79}{192}e_1^4+e_0^2e_1^2-\frac{31}{24}e_0e_1^3)$$
$$B(\bar{y}_{BTR}) = \bar{Y}\frac{N-n}{N-1}\frac{1}{n}\left[(\frac{3}{8}C_{02}-\frac{1}{2}C_{11})\right]$$

$$+\bar{Y}\frac{N-n}{N-1}\frac{1}{n}\left[\frac{A}{n}(\frac{3}{8}C_{12}-\frac{13}{48}C_{03})+\frac{B}{n^2}(\frac{73}{384}C_{04}-\frac{13}{48}C_{13})\right.$$
$$\left.+\frac{D}{n^2}(\frac{73}{128}C_{02}^2-\frac{13}{16}C_{02}C_{11})\ \right] \tag{2.8}$$

$$MSE(\bar{y}_{BTR})=E(\bar{y}_{BTR}-\bar{Y})^2=\bar{Y}^2E(e_0^2+\frac{1}{4}e_1^2-e_0e_1)$$
$$+\bar{Y}^2E(\frac{5}{4}e_0e_1^2-e_0^2e_1-\frac{3}{8}e_1^3+\frac{79}{192}e_1^4+e_0^2e_1^2-\frac{31}{24}e_0e_1^3)\ =\bar{Y}^2\frac{N-n}{N-1}\frac{1}{n}\left[(C_{20}+\frac{1}{4}C_{02}-C_{11})+\frac{1}{n}A(\frac{5}{4}C_{12}-C_{21}-\frac{3}{8}C_{03})+\right.$$
$$\left.\frac{1}{n^2}B(\frac{79}{192}C_{04}+C_{22}-\frac{31}{24}C_{13})+\frac{1}{n^2}D(C_{20}C_{02}+2C_{11}^2-\frac{31}{8}C_{11}C_{02}+\frac{79}{64}C_{02}^2)\ \right] \quad (2.9))$$

2.3. Bias and Mean Square Error of $\bar{y}_R$

We find (see Sukhatme , Sukhatme and Asok,1984) to second order approximation

$$B(\bar{y}_R)=\bar{Y}\frac{N-n}{N-1}\frac{1}{n}(C_{02}-C_{11})+\bar{Y}\frac{N-n}{N-1}\frac{1}{n}\left[A\frac{1}{n}(C_{12}-C_{03})+B\frac{1}{n^2}(C_{04}-C_{13})+3D\frac{1}{n^2}(C_{02}^2-C_{02}C_{11})\right] \quad (2.10)$$

$$MSE(\bar{y}_R)=\bar{Y}^2\frac{N-n}{N-1}\frac{1}{n}\left[(C_{20}+C_{02}-2C_{11})+\frac{2}{n}A(2C_{12}-C_{21}-C_{03})\right]$$

$$+\bar{Y}^2\frac{N-n}{N-1}\frac{1}{n}\left[\frac{3}{n^2}B(C_{04}+C_{22}-2C_{13})+\frac{3}{n^2}D(C_{20}C_{02}+2C_{11}^2-6C_{11}C_{02}+3C_{02}^2)\right] \quad (2.11)$$

Under the assumption of bivariate normality of (y,x) and equality of coefficients variation of y and x ,i.e. $C_{20}=C_{02}$ it may be shown that

$$MSE(\bar{y}_R)=\bar{Y}^2\frac{C_{02}}{n}\left[2(1-\rho)+\frac{C_{02}}{n}(12-18\rho+6\rho^2)\right] \quad (2.12)$$

$$MSE(\bar{y}_{BTR})=\bar{Y}^2\frac{C_{02}}{n}\left[(\frac{5}{4}-\rho)+\frac{C_{02}}{n}(\frac{143}{64}-\frac{31}{8}\rho+2\rho^2)\right] \quad (2.13)$$

$$MSE(\bar{y}_{SQR})=\bar{Y}^2\frac{C_{02}}{n}\left[(\frac{5}{4}-\rho)+\frac{C_{02}}{n}(\frac{151}{64}-\frac{33}{8}\rho+2\rho^2)\right] \quad (2.14)$$

