INVESTMENTS AND ENVIRONMENTAL LIABILITY LAW
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ABSTRACT
According to the “polluter pays” principle, environmental liability law allows for internalisation of the cost of pollution. In order to adjust to these policy instruments under the conditions of imperfect markets, this paper presents an approach to valuing investments in environmental protection technologies and to examining the determinants of their price ceiling. The latter depends on the (corrected) net present values of the payments and on the interdependencies arising from changes in the optimal investment and production programmes. Though we can confirm the well-established results of environmental economics for a single investment, tightening environmental liability law may have counterproductive effects on investments in environmental protection technologies. In effect, all the (sometimes contradictory and unexpected) consequences of such policy changes can be interpreted in an economically comprehensible manner.


MSC: 91B24, 91B25, 91B38, 91B76

1. INTRODUCTION
Unlike administrative law that forces companies (not) to react in a certain way (which could be necessary in critical situations to react rapidly to occurring dangers), market oriented environmental policy leaves flexibility and the possibility to dispose on their own to its addressees: via its influence on companies’ earnings it still allows them to make their own decisions regarding how much they want to reduce environmentally harmful behaviour and to bear the financial consequences (e.g. paying taxes, buying or not being able to sell emissions allowances, not obtaining subsidies) for the remaining emissions. Similar is the effect of environmental liability law: polluters should prevent and limit emissions, but be obliged to pay for remediation in case of environmental damage occurring.1

As usual in tort law, we can distinguish between strict liability and negligence liability. For the polluter to be strictly liable, it is only necessary that his acts or omissions allow a damage or loss to occur. Under a negligence liability scheme, in addition culpability is a prerequisite for being liable.

Thus, if the company *knows* that it has to pay for compensation of damages, then in a similar manner to that already described by Pigou (1932, pp. 172, 174, 183, 224) for social policy using taxes and subsidies, tightening liability conditions for environmental damages leads to their consideration in economic decisions: the individual compares prevention costs to the costs of compensation and, consequently, may avoid environmentally harmful behaviour. Hence, investments in environmental protection technologies can become economically significant.

In this context, it is the **first objective** of this paper to derive an approach to evaluate investments in environmental protection in order to cope with the needs resulting from environmental liability law. Since production is the major source of (potential) environmental harm and, therefore, investments in environmental protection affect production, such a model should combine both investment appraisal and production. Having developed such an approach the **second objective** is to examine whether and to what degree changes in environmental liability law can actually provide financial incentives for investing in environmental protection.

Although a variety of traditional and neoclassical investment appraisal approaches in finance theory exist (e.g. cost oriented approaches, NPV- or DCF- methods, real options approaches), unfortunately, these can only partially cover the particular nature of investments in environmental protection: Because of their relevance for production, it is necessary to derive the payments and constraints required for a financial valuation from production planning with special regard to environmental liability law and joint production. On this basis, we will develop a valuation model and examine the determinants of the price ceiling for an investment in emissions reductions technologies. This model considers activity level-dependent and -independent payments and treats the indivisibility of the investment to be valued. Due to the fact that many of the impacts of pollution on the environment have yet to be explored and because of changes in environmental policy and in ecological awareness, this analysis also takes uncertainty into consideration.

Employing duality theory of linear programming, it can be shown that the price ceiling depends on the (corrected) net present values of the payments and on the interdependencies due to changes in the optimal programme. Sensitivity analysis provides information about the sometimes contradictory and unexpected consequences of legislative changes. Nevertheless, all these effects can be interpreted in an economically comprehensible manner and are demonstrated in an example. A conclusion summarises the main results.

2. FINANCIAL VALUATION OF INVESTMENTS IN ENVIRONMENTAL PROTECTION TECHNOLOGIES

2.1. Background – Financial Evaluation on Imperfect Markets

In economic literature, several studies examine the consequences of environmental policy on investments in environmental protection technologies (Reinaud 2003; Zhao 2003; Chakraborty 2004; Knutsson et al. 2006; Laurikka 2006; Laurikka and Koljonen 2006; Buchner 2007; Sekar et al. 2007; Yang and Blyth 2007; Blanco and Rodrigues 2008). Some of these refer to a single sector or the whole economy. Others adopt an enterprise point of view and employ different techniques for project appraisal. Very common are cost-based approaches and the use of discounted cash flow (DCF) models, which calculate the present value of an investment by discounting future cash flows at an appropriate discount rate, or real options analyses and simulations.

However, these models refer to **perfect markets** – a condition that does not apply to most companies (Klingelhöfer 2009, p. 371): borrowing and lending conditions are restricted and differ, and the best opportunity is not always determined on financial markets. Instead, for manufacturing companies it will often be an investment in other technologies, producing more or less of the desired outputs, or trading of assets, stocks or emissions allowances. Hence, in imperfect markets the discount rates are endogenous to the investment programme, and the (net) present values in most cases have to be corrected for restricted capacities, as will be shown in section 3.1. Under these circumstances neither “ordinary” (net) present values merely calculated with exogenous interest rates (even if adjusted to uncertainty), nor real options values, are adequate methods for appraising technology investments – all the more because the characteristics of investments in environmental protection technologies normally may not fulfil all the other prerequisites for applying option pricing models either (Klingelhöfer, 2009, p. 371): markets are not complete; short selling of (installed) machines and production lines is generally not possible; the underlying asset (i.e. the investment) is
normally neither divisible nor follows a distinct (stochastic) price process; the effects of technology investments and the consequences of interdependencies on the entire production are individually different. Thus, both alternatives, the construction of a replicating portfolio with production lines as well as finding a twin asset that is perfectly correlated with the underlying, are likely to fail. (Nevertheless, from a mathematical point of view, it can be shown that certain discrete option pricing models can be derived as special cases of the model presented in this paper if their prerequisites are fulfilled; Klingelhöfer (2009)).

Consequently, to assess environmentally beneficial investments and to examine the impact of environmental liability law, the investor must consider the following restrictions:

Resulting from restricted capacities (due to budget constraints, production constraints or environmental policy), every activity on imperfect markets can display interdependencies with other decisions. Especially for capital intensive projects, budget constraints may force the investor to give up alternative investments or to limit production. On the other hand, the revenues of production may extend the possibilities for investments and finance – and, thus, for environmentally beneficial investments as well. Hence, a financial valuation needs to derive the required payments and constraints from production theory and production planning with special regard to environmental policy. This makes it impossible to calculate the value of an investment only by discounting its payments using a single market interest rate. Instead, the endogenous marginal rates of return of the best alternatives are necessary for theoretically correct valuation. Also, it is not possible to determine the profitability of an additional object merely by calculating net present values: the realisation of additional objects may lead to capacity shortages and, therefore, to changes in the relevance of other objects or capacities to making a decision (i.e. the binding restrictions may alter).

Consequently, to assess the degree of profitability of an additional single investment or activity within imperfect markets, we have to compare the situation after investing (i.e. the valuation programme) to the situation before doing so (i.e. the basic programme) (Hering, 2006; Jaensch, 1966; Klingelhöfer, 2006; Matschke, 1975). If the maximum value of the valuation programme is greater than in the basic programme, it is reasonable to invest. Ensuring this by means of a minimum withdrawal constraint, the valuation programme determines the maximum payable price \( p \) for the investment.

**Uncertainty**, resulting from production itself, from individual actions or omissions with regard to environmental harassment as well as from either reiterative changes of environmental policy, shifts in ecological awareness or altered conditions on liberalised markets, etc., can be taken into account by using decision trees (Klingelhöfer 2003, 2006; Magee 1964a; 1964b; Mao 1969). Starting with the realised and, therefore, known state \( s = 0 \) (denoting the state actually realised in \( t = 0 \)), we obtain a set \( S = \{0; 1; \ldots; S\} \) of possible states \( s \) – “organised” in a tree structure until time horizon \( t = T \). However, the states being consecutively numbered from \( s = 0 \) to \( s = S \), the two-dimensional tree of states for each point in time can be transformed into a one-dimensional mathematical structure. Subsequently, the valuation considers payments in all possible states. Information on probabilities, means or variances is not necessary, as simple dominance considerations are sufficient (it only needs to be known which states can possibly occur; the probability must be greater than zero, but smaller than 1). Therefore, the Bernoulli principle and its axioms are not needed.

**2.2. Derivation of the Payments from Production Planning.**

Every production, especially with regard to the environment, is characterised as joint production: Using activity analysis (Debreu 1959; Klingelhöfer 2000; Koopmans 1957, 1959; Nikaido 1968), a singular realisation of the production process \( \beta \) (for example, one hour) – the so-called basic activity \( B_\beta \) – consumes a combination of several kinds of \( m \) inputs \( r_\mu \) (e.g., fuel, labour) and produces a combination of \( n \) wanted and

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2 Even if a market price exists, the individual value of a machine need not match it: if, for example, a production line allows for earnings of R100,000 a day and a machine failure (market value R20,000) occurs, consequently the losses during an assumed one-day replacement of this machine accrue to R100,000. Thus, the producer would even pay e.g. R110,000 for immediate replacement of the machine, which amounts to R90,000 more than its market value, because in this case he would still gain back R100,000 from production this day and, therefore, have R10,000 more than replacing this machine one day later for R20,000. Thus, in use, the value of the machine for the producer is very different from its market value.
unwanted outputs \( x_v \) (e.g., products, electric power, heat, emissions, waste). Thus, a basic activity is defined as a vector of \( m \) input and \( n \) output commodities \( \varphi_\epsilon \):^3

\[
\varphi^{B,\beta} = (\varphi_1, \ldots, \varphi_{m+n})' = (r_1', x_1', \ldots, r_m', x_n') \geq 0
\]  

(2.1)

Then, every possible production of a technology set \( T \) is a linear combination of the \( q \) basic activities with non-negative coefficients \( \lambda^\beta \) describing their levels.

\[
\forall \varphi = (r'_v, x'_v) \in T: \quad \varphi = \sum_{\beta=1}^{q} \varphi^{B,\beta} \cdot \lambda^\beta
\]

(2.2)

Introducing a price system with positive prices \( p_\epsilon \) for the (desired) input of waste and the output of products, prices equal to zero for neutral inputs and outputs (e.g. air and water in certain cases) and negative prices for the input of (traditional) factors of production (primary commodities such as material, labour, or fuel) and the output of waste and emissions delivers the contribution margin \( CM \):

\[
CM(\varphi) = p'_v \cdot \varphi = p'_v \cdot \sum_{\beta=1}^{q} \varphi^{B,\beta} \cdot \lambda^\beta = \sum_{\beta=1}^{q} \sum_{\epsilon=1}^{m+n} p_\epsilon \cdot \varphi^{B,\beta} \cdot \lambda^\beta = CM(\lambda)
\]

(2.3)

An environmental damage or loss typically occurs for two reasons:

1. Even without a failure, given constraints are violated either because
   a) the operators, hoping that there will be no damage, allow production (more or less consciously) to exceed its allowed level or
   b) some of the production coefficients relevant to the restriction or some of the limits are varying.
2. A failure/accident.\(^4\)

Since for environmental damages the claimant normally does not have the opportunity to react and since, therefore, his behaviour in many cases is a datum for the potential polluter, a unilateral view will be assumed in the following. Then, the potential polluter’s costs \( C \) resulting from an existing liability scheme consist of the expected costs of a damage/loss \( ECD \) and his costs for damage prevention \( CDP \) (Klingelhöfer 2000, pp. 205-209). Of these, the expected costs of damage\(^5\) will decline with the level of his preventative activities \( pa \) (the standard of care)

\[
\partial ECD/\partial pa \leq 0,
\]

(2.4)

while the prevention costs will increase with \( pa \):

\[
\partial CDP/\partial pa \geq 0
\]

(2.5)

Punitive elements and compensations for immaterial damage, for pain and suffering can be included via a factor or a summand to the amount of damage. Thus, for the potential polluter’s total costs \( C \) from an existing liability scheme we will obtain:

\[
C(pa) = CDP(pa) + ECD(pa)
\]

(2.6)

In the case of strict liability, where the polluter is liable regardless of culpability, the function’s minimum \( C^{min} \) determines the optimal level \( pa^{opt} \) of preventative activities, while in the case of negligence liability, depending on the level of preventative activities, the costs incurred by the polluter may be even lower than the sum of expected costs of damage and prevention costs: exercising the necessary standard of care \( pa^{pec} \), the

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\(^3\) Underlining a variable denotes a vector and the prime (the symbol ‘) the transposition of a vector.

\(^4\) For example, the question of a failure (or normal operation), according to § 6 Umwelthaftungsgesetz (§ 6 of the German Environmental Liability Law) is essential for the application of the far-ranging assumption of reason, which leads to massive alleviations of the claimant’s burden of proof (cp. Klingelhöfer 2000, pp. 187-196).

\(^5\) While we use decision trees to model uncertain future, the probability of a damage in a particular state (depending on the level of preventative activities) is considered via the expected cost of damage in this case.
polluter is no longer liable for the damage and only prevention costs occur. Thus, the cost function \( C(pa) \) will have a jump discontinuity in \( p_{a,nec} \):

\[
C(pa) = \begin{cases} 
CDP(pa) + ECD(pa) & \text{for } pa < p_{a,nec} \\
CDP(pa) & \text{for } pa \geq p_{a,nec}
\end{cases}
\] (2.7)

Regarding the analysis, according to the above mentioned distinction we have to consider whether liability depends on a failure or not:

**Ad 1: Given Constraints Are Violated Without a Failure/Accident**

In this case it is normally not necessary to take the costs for damage prevention \( CDP \) explicitly into account, since they are part of the opportunity cost of production for violation of allowed limits. Thus, they are already considered via the quantities valuated with prices (Klingelhöfer 2000, pp. 503-508): \( p' \cdot \varphi \). Since constraint violating (and in case 1b also constraint approximating) production can be understood as a (conscious or unconscious) reduction of the enterprise’s standard of care (i.e. \( \varphi_{pa}/\varphi_{x,\nu} \leq 0 \)), the probability and the amount of a damage and, therefore, the expected cost of damage (including compensation for pain and suffering) for the enterprise tend to increase:

\[
\frac{\partial ECD}{\partial \varphi_{x,\nu}} \geq 0
\] (2.8)

The same may be true for using \( r_{\mu} \) environmentally harmful inputs (and also for taking of – perhaps not even dangerous, but useful – resources from nature):

\[
\frac{\partial ECD}{\partial r_{\mu}} \geq 0
\] (2.8*)