3. COMPARISON OF EFFICIENCY

(i) Considering terms up to $O(1/n)$,

$$MSE(\bar{y}_R)=\bar{Y}^2\theta(C_{20}+C_{02}-2C_{11})$$

$$MSE(\bar{y}_{SQR})=MSE(\bar{y}_{BTR})=\bar{Y}^2\theta(C_{20}+\frac{1}{4}C_{02}-C_{11})$$

$$B=B(\bar{y}_R)=\bar{Y}\ \theta(C_{02}-C_{11})$$

$$B_1=B(\bar{y}_{SQR})=B(\bar{y}_{BTR})=\bar{Y}\ \theta(\frac{3}{8}C_{02}-\frac{1}{2}C_{11})$$

Now,

$$MSE(\bar{y}_R)-MSE(\bar{y}_{SQR})$$
$$=MSE(\bar{y}_R)-MSE(\bar{y}_{BTR})$$
$$=\bar{Y}^2\theta(\frac{3}{4}C_{02}-C_{11})$$

Thus, to $O(1/n)$, $\bar{y}_{BTR}$ and $\bar{y}_{SQR}$ are equally efficient. They are more efficient than $\bar{y}_R$

if $\dfrac{C_{11}}{C_{02}}<\dfrac{3}{4}$

or $k<\dfrac{3}{4}$,

and more efficient than $\bar{y}$ if $k>\dfrac{1}{4}$.

Hence , $\bar{y}_{SQR}$ and $\bar{y}_{BTR}$ are more efficient than both $\bar{y}_R$ and $\bar{y}$ if $\frac{1}{4}<k<\frac{3}{4}$.

Further, to $O(1/n)$ the biases of $\bar{y}_{SQR}$ and $\bar{y}_{BTR}$ are equal and further they are less biased than $\bar{y}_R$ if k is either greater than 5/4 or less than 11/12..

(ii) To $O(1/n^2)$,for large N,

$$MSE(\bar{y}_{SQR}) - MSE(\bar{y}_{BTR}) = \left[(\frac{87}{192} - \frac{79}{192})E(e_1^4) - (\frac{33}{24} - \frac{31}{24})E(e_0 e_1^3) \right] = \bar{Y}^2 \frac{1}{4n^2}(\frac{1}{2}C_{02}^2 - C_{11}C_{02})$$

Hence, $\bar{y}_{SQR}$ will be more efficient than $\bar{y}_{BTR}$ to $O(1/n^2)$

$$\text{if } \frac{1}{2}C_{02}^2 - C_{11}C_{02} < 0$$

$$\text{or } \frac{C_{11}}{C_{02}} > \frac{1}{2}$$

$$\text{or } \rho > \frac{1}{2} \text{ when } C_{20} = C_{02}$$

Under bivariate normality of y and x with the assumption that $C_{20} = C_{02}$ and the correlation coefficient between y and x equal to ρ we have

$$MSE(\bar{y}_{BTR}) - MSE(\bar{y}_R) = \frac{1}{n}\bar{Y}^2 C_{02}\left[(\rho - \frac{3}{4}) - \frac{C_{02}}{n}(\frac{625}{64} - \frac{113}{8}\rho + 4\rho^2) \right]$$

(see Upadhyaya et al,2011 and correct the coefficient of ρ as -113/8 instead of -103/8)

$$MSE(\bar{y}_{SQR}) - MSE(\bar{y}_R) = \frac{1}{n}\bar{Y}^2 C_{02}\left[(\rho - \frac{3}{4}) - \frac{C_{02}}{n}(\frac{617}{64} - \frac{111}{8}\rho + 4\rho^2) \right]$$

If $\rho < \frac{3}{4}$, both $MSE(\bar{y}_{BTR}) - MSE(\bar{y}_R)$ and $MSE(\bar{y}_{SQR}) - MSE(\bar{y}_R)$ are negative.Hence, to $O(1/n^2)$ the sufficient condition that both $\bar{y}_{BTR}$ and $\bar{y}_{SQR}$ are more efficient than $\bar{y}_R$ is that

$$0 < \rho < \frac{3}{4}$$

Further , $\bar{y}_{SQR}$ will be more efficient than $\bar{y}_{BTR}$ if

$$\rho > \frac{1}{2}$$

Hence, $\bar{y}_{SQR}$ will be more efficient than $\bar{y}_{BTR}$ and $\bar{y}_R$ if

$$\frac{1}{2} < \rho < \frac{3}{4}$$

To sum up under the assumptions of bivariate normality of y and x and the equality of coefficients of variation of y and x ,the preferred estimators for ranges of ρ re shown below in Table-1.