Since all expected costs of damage diminish the economic success of production, they modify the function (2.3) for the contribution margin of production:

\[
CM(\varphi) = p' \cdot \varphi - \sum_{\nu} ECD(\varphi_{\nu}) - \sum_{\mu} ECD(\varphi_{\mu}) = p' \cdot \varphi - \sum_{\nu} ECD(\varphi_{\nu})
\] (2.9)

On the other hand, the constraint for the maximum allowed emissions/input \( \varphi_{e,max,old}^{\mu} \) of the dangerous substances to be exempted from liability has to be adjusted for the (maximum) excess \( \varphi_{e,max,exc}^{\mu} \) until the new limit \( \varphi_{e,max,new}^{\mu} \):

\[
\varphi_{e} \leq \varphi_{e,max,old}^{\mu} + \varphi_{e,max,exc}^{\mu} = \varphi_{e,max,new}^{\mu} \quad \forall \nu
\] (2.10)

Depending on the choice of \( \varphi_{e,max,exc}^{\mu} \) (e.g. as a limit that provides actions leading to excessive damages from becoming relevant to criminal law, too) this may even mean that any new maximum level no longer exists. Also, depending on the severity of consequences and modelled in the same way, several levels of excessive production \( \varphi_{e,max,exc1}^{\mu} \), \( \varphi_{e,max,exc2}^{\mu} \) etc., which lead to staggered new upper limits \( \varphi_{e,max,new1}^{\mu} \), \( \varphi_{e,max,new2}^{\mu} \) etc., are possible.

Unfortunately the modifications of the expected costs of damage ECD and the contribution margin CM according to (2.9) and (2.10) need not be of a linear nature. Nevertheless, often it is feasible to approximate such a function by a discrete one using intervals \( h \) where it is possible to assign a constant rate of expected costs of damages \( ecd_{eh} \) to the partial amounts \( \varphi_{eh} \) of \( \varphi_{e} \). Thus, we will receive for each interval \( h \) a function \( ECD(\varphi_{eh}) = ECD(\lambda_{h}^{\beta}) \) which is linearly dependent on \( \varphi_{eh} \) and, therefore, on \( \lambda_{h}^{\beta} \). Hence, instead of (2.9) and (2.10) we will obtain:

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\( ^6 \) This implies that the expected cost of damage are additively separable with regard to the amounts \( \varphi_{e} \) in the sense that each of the summed terms depends only on one variable \( \varphi_{e} \). Thus, progressively increasing damages which result from synergetic interaction of different environmentally harmful substances of different processes cannot be modelled. Nevertheless, since under the assumption of joint production the expected costs of damage (and, therefore, the contribution margin) are a function depending only on \( \lambda_{h}^{\beta} \), the following not only allows one to model additive impacts of harmful substances, but also those progressively increasing damages.
\[ CM(\varphi) = p' \varphi - \sum_{\varepsilon} ECD(\varphi_{\varepsilon}) = p' \varphi - \sum_{\varepsilon} \sum_{h} ECD(\varphi_{ch}) = p' \varphi - \sum_{\varepsilon} \sum_{h} ecd_{ch} \cdot \varphi_{ch} = CM(\lambda_{ch}) \]  \hfill (2.11)

\[ \varphi_{\varepsilon} = \sum_{h} \varphi_{ch} = \sum_{\beta=1}^{q} \sum_{h} \varphi_{\varepsilon}^{B, \beta} \cdot \lambda_{h}^{\beta} \leq \varphi_{\varepsilon}^{\text{max,old}} + \varphi_{\varepsilon}^{\text{max,exc}} = \varphi_{\varepsilon}^{\text{max,new}} \quad \forall \varepsilon \]  \hfill (2.12)

Additionally, we have to consider constraints for the amounts in the partial intervals \( h \):

\[ 0 \leq \varphi_{ch} = \sum_{\beta=1}^{q} \varphi_{\varepsilon}^{B, \beta} \cdot \lambda_{h}^{\beta} \leq \varphi_{\varepsilon}^{\text{max}} \quad \forall \varepsilon, \forall h \]  \hfill (2.13)

where

\[ \sum_{h} \varphi_{ch}^{\text{max}} = \varphi_{\varepsilon}^{\text{max,new}} \quad \forall \varepsilon \]  \hfill (2.14)

However, since we have a double sum in (2.11) and (2.12), which allows one to regulate the inputs and outputs of several processes \( \beta \) together (i.e. the amounts of one process \( \beta \) can compensate for those of another process \( \beta^* \)), it is generally not possible to derive other upper limits \( \lambda_{h}^{\beta,\text{max}} \) than \( \lambda_{h}^{\beta,\text{max}} \) for the (partial) levels \( \lambda_{h}^{\beta} \) of production. Therefore, the choice of one process \( \beta \) and of its upper limits \( \lambda_{h}^{\beta,\text{max}} \) depends on the choice of the other processes \( \beta^* \): a greater production of a harmful substance using process \( \beta^* \) leaves less space for production with process \( \beta \) before reaching the next interval \( h \) with a higher rate \( ecd_{ch} \) of expected costs of damages.

For investment assessment in the next sections this leads to consequences for the corrected net present values: the primal constraint for dividing process \( \beta \) into partial processes \( h \)

\[ \sum_{h} \lambda_{h}^{\beta} \leq \lambda_{h}^{\beta,\text{max}} \quad \forall \varepsilon \in \{1; 2; \ldots; q\} \]  \hfill (2.15)

leads to a common dual variable \( r_{s}^{\beta} \), and therefore, (after division by \( l_{0} \)) to the same upper limit – for all the used partial processes \( h \) in state \( s \). However, this is understandable because process \( \beta \) is divided into partial processes only for mathematical reasons. Actually, it is still the same.

Furthermore, if the rates \( ecd_{ch} \) of the expected costs of damages do not decline with the emission/input of \( \varphi_{\varepsilon} \) (ECD is then convex), special order conditions for the intervals \( h \) are not necessary, and we only need the conditions (2.11), (2.13), and (2.15). As a result both strict liability and negligence liability are modelled:

- For strict liability, where it is only necessary that his acts or omissions permit a damage or loss to occur, we have to set \( \varphi_{\varepsilon}^{\text{max,old}} = 0 \) and \( ecd_{ch} > 0 \ \forall h \).
- In a scheme of negligence liability, the polluter only has to compensate if the emissions caused by his production violates \( \varphi_{\varepsilon}^{\text{max,old}} > 0 \). Then \( ECD(\varphi_{\varepsilon}) = 0 \) and, therefore, \( ecd_{ch} = 0 \) for \( \varphi_{\varepsilon} \leq \varphi_{\varepsilon}^{\text{max,old}} \). (Hence, usually \( \varphi_{\varepsilon}^{\text{max}} = \varphi_{\varepsilon}^{\text{max,old}} \)). Violation of \( \varphi_{\varepsilon}^{\text{max,old}} \) leads to \( ecd_{ch} > 0 \). Thus, we can interpret \( ecd_{ch} \) as the rate of punitive costs for exceeding emissions/input of the dangerous substances and the similarity of environmental liability law to the effects of a tax system becomes evident. Moreover, varying the value of the upper bound \( \varphi_{\varepsilon}^{\text{max,old}} \) demonstrates that strict liability can be interpreted as a borderline case of negligence liability.

Obviously, after these discussions of the effects of emission/input of \( \varphi_{\varepsilon} \) case 1a is explained. For varying production coefficients (relevant to the restriction) or limits as ecological risks (case 1b), chance constrained programming and optimisation using fuzzy sets could be combined with the above described adjustments of

resulting from synergetic interaction of different environmentally harmful substances \( \varphi_{\varepsilon}^{\beta} \) which are all inputs or outputs of the same process \( \beta \).
the contribution margin and the constraint system (cp. e.g. Steven 1994, pp. 133-161; Bogaschewsky 1995, pp. 302-346). Alternatively, theoretically it would be possible to discretise the production processes for the said production coefficients (Klingelhöfer 2000, pp. 250-252). However, the problem would be much more complex then.

**Ad 2: A Failure/Accident**

To be protected against failures and accidents which might occur independently of the current level of production, it is necessary to include **preventative activities independent from** \( \lambda \) (Terms varying with \( \lambda \) could be treated as previously.) Assuming a unilateral view which is independent of the claimant’s behaviour, the damage prevention costs \( CDP \) and the expected costs of damage \( ECD \) can again be written as independent variables of the preventative activities \( pa \) (Klingelhöfer 2000, pp. 250-252; Klingelhöfer 2005, pp. 142 f.):

\[
CDP = CDP(pa) \quad \text{and} \quad ECD = ECD(pa)
\]

Considered in the objective function (2.3), we obtain:

\[
CM(\varphi, pa) = p' \cdot \varphi - ECD(pa) - CDP(pa)
\]

These formulas already demonstrate that in the case of damage prevention costs and the expected costs of damage independent of the current level of production, with regard to investment assessment it is often easier to determine these costs, isolated, directly and then to integrate them as (production decision irrelevant) fixed costs. Thus, **separability** from the rest of the production planning problem allows one to regard such a prevention of failures/accidents as an **other investment** in the investment assessment model to be developed in the next sections, because the stream of necessary payments to realise such an “insurance” could be compared directly to the activity level independent payments of the savings from fewer failures/accidents.

If it is not possible to separate activity level dependent parts from the activity independent parts for the purpose of prevention, this prevention can be treated formally in the same way as a “normal” investment in environmental protection (for the assessment of such investments cp. Klingelhöfer 2006).

### 2.2. Model for the Financial Valuation of Investments into Environmental Protection Technologies

According to section 2.1, the first step to assess the degree of profitability of an investment on imperfect markets, the **basic programme**, calculates the maximum value of the situation without realising this investment. This maximum value can be operationalised by maximising the sum \( SWW \) of weighted withdrawals \( w_s \cdot W_s \) subject to the constraints of investment and production, where \( s \in S = \{0; 1; 2; \ldots; S\} \) denotes the present state and the \( S \) future states (Klingelhöfer, 2009, pp. 373-374). Deriving the constraint system, we have to consider that investments in environmental protection technologies affect production. Therefore, it is necessary to integrate contribution margins, production constraints and the payments resulting from environmental liability law. While the production constraints directly become part of the constraint system, the contribution margins \( CM \) according to (2.11) modify the investment programme’s liquidity constraints (Klingelhöfer, 2006, pp. 127-130, 271 f.): liquidity must be guaranteed with respect to all the payments from production and environmental liability law, \( z_{js} \) from the other projects \( inv_j \) (e.g. credits or loans), the payments \( uz_s \) which are independent of production quantities and the investment programme (e.g. additional individual deposits, fixed rents, taxes or payments determined in former periods), and the withdrawals \( W_{s'} \), – otherwise the company becomes insolvent. Thus, we can derive the following linear programming problem as the basic programme:

\[
\text{max. } SWW, \quad SWW := \sum_{s=0}^{S} w_s \cdot W_s
\]

Subject to:

\[
\text{7 The weights in the objective function express the decision maker’s individual relative preferences for payments in the states regarded. Although, at first sight, this seems to be similar to using expected values, weighting the payments of each possible states does not imply considering probabilities and, therefore, the sum of weights does not have to equal 1 (cp. Klingelhöfer 2003, pp 290-291, Klingelhöfer 2006, pp 75-81).}
Liquidity constraints (capital budget constraints) for the $S+1$ states $s$ (cp. (2.11)):

$$\sum_{j=1}^{J} z_{js} \cdot \text{inv}_j - \sum_{\beta=1}^{q} \sum_{\varepsilon=1}^{m+n} \sum_{h} \left( p_{es} - \text{ecd}_{eh} \right) \cdot \phi_{e}^{B,\beta} \cdot \lambda_{hs}^{\beta} + W_s \leq u z_s \quad \forall s \in S$$

Staggered production constraints from environmental liability law (cp. (2.13)):

$$\sum_{\beta=1}^{q} \phi_{e}^{B,\beta} \cdot \lambda_{hs}^{\beta} \leq \phi_{ehs}^{\max} \quad \forall s \in \{1; 2; \ldots; m+n\} \quad \forall s \in S$$

$\Gamma_s$ other production and environmental constraints $\gamma$ for the $S+1$ states $s$:

$$\sum_{\beta=1}^{q} \sum_{e=1}^{m+n} \sum_{h} a_{eys} \cdot \phi_{e}^{B,\beta} \cdot \lambda_{hs}^{\beta} \leq b_{\gamma s} \quad \forall \gamma \in \{1; 2; \ldots, \Gamma_s \} \quad \forall s \in S$$

Restrictions of quantity of $J$ other investment objects and financial transactions:

$$\text{inv}_j \leq \text{inv}_j^{\max} \quad \forall j \in \{1, \ldots, J\}$$

Non-negativity conditions:

$$\lambda_{hs}^{\beta}, \text{ inv}_j, W_s \geq 0 \quad \forall \beta \in \{1; \ldots; q\} \quad \forall \gamma \quad \forall j \in \{j = 1; \ldots; J\} \quad \forall s \in S.$$  

With the known solution of the basic programme (2.18), the valuation programme calculates the maximum payable price $p_I$ which can be paid for an investment $I$ in environmental protection technologies under the condition that the investor’s utility may not be lower than in the basic programme (minimum withdrawal constraint). Besides this different objective function VAL, it exhibits nearly the same structure as the basic programme, but it has to take into account all the activity level-dependent and -independent payments caused by this investment. This means that we have to consider not only the adjusted contribution margins according to (2.11), but also the price $p_I$ of the investment and other activity level-independent payments $z_{Is}$ (e.g. for its installation). Thereafter, the valuation programme is derived as follows:

$$\text{max. VAL; } \text{ VAL} := p_I$$

Subject to:

Liquidity constraints (capital budget constraints) for the $S+1$ states $s$ (cp. (2.11)):

$$-\sum_{j=1}^{J} z_{js} \cdot \text{inv}_j - \sum_{\beta=1}^{q} \sum_{\varepsilon=1}^{m+n} \sum_{h} \left( p_{es} - \text{ecd}_{eh0} \right) \cdot \phi_{e}^{B,\beta} \cdot \lambda_{h0}^{\beta} + W_0 + p_I$$

$$\leq u z_{0} + z_{10} + \sum_{\varepsilon=1}^{m+n} \sum_{h} \left( p_{es} - \text{ecd}_{eh0} \right) \cdot \phi_{e}^{B,1} \cdot \lambda_{h0}^{1}$$

$$-\sum_{j=1}^{J} z_{js} \cdot \text{inv}_j - \sum_{\beta=1}^{q} \sum_{\varepsilon=1}^{m+n} \sum_{h} \left( p_{es} - \text{ecd}_{ehs} \right) \cdot \phi_{e}^{B,\beta} \cdot \lambda_{hs}^{\beta} + W_s$$

$$\leq u z_s + z_{1s} + \sum_{\varepsilon=1}^{m+n} \sum_{h} \left( p_{es} - \text{ecd}_{ehs} \right) \cdot \phi_{e}^{B,1} \cdot \lambda_{hs}^{1} \quad \forall s \in S \setminus \{0\}$$