Table-1 Comparison of Estimators

ρ	**Preferred estimator**
$0 - \frac{1}{4}$	$\bar{y}$
$\frac{1}{4} - \frac{1}{2}$	$\bar{y}_{BTR}$
$\frac{1}{2} - \frac{3}{4}$	$\bar{y}_{SQR}$

$\frac{3}{4}-1$	$\bar{y}_R$

4. NUMERICAL ILLUSTRATION:

Illustration 1

Consider 12 natural populations described in Table-2 to compare the ratio type estimator with square root transformation estimator with classical ratio estimator as regards bias and mean square error.Bias and mean square error have been computed excepting the common constants(Table-3):

Table-2 Natural Populations

Pop no.	*Description*	N	y	x	ρ_{yx}	C_x	C_y
1	*Sampford (1962)*	17	Acreage under oats in 1957	Acreage of crops and grass in 1947	0.4	0.22	0.45
2	*Singh and Chaudhary(1986)*	16	Area under wheat !979-80	Total cultivated area during 1978-79	0.96	0.74	0.69
3	*Konijn(1973)*	16	Food expenditure	Total expenditure	0.95	0.08	0.11
4	*Murthy(1967)*	16	Area under winter paddy(in acres)	Geographical area (in acres)	0.29	0.09	0.47
5	*Singh and Chaudhary(1986)*	17	No. of milch animals in survey1977-78	No. of milch animals in census1976	0.72	0.02	0.01
6	*Murthy(1967)*	16	Output for factories(000Rs)	Fixed capital(000Rs.)	0.84	0.15	0.09
7	*Panse and Sukhatme(1967)*	16	Progeny mean(mm)	Parental plant value(mm)	0.68	0.07	0.06
8	*Panse and Sukhatme(1967)*	20	Parental plot mean(mm)	Parental plant value(mm)	0.0.56	0.07	0.04
9	*Singh and Chaudhary((1986)*	15	Area under wheat in 1973	Area under wheat in 1971	0.23	0.82	0.67
10	*Panse and Sukhatme(1967)*	25	Parental plot mean(mm)	Parental plant value(mm)	0.54	0.07	0.03
11	*Panse and Sukhatme(1967)*	20	Progeny mean(mm)	Parental plant value(mm)	0.68	0.07	0.05
12	*Swain(2003)*	19	No. of milch cows in 1957	No. of milch cows census 1956	0.72	1.14	1.12

Table 3 Comparison of Bias and Mean square Error

Pop. no	$\frac{\lvert Bias(\bar{y}_{SQR})\rvert}{\theta\bar{Y}}$	$\frac{\lvert Bias(\bar{y}_R \rvert}{\theta\bar{Y}}$	$\frac{MSE(\bar{y}_{SQR})}{\theta\bar{Y}^2}$	$\frac{MSE(\bar{y}_R)}{\theta\bar{Y}^2}$	$\frac{MSE(\bar{y}_R)}{MSE(\bar{y}_{SQR})}100$
1	0.34	0.18	0.174912	0.171524	98.06
2	0.07	0.11	0.125636	0.048972	38.98
3	0.28	0.31	0.005316	0.001732	32.58
4	0.38	0.51	0.210694	0.204538	97.08
5	0.20	0.64	0.000056	0.000212	378.57
6	0.12	0.50	0.002475	0.0081	327.27
7	0.09	0.42	0.001983	0.002816	142.01

8	0.04	0.33	0.001257	0.003364	267.62
9	0.28	0.81	0.489244	0.865788	176.96
10	0.26	0.77	0.000998	0.003546	355.31
11	0.13	0.51	0.001324	0.002598	196.22
12	0.02	0.37	0.654131	0.703661	107.57

Comments : For the populations under consideration,the ratio type estimator with square root transformation is less biased than the classical ratio estimator.Further, For populations 5-12 , the ratio type estimator with square root transformation is more efficient than the classical ratio estimator in the sense of having lesser mean square error.

Illustration 2

Consider a hypothetical population of size N =5 as follows:

$$y: 3\ 5\ 8\ 4\ 5$$
$$x: 5\ 2\ 6\ 4\ 3$$

For a simple random sample (without replacement) of size n =2,the exact biases and mean square errors of $\bar{y}_R$, $\bar{y}_{SQR}$ and $\bar{y}_{BTR}$ are given in Table-4.