Staggered production constraints from environmental liability law (cp. (2.13)):

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Mathematically, the new basic activity $\phi_{e}^{B,1}$ differs from the old ones $\phi_{e}^{B,\beta}$ only in the more environmentally friendly relation between inputs and outputs. If the investment allows several new processes, they can be integrated analogously to the following. For the mathematical formulation of the effects of different kinds of environmental protection technologies cp. e.g. Klingelhofer (2000), S. 252-305, S. 396-416, 446-472.
\[
\sum_{\beta=1}^{q} \Phi_c^B \cdot \lambda^\beta_{hs} + \Phi_c^I \cdot \lambda^1_{hs} \leq \Phi_{\text{ch}}^{\max} \quad \forall \varepsilon \in \{1; 2; \ldots; m+n\} \quad \forall h \quad \forall s \in S
\]

Other production and environmental constraints \(\gamma\) for the \(S+1\) states \(s\):

\[
\sum_{\beta=1}^{q} \sum_{\varepsilon=1}^{m+n} a_{\varepsilon} \cdot \Phi_c^B \cdot \lambda^\beta_{hs} + \sum_{\varepsilon=1}^{m+n} a_{\varepsilon} \cdot \Phi_c^I \cdot \lambda^1_{hs} \leq b_{\gamma} \quad \forall \varepsilon \in \{1; 2; \ldots, \Gamma_s\} \quad \forall s \in S
\]

\(q+1\) activity level constraints, which also consider the partial processes, for the \(S+1\) states \(s\) (cp. (2.15)):

\[
\sum_{h} \lambda^\beta_{hs} \leq \lambda^\beta_{s}^{\max} \quad \forall \beta \in \{1; 2; \ldots; q\} \quad \forall s \in S
\]

Restrictions of quantity of \(J\) other investment objects and financial transactions:

\[
\text{inv}_{j} \leq \text{inv}^{\max}_{j} \quad \forall j \in \{1, ..., J\}
\]

Minimum withdrawal constraint (ensuring that the utility is not less than before):

\[-\sum_{s=0}^{S} W_s \cdot W_s \leq -SWW^{\text{opt}}
\]

Non-negativity conditions:

\[
\lambda^\beta_{hs}, \text{inv}_{j}, W_s \geq 0 \quad \forall \beta \in \{1; \ldots; q\} \quad \forall h \quad \forall j \in \{j = 1; \ldots; J\} \quad \forall s \in S
\]

Besides the maximum payable price \(p_f\) for the investment and the activity levels \(\lambda^1_{hs}\) of the partial processes of the environmentally friendly process, the basic programme and the valuation programme contain the same decision variables: the activity levels \(\lambda^\beta_{hs}\) of the partial processes \(h\) of the \(q\) old processes, the quantities \(\text{inv}_{j}\) of the other investment objects and financial transactions, and the withdrawals \(W_s\). We find that the contribution margins (2.11) constitute part of the liquidity constraints, and that (2.13) and (2.15) are constraints of either programme.

3. ENVIRONMENTAL LIABILITY LAW AND THE WILLINGNESS TO INVEST IN ENVIRONMENTAL PROTECTION

3.1. (Corrected) Net Present Values and the Price Ceiling for the Investment

Using complementary slackness conditions enables us to interpret the restrictions of both the duals of the basic programme and the valuation programme in an economic manner:

By introducing the dual variables
- \(l_{s}\) for the liquidity constraints (and the resulting endogenous discount factors \(\rho_{s,0} = l_{s}/l_{0}\) to discount payments in state \(s\) to state \(0\));
- \(\pi_{\gamma}\) for the production and environmental constraints;
- \(\zeta^{\text{ELL}}_{c}\) for the constraints of the intervals \(h\) with constant rate of expected costs for damages resulting from environmental liability law;
- \(\zeta^{\beta}_{s}\) for the activity level constraints;
- \(\xi_{j}\) for the limits of the other investment objects and financial transactions;
and dividing the dual constraints of the decision variables by \( l_0 \), we obtain the (corrected) net present values \( NPV^{(corr)} \) of:

- using the partial processes \( h \) of the processes \( \beta \in \{ 1; 2; \ldots; q; I \} \) in the states \( s \):

\[
NPV_{\beta,h,s}^{corr} := NPV_{\beta,h,s} - Correction
= \sum_{s=1}^{m+n} (p_{\epsilon,s} - ecd_{\epsilon,h,s}) \cdot \varphi_{\epsilon,s}^{\beta} \cdot l_s \cdot l_0 - \left( \sum_{\gamma=1}^{m+n} \sum_{\epsilon=1}^{m+n} a_{\epsilon,s} \cdot \varphi_{\epsilon,s}^{\beta} \cdot \pi_{s,\gamma}^{\beta} \cdot l_0 + \sum_{\epsilon=1}^{m+n} \varphi_{\epsilon,s}^{\beta} \cdot \zeta_{\epsilon,h,s} \right) \\
\leq \frac{\beta}{l_0}, \forall s \in S
\]

(3.1)

- discounted monetary equivalent of the required capacity of the production and environmental constraints (including the relevant restrictions of environmental liability law)

- the realisation of other investment objects and financial transactions \( j \):

\[
NPV_{inv,j} := \sum_{s=0}^{S} z_{j,s} \cdot l_s \cdot l_0 = \sum_{s=0}^{S} z_{j,s} \cdot \rho_{s,0} \leq \frac{\xi_{j}}{l_0}, \forall j
\]

(3.2)

Because of the complementary slackness conditions existing between the primal decision variables and the corresponding dual restrictions, the known decision rules for \( NPV \) on perfect markets can only be applied to these (corrected) \( NPV \) on imperfect markets. This is also intuitively understandable: since the use of restricted capacities diminishes the available amount for other advantageous processes, the \( NPV \) of the (partial) processes not only have to take into account the discounted contribution margins, but also the required capacities. Thus, we can see that environmental liability law exerts three, partly opposing effects on the use of a (partial) process:

1. The expected costs of damages reduce the possible cash flow surplus.
2. The discounted monetary equivalents of the required capacity intervals \( h \) resulting from environmental liability law increase the correction terms.
3. Violation of the restriction relevant for environmental liability law allows for employing (partial) processes which otherwise (fulfilling the restrictions) could not be used – with the result of additional positive corrected net present values.

Hence, ceteris paribus, tightening of environmental liability law leads to a loss of profitability of a process. However, resulting from interdependencies the need to restructure production may arise. Then, changes in the dual variables may allow processes which have thus far been disadvantageous to become profitable because others are charged greater penalties. To gain further information on this effect, sensitivity analysis will be employed in the next section.

Moreover, since according to (3.1) the \( NPV^{corr} \) of all partial processes \( h \) of one process \( \beta \in \{ 1; 2; \ldots; q; I \} \) are restricted by the same dual variable \( r_{\epsilon,h,s}^{\beta} \) which is independent of the intervals \( h \) and can only be positive if the corresponding primal constraint \( \lambda_{h,s}^{\beta} \leq \lambda_{s,0}^{\beta,\max} \) is fulfilled as an equation, we can derive for the partial processes employed in the optimal solution:

\[
\text{All the following (corrected) net present values } NPV^{(corr)} \text{ can be derived from both the basic programme (2.18) and the valuation programme (2.19). However, the dual variables and, consequently, the factors } \rho_{s,0} = l_s / l_0 \text{ normally differ between both programmes. Especially, in the case of an existing finite positive solution of both the primal and dual valuation programme, we can deduce } l_0 = 1 \text{ and, therefore, } \rho_{s,0} = l_s \text{ from the complementary slackness condition } p_{h} \cdot (1 - l_0) = 0 \text{ for all the (corrected) NPVs derived from the valuation programme.}
\]
• Either only one partial process $h$ of process $\beta$ has a positive corrected net present value (meaning that process $\beta$ is not divided into partial processes in the end and that the rate $ecd_{\beta}$ of the expected costs of damages stays constant until the maximum activity level $\lambda_{s}^{\beta,\max}$)

• or – in the case of more than one partial process with $NPV_{\lambda,h,\beta,s}^{corr} > 0$ – all partial processes $h$ have the same positive corrected net present value which is also the corrected net present value of the whole process $\beta$.

\[
NPV_{\lambda,\beta,s}^{corr} = NPV_{\lambda,h,\beta,s}^{corr} = \epsilon_{s}^{\beta} \quad \forall NPV_{\lambda,h,\beta,s}^{corr} > 0
\]  

(3.3)

This outcome can be understood, because the net present value is a partial model calculating by using marginal values. Similar to communicating tubes where the pressure is equal in each of them, it does not matter which of the used partial processes $h$ (partially) replaces the best opportunity by weakening the constraint or which of the used partial processes $h$ is (partially) replaced by the best opportunity in case of a tighter upper limit – the result is the same (and therefore even the same for the whole process $I$) because it is always the same best opportunity.

With these results, it is possible to obtain information about the determinants of the maximum payable price. According to the duality theory of linear programming in the case of an existing finite positive solution, the optimal solutions of the primal and the dual problem are equal. Therefore, the optimal solution of the dual of the valuation programme provides information about the price ceiling. Since the withdrawal constraint is part of the valuation programme’s constraint system, it also takes into account the optimal solution $SWW^{opt}$ of the basic programme. Ergo, in the case of an existing finite positive solution of this programme, the optimal solution of its dual can be inserted for $SWW^{opt}$ in the minimum withdrawal constraint of the valuation programme. Consequently, the equation of the price ceiling contains several corresponding dual variables of both programmes.

Nevertheless, using the (corrected) net present values $NPV^{corr}$ (3.1)-(3.3) allows one to interpret this equation in an economic context. If one of the primal variables $\lambda_{s}^{\beta}$ of the activity levels or $inv_{j}$ of the other investment objects and financial transactions is positive, then – by reason of “complementary slackness” – the corresponding inequality (3.1) or (3.2) is satisfied as an equation. Therefore, taking into account the remarks regarding the $NPV^{corr}$ (3.1) which led to (3.3), we can use the $NPV_{\lambda,\beta,s}^{corr}$ and $NPV_{inv,j}$ to substitute the corresponding positive dual variables $\xi_{s}$ and $\xi_{j}$ of the valuation (VP) and the basic programme (BP).

Introducing the dual variable $\delta$ of the withdrawal constraint, we then obtain the maximum payable price for an investment in environmental protection technologies as a sum of several (partly corrected) net present values:

\[
\begin{align*}
\hat{p}_{i}^{opt} &= I + II + III + IV + V + VI + VII \\
&= \left( \sum_{s=0}^{S} z_{s} \cdot l_{s,\delta}^{VP} \right) + \left( \sum_{\lambda_{j},\delta,\beta} \lambda_{j}^{max} \cdot NPV_{\lambda,\delta,\beta}^{corr,VP} \right) + \left( \sum_{\delta,j} \left( \sum_{s=0}^{S} \left( \rho_{s,0}^{VP} - \delta \cdot l_{s}^{BP} \right) \right) \right) + \left( \sum_{\lambda_{j},\delta,\beta} \lambda_{j}^{max} \cdot NPV_{\lambda,\delta,\beta}^{corr,BP} \right) + \left( \sum_{\delta,j} \left( \sum_{s=0}^{S} \left( \sum_{s=1}^{S} \left( l_{s}^{BP} \cdot \pi_{\delta}^{VP} - \delta \cdot \sum_{s=1}^{S} b_{\delta}^{BP} \right) \right) \right) \right) + \left( \sum_{\delta,j} \left( \sum_{s=0}^{S} \left( \sum_{\lambda_{j},\delta,\beta} \lambda_{j}^{max} \cdot NPV_{\lambda,\delta,\beta}^{corr,BP} \right) \right) \right)
\end{align*}
\]

(3.4)

\[
\begin{align*}
p_{i}^{opt} &= NPV \text{ of all activity level-independent payments of the investment in environmental protection technologies (without } \hat{p}_{i}^{opt} \text{)}
\end{align*}
\]

(I)

10 Compare footnote 9.

11 The dual variable $\delta$ of the withdrawal constraint calculates the value of a marginal increase in $SWW^{opt}$ referring to the objective function of the valuation programme (the price ceiling).
+ $NPV_{corr}$ of operating the profitable environmentally friendly processes at their maximum activity levels $\lambda_{n,max}$

(II)

+ $NPV$ of the changes between $VP$ and $BP$ regarding the valuation of the payments that are independent of activity levels and the investment programme

(III)

+ $NPV_{corr}$ of the changes between $VP$ and $BP$ regarding the use of the other profitable production processes $\beta$

(IV)

+ $NPV$ of the changes between $VP$ and $BP$ regarding the monetary equivalents of the staggered production constraints stemming from environmental liability law

(V)

+ $NPV$ of the changes between $VP$ and $BP$ regarding the monetary equivalents of the other production and environmental constraints

(VI)

+ $NPV$ of the changes between $VP$ and $BP$ regarding the realised other investment objects and financial transactions

(VII)

This price ceiling for an investment in environmental protection technologies depends on the (corrected) $NPV$s of its payments and on the interdependencies occurring because of changes in the optimal investment programme. Under uncertainty it includes the discounted payments of all states – even those which, in fact, will not occur.

3.2. Environmental Liability Law and the Willingness to Invest in Environmental Protection Technologies

The economic interpretation of the terms (II), (IV) and (V) of (3.4) demonstrates the effect of environmental liability law on the price ceiling for environmental technology investments: via the changes to the corrected net present values of the processes which were explained regarding (3.1), and via the monetary equivalent of the allowed emissions/inputs of dangerous substances in the intervals which are relevant for environmental liability law. Thus, the known results of environmental economics are confirmed for a single investment. Nevertheless, sensitivity analysis of the left hand side coefficients of both the basic and valuation programmes shows that tightening the law may be counterproductive even for environmentally beneficial investments. The maximum payable price $P_1^{opt}$ may increase, decline or remain constant, for several reasons:

a) Mathematical proof regarding the structure of evaluation on imperfect markets (comparing the situation before and after realising the investment):

- The rates $ecd_{hs}$ of expected costs of damages are coefficients for decision variables which are basis or non-basis variables. This may differ between the basic programme and the valuation programme.
- The minimum withdrawal constraint connects the basic and the valuation programmes.
- Negative $NPV(corr)$ are not part of the optimal solution – in either of the programmes.