Table 4: Comparison of Biases and Mean square errors

Estimators	Absolute Bias	M. S .E	Relative Efficiency
$\bar{y}$	0	1.05	100
$\bar{y}_R$	0.1903	1.8667	56.25
$\bar{y}_{SQR}$	0.0592	0.9996	105.04
$\bar{y}_{BTR}$	0.0569	0.9922	105.82

Comments: As per Illustration 2

(i)The ratio type estimator with square root transformation and ratio type exponential estimator have nearly equal bias and efficiency and further they are less biased and more efficient than the classical ratio estimator..

(ii) The classical ratio estimator is less efficient than the simple mean per unit estimator $\bar{y}$,where as the ratio type estimator with square root transformation and the ratio type exponential estimator are more efficient than $\bar{y}$.

5. CONCLUSIONS:

(i)To $O(1/n)$ $\bar{y}_{SQR}$ and $\bar{y}_{BTR}$ are equally efficient and more efficient than both $\bar{y}_R$ and $\bar{y}$ if $\frac{1}{4} < k < \frac{3}{4}$. To same order of approximation both $\bar{y}_{SQR}$ and $\bar{y}_{BTR}$ are less biased than $\bar{y}_R$ if k is either greater than $\frac{5}{4}$ or less than $\frac{11}{12}$.

(ii) Under the assumption of bivariate normality of (y, x) and equality of coefficients of variation of y and x ,to $O(1/n^2)$ $\bar{y}_{SQR}$ is more efficient than both $\bar{y}_R$ and $\bar{y}_{BTR}$ for $\frac{1}{2} < \rho < \frac{3}{4}$. However , for the values of ρ in the range $\frac{3}{4} < \rho < 1$, $\bar{y}_R$ is to be preferred over $\bar{y}$, $\bar{y}_{SQR}$ and $\bar{y}_{BTR}$.

(iii) Numerical illustration with the help of twelve natural populations shows that there might arise situations when $\bar{y}_{SQR}$ is less biased and more efficient than $\bar{y}_R$ under large sample approximations. Also, for the artificial population under consideration $\bar{y}_{SQR}$ happens to be less biased and more efficient than $\bar{y}_R$. However , the difference between $\bar{y}_{SQR}$ and $\bar{y}_{BTR}$ as regards bias and mean square error is only marginal.

RECEIVED APRIL, 2013
REVISED NOVEMBER, 2014

REFERENCES

[1] BAHL,S. and TUTEJA,R.K. (1991): Ratio and Product Type Exponential Estimator. **,Information and Optimization Sciences**,12,159-163.
[2] COCHRAN,W.G.(1940):The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce. **Journ.Ag.Sc**.,30,262-275.
[3] KONIJN,H.S.(1973): **Statistical Theory of Sample Survey Design and Analysis**. North Holland Publishing Co. Amsterdam.
[4] MURTHY,M.N.(1967): **Sampling: Theory and Methods**. Statistical Publ. Society,ISI, Calcutta.
[5] NEYMAN, J. (1934):On the two different aspects of the representative method: The method of stratified sampling and the method of purposive selection. **J. Royal Stat. Soc**.,97,558-625.
[6] PANSE,V.G. and SUKHATME,P.V.(1967): **Statistical Methods for Agricultural Workers**. Indian Council of Agril.Res, New Delhi
[7] SAMPFORD,M.R.(1962):**An Introduction to Sampling Theory**. Oliver and Boyd. London.
[8] SINGH,D and CHAUDHARY,F.S.(1986):**Theory and Analysis of Sample Survey Designs**, First Edition,Wiley Eastern Ltd. Calcutta.
[9] SUKHATME, P.V. and SUKHATME , B.V.(1970): **Sampling Theory of Surveys with Applications**, Asia Publishing House, Calcutta.
[10] SUKHATME, P.V., SUKHATME, B.V. and ASOK, C.(1984):**Sampling Theory of Surveys with Applications**,Indian Society of Agricultural Statistics, New Delhi.
[11] SWAIN, A.K.P.C.(2003): **Finite Population Sampling: Theory and Methods**. South Asia Publishers, New Delhi.
[12] UPADHYAYA,L.N.,SINGH,H.P.,CHATTERJEE,S. and YADAV,R.(2011): Improved Ratio and Product Exponential Type Estimator, **Journ. Stat.Theo. and Pract**. 5, 285-293.