Therefore, (over-) compensation for the effects of a tightened scheme of environmental liability law between the two programmes is possible because some of the processes contributing to the terms IV and V of (3.4) may be dispensed with earlier. Hence, while the (positive) terms II of (3.4) are still decreasing, the absolute value of some of the negative terms (IV) and (V) can no longer be diminished.

b) Economic interpretation:

Higher expected costs of damages may initially ameliorate the conditions of environmentally beneficial processes in comparison to the older ones in the situation without investment: the producer employs an environmental protection technology to reduce the expected harm of his production to the environment. Therefore, the new processes after realising the investment (i.e. in the valuation programme) are relatively less affected by higher expected costs of damages than the previous ones before investing (i.e. in the basic programme). Hence, without realising the investment in environmental protection technology, the optimal investment and production programme may lose its profitability faster than in the case of employing such a technology – with the predicted result that investments in environmental protection technologies would be encouraged.
However, if the expected costs of damages still continue to rise, this different rate of losing profitability may cause that “dirty” processes, which were used in the optimal solution of the basic programme with lower expected costs of damages before, gain negative corrected NPVs more quickly than the employed (cleaner) ones in the valuation programme. Since processes and other objects with negative (corrected) NPVs will no longer be chosen in the optimal solution, they will no longer diminish $SWW_{\text{opt}}$ either. The optimal solution of the dual valuation programme (and consequently $P_1^{\text{opt}}$) may then deteriorate because, according to term II of (3.4) in conjunction with (3.1), terms with positive corrected NPVs may decline with the rising expected cost of damages, while the corresponding (former positive) ones in the terms IV and V can no longer be diminished because processes not chosen can no longer enter (3.4).

This situation may especially be encountered, when without investment in environmental protection production would be stopped (therefore, tightened environmental liability could no longer affect production in the basic programme and harm to the environment would no longer occur), while it still delivers a positive contribution margin when producing with the clean new processes after realising the investment. Then, we would still produce in the valuation programme to cover fixed costs – and, consequently, there would still be harm to the environment, while the profitability of production would be more and more affected by tightened environmental liability law.

Consequently, both the mathematical derivation as well as its economic interpretation prove that a tightened environmental liability law sometimes may lead to the paradoxical situation that: 1. it is unprofitable to invest in an environmentally beneficial technology; 2. the marginal incentive to invest is negative; and 3. the danger/harm for the environment even increases. Before this is demonstrated by employing a simple example in the following section, it should be stated that this – perhaps unexpected – outcome does not result from employing a linear programming approach but rather from the interdependencies between (constrained) production, restricted environmental capacities, and investments on imperfect markets.\(^\text{12}\)

### 4. EXAMPLE – EFFECTS OF ENVIRONMENTAL LIABILITY LAW ON THE WILLINGNESS TO INVEST IN ENVIRONMENTAL PROTECTION TECHNOLOGIES

Given an imperfect market under certainty with a lending opportunity at the interest rate $i_L = 50\%$ (investment object $inv_L$), but without the possibility of borrowing money, an investor with the initial amount of cash $uz_0 = 50$ [$\text{S}$] in $t = 0$ wants to maximise his withdrawals in $t = 1$.\(^\text{13}\) Therefore, he can produce with the basic activity $\nu_2^{B,\text{old}} = \begin{pmatrix} B_{1,\text{old}}^2; B_{2,\text{old}}^2; x_{p,\text{old}}^2; x_{E,\text{old}}^2 \end{pmatrix} = \begin{pmatrix} 4; 5; 8; 10 \end{pmatrix}$ up to a maximum activity levels of $\lambda_{\text{old,max}} = 10$ at the current prices $p = (p_{r1}; p_{r2}; p_{xP}; p_{xE})' = (-4; -2; 10; 0)$. Suppose the government intends to promulgate a new environmental liability law, and the producer must consider compensation for damages resulting from his emissions $E$. In the first stage he calculates for both points in time with the following discrete function $ecd_{xE}$, which leads to 3 intervals $h$ for the partial processes $\lambda_{h}^{\text{old}}$ (with [QU] for “quantity units”):

\[
ecd_{xE,1} = 0 \quad \text{for} \quad 0 \ [\text{QU}] \leq x_E \leq 30 \ [\text{QU}] \quad \Rightarrow \quad 0 \leq \lambda_{1}^{\text{old}} \leq \lambda_{1,\text{max}}^{\text{old}} = 3
\]
\[
ecd_{xE,2} = 1 \quad \text{for} \quad 30 \ [\text{QU}] < x_E \leq 60 \ [\text{QU}] \quad \Rightarrow \quad 0 < \lambda_{2}^{\text{old}} \leq \lambda_{2,\text{max}}^{\text{old}} = 3
\]

\(^{12}\) Thus, this outcome can also be derived by employing other approaches (e.g. convex programming techniques which also take these interdependencies into account).

\(^{13}\) The reader may suspect that these assumptions may seem not very realistic. However, similar results can be derived with other numbers, a longer time horizon, a different structure for the desired withdrawals, and more complex assumptions regarding the borrowing and lending conditions on the market as well. The purpose of choosing $i_L = 50\%$ and not allowing for credits is merely to simplify the example as much as possible, while still focussing on demonstrating the main outcomes which were derived in chapter \(^{3}\).
\[ ecd_{3,E,3} = 2 \quad \text{for} \quad 60 \, [\text{QU}] < x_E \quad \Rightarrow \quad 0 < \lambda_3^{\text{old}} \leq \lambda_3^{\text{old, max}} = 4 \]

Therefore, he wants to replace his current process by a new one with the input/output vector \( q \, B, I = \left( B_1, B_2, B_3, B_4 \right) = (5, 5, 8, 7) \). This means: for 3 [QU] (quantity units) less of emissions than the old process the new one needs 1 [QU] more of input 1. Since it can also be operated at maximum activity levels of \( A^{\text{l, max}} = 10 \), we will derive \( \lambda_1^{\text{l, max}} = 30/7 \), \( \lambda_2^{\text{l, max}} = 30/7 \), and \( \lambda_1^{\text{l, max}} = 10/7 \) for its partial processes. For the purpose of uninstalling the old process and installing the new one, the investor has to spend \( z_{I,0} = -50 \, [\text{£}] \) in \( t = 0 \).

Depending on the rates of expected costs of damages \( ecd_{s,E,h} = ecd_{s,E,h0} = ecd_{s,E,h1} \) we arrive at the optimal solutions \( SWW^{\text{opt}} \) of the basic and \( p_1^{\text{opt}} \) of the valuation programme as given in table 1.

With no or only little compensation for environmental damages, the investment is not worthwhile: at both points in time, the resulting contribution margin is smaller (50 [£] instead of 54 [£]), and the initial amount of cash (and the interest) for uninstalling the old process and installing the new one is lost. Thus, the maximum payable price \( p_1^{\text{opt}} = -116 \, 2/3 \, [\text{£}] \) for realising the investment is negative, i.e., the investor only installs clean(er) technologies if someone else pays for this.

As expected, tightening environmental liability law affects the maximum payable price for the environmental beneficial investment; although not always in the politically desired manner as shown in section 3.2. Starting from \( ecd_{s,E} = 0 \, [\text{£}/\text{QU}] \), the contribution margins of production will decline in both the basic and the valuation programme. Since production with the cleaner process leads to fewer emissions than with the old one, the contribution marginal of the old process declines faster – therefore, the investment becomes more and more profitable. At \( ecd_{s,E} = (1/3; 1 1/3; 2 1/3) \, [\text{£}/\text{QU}] \), the investment reaches its break-even point. Now, the investor is even willing to pay for it. Up to \( ecd_{s,E} = (3.4; 4.4; 5.4) \, [\text{£}/\text{QU}] \), this advantage is increasing, so that the investor is able to pay increasing amounts for the investment and still obtains, at the least, the same sum of weighted withdrawals as in the situation without investment.

For \( ecd_{s,E} > 5.4 \, [\text{£}/\text{QU}] \), the contribution marginal of the production using the old process is too low to continue producing. Hence, for \( ecd_{s,E} > (3.4; 4.4; 5.4) \, [\text{£}/\text{QU}] \) only production in the 1st and 2nd interval allows for \( SWW^{\text{opt}} \), while it is stopped in the 3rd interval (and no longer pollutes the environment either). In the valuation programme, however, it is still worthwhile to produce in all 3 intervals. Hence, pollution after realising the investment is higher than before. Furthermore, since in both programmes the first two partial processes are charged, but tightening environmental liability \( ecd_{s,E,3} \) only diminishes the contribution margins of production using process I, the price ceiling \( p_1^{\text{opt}} \) for the investment in environmental protection technologies begins to fall.

Both effects described even accelerate for \( ecd_{s,E} > (4.4; 5.4; 6.4) \, [\text{£}/\text{QU}] \), because the old process becomes disadvantageous in the 2nd interval, too – until, beyond \( ecd_{s,E} = (5 1/7; 6 1/7; 7 1/7) \, [\text{£}/\text{QU}] \), production with the cleaner process becomes unprofitable in the 3rd interval as well. But with \( ecd_{s,E} > (5.4; 6.4; 7.4) \, [\text{£}/\text{QU}] \), the whole production with old2 becomes unprofitable, and the sum of weighted withdrawals \( SWW^{\text{opt}} \) remains constant (still, the initial amount of cash \( u_{z0} = 50 \, [\text{£}/\text{QU}] \) can be invested at the lending opportunity \( v_{I,1} \)). Hence, only production with the cleaned process I may lead to environmental damages at both points in time. Consequently, constant withdrawals in the situation without investment, and decreasing contribution margins in the situation with investment, cause the maximum payable price \( p_1^{\text{opt}} \) for the filter investment to continue to decline.
Table 1: Optimal solutions of BP and VP with respect to environmental liability

<table>
<thead>
<tr>
<th>ecd_{xE,h} (with h = 1; 2; 3)</th>
<th>SWW_{opt}</th>
<th>P_{opt}</th>
<th>Emissions x_E before investment</th>
<th>after investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0; 0; 0</td>
<td>1,425</td>
<td>-116 2/3</td>
<td>100</td>
<td>70</td>
</tr>
<tr>
<td>0; 1; 2</td>
<td>1,150</td>
<td>-16 2/3</td>
<td>100</td>
<td>70</td>
</tr>
<tr>
<td>1/3; 1 1/3; 2 1/3</td>
<td>1,066 2/3</td>
<td>0</td>
<td>100</td>
<td>70</td>
</tr>
<tr>
<td>1; 2; 3</td>
<td>900</td>
<td>33 1/3</td>
<td>100</td>
<td>70</td>
</tr>
<tr>
<td>2; 3; 4</td>
<td>650</td>
<td>83 1/3</td>
<td>100</td>
<td>70</td>
</tr>
<tr>
<td>3; 4; 5</td>
<td>400</td>
<td>133 1/3</td>
<td>100</td>
<td>70</td>
</tr>
<tr>
<td>3.4; 4.4; 5.4</td>
<td>300</td>
<td>153 1/3</td>
<td>60</td>
<td>70</td>
</tr>
<tr>
<td>4; 5; 6</td>
<td>210</td>
<td>143 1/3</td>
<td>60</td>
<td>70</td>
</tr>
<tr>
<td>4.4; 5.4; 6.4</td>
<td>150</td>
<td>96 2/3</td>
<td>30</td>
<td>70</td>
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<td>105</td>
<td>94 6/7</td>
<td>30</td>
<td>70</td>
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<td>5 1/7; 6 1/7; 7 1/7</td>
<td>82 5/7</td>
<td>82 5/7</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
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<td>75</td>
<td>74 2/7</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
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<td>75</td>
<td>14 2/7</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
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<td>75</td>
<td>0</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>7; 8; 9</td>
<td>75</td>
<td>-42 6/7</td>
<td>0</td>
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</tr>
<tr>
<td>8; 9; 10</td>
<td>75</td>
<td>-30</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

For \( ecd_{xE} \geq (6 1/7; 7 1/7; 8 1/7) \ [S/QU] \), the investment will even lose its profitability: though production with process I is still worthwhile because the contribution margins are still positive, they do not cover the activity level-independent payments \( z_{I,0} = -50 [S] \) for (un)installing the processes in \( t = 0 \). If finally \( ecd_{xE} \leq (7 1/7; 8 1/7; 9 1/7) \ [S/QU] \), there will not be any production in the valuation programme either. For this reason, the investor will lose \( z_{I,0} = -50 [S] \) overall.

Nevertheless, although a tightened scheme of environmental liability in this example may lead to an investment in cleaner technologies becoming unprofitable, it could be argued that environmental protection sometimes costs money. But even this argument does not hold: Starting at \( ecd_{xE} = (3.4; 4.4; 5.4) \ [S/QU] \), higher rates of expected costs of damages lead to more undesired output, and for \( (6 1/7; 7 1/7; 8 1/7) \ [S/QU] \ < ecd_{xE} < (7 1/7; 8 1/7; 9 1/7) \ [S/QU] \), investing in environmental protection technology results in a negative outcome not only in terms of profitability, but also for environmental protection (cp. dark shading in table 1). Taking into account that table 1 furnishes information on the maximum price payable for the clean investment, the actual price to pay will normally be greater than 0 [S] (the difference between the price...
ceiling and the actual price leads to the actual profitability of the investment), this dark shaded area of negative outcome for both the investment’s profitability and environmental protection will be even larger.

5. CONCLUSION

This paper has offered a general approach to valuate investments in environmental protection technologies with particular regard to the effects of environmental liability law. Since these investments affect production, the payments required for a financial valuation have to be derived from production theory and production planning. With respect to the environment, production processes are characterised by joint production. Environmental liability modifies the contribution margins and the constraint system. The model considers activity level-dependent and -independent payments and handles the indivisibility of the investment in two steps.

Applying duality theory allows one to identify the determinants of the price ceiling for an investment with regard to environmental liability, and uncertainty. Also, the model provides exact information on the determinants of the maximum payable price. This price ceiling can be interpreted as a sum of (sometimes corrected) net present values; information on probabilities, means and variances is not required. Surprisingly, if more than one partial process \( h \) of a process \( \beta \) is chosen, then all these partial processes have the same nonnegative corrected net present value, which is also the corrected net present value of the whole process \( \beta \).

(Thus, the corrected net present values of the partial processes are not value additive.) Using sensitivity analysis, we have been able to demonstrate that a tightened scheme of environmental liability does not always encourage environmentally beneficial investments. In particular cases, it may even lead to the paradoxical situation that: 1. it is unprofitable to invest in an environmental protection technology; 2. the marginal incentive to invest is negative; and 3. the harm for the environment even increases.

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