THE FINITE HORIZON TRENDED CONTINUOUS INVENTORY REPLENISHMENT AND PRICING PROBLEMS FOR DETERIORATING ITEMS WITH STOCK AND PRICE SENSITIVE DEMAND

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ABSTRACT
In this paper, an inventory model for deteriorating items with price and stock dependent demand rate over a finite planning horizon is reconciled. In contrast to the traditional deterministic inventory model with static price over the entire planning horizon or fixed number of price changes over the finite time horizon, an alternative continuous model is derived in which prices selling and setting are to be the decision variables. It is shown that the total profit function is concave. With the concavity, a solution procedure is presented to determine the optimal order replenishment, optimal price selling and setting strategy and optimal profit for deteriorated seasonal items. Numerical examples and sensitivity analyses are given to validate the results of the inventory model. The analysis shows the influence of key model parameters.

KEYWORDS: Pricing, Ordering, Stock dependent demand, Continuous model, Deterioration

MSC: 90B05

RESUMEN
En este documento, un modelo de inventario para el deterioro de los elementos con los precios y tipos de acciones depende de la demanda en un horizonte de planificación finitos se ha reconciliado. En contraste fijo de los cambios de precios en el horizonte de tiempo finito, un modelo continuo alternativa se deriva en que los precios de venta y el establecimiento deben ser las variables de decisión. Se demuestra que la función de beneficio total es cóncava. Con la concavidad, un procedimiento de solución se presenta para determinar la reposición del orden óptimo, el precio de venta óptimo y las estrategias de ajuste y el beneficio óptimo de deterioro artículos de temporada. Ejemplos numéricos y análisis de sensibilidad se dan para validar los r esultados del modelo de inventario. El análisis muestra la influencia de los parámetros del modelo clave.

1. INTRODUCTION

Classical inventory models are usually developed over infinite planning horizon. The assumption of an infinite planning horizon is not realistic due to several reasons such as variations of inventory costs, changes in product specifications and designs, technological changes etc., the business period is not finite. Seasonal items are an important part of stocks carried in practice. The problem of managing inventory of a seasonal product is complex for a variety reasons. The product can usually produced by the vendor only at finite rate. In many instances, the demand for the product is sensitive to the price charged by the vendor to the customer.

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This paper establishes and analyzes three inventory models under profit maximization which extends the classical economic order quantity (EOQ) model. An efficient EOQ does more than just reduce cost. It also creates revenue for the retailer and the manufacturer. The evolution of the EOQ model concept tends toward revenue and demand focused strategic formation and decision making in business operations. Evidence can be found in the increasingly prosperous revenue and yield management practices and the continuous shift away from supply-side cost control to demand-side revenue stimulus. This paper focuses on the profit maximizing issues in a continuous production model, based on cost reduction mechanisms and a revenue improvement stimulus and formulates three decision models by considering the effects of price setting/changing and the replenishment policy, taking into account the effect of an on demand pricing scheme and revenue increments.

Many inventory models have been proposed to deal with a variety of inventory problems. Comprehensive review of inventory models can be found in Khouja (1999) and Petruzzi and Dada (1999). To control an inventory system, one cannot ignore demand monitoring since inventory is partially driven by demand, and as suggested by Lau and Lau (2003). Tripathy and Pattnaik (2011) developed a fuzzy inventory model where unit cost of production is a function of reliability and constant demand. Researchers and practitioners are of opinion that one of the major factors for the occurrence of variability of demand rate is due to its time dependency. As pricing is an obvious strategy to influence demand, studies on inventory models with price-dependent demand have received much attention. Polatoglu (1991) proposed an inventory model for developing pricing and procurement decisions simultaneously. You (2005) developed an inventory model in which the demand is price and time dependent and the number of price changes can be controlled. Khouja (2000) investigated a newsboy problem in which discount prices are decision variables and discount strategies are used to sell excess inventory. Shinn and Hwang (2003) dealt with the problem with determining the order quantity under the condition that the demand is a convex function of price and delay in payments is order-size dependent. In addition to the researches on inventory models with price-dependent demand many researchers have observed the phenomenon that demands for an item may increase with the presence of quantity goods is investigated by Zhou and Yang (2003). You (2007) extended an inventory model with stock and price sensitive demand where multiple price changes are allowed.

There is considerable literature on problem of determining the lot size of seasonal products with deterioration under additions of finite productions and fixed demand. The production-inventory systems of deteriorated seasonal products are most common in reality and a number of researchers have investigated the problem of determining economic replenishment policy of such items. Bhunia and Maiti (1998) investigated a deteriorating inventory model with linear stock and time dependent demand. Mandal and Maiti (1999) and Giri and Chaudhuri (1998) dealt with an inventory model with power form stock dependent demand. Chung et al. (2000) studied a deteriorating inventory with linear stock dependent demand. Chung (2003) developed an algorithm for an inventory system with a power form stock dependent demand. Balkhi and Benkherouf (2004) discussed a deteriorating inventory model with a power form stock and time dependent demand for a finite planning horizon. Teng and Chang (2005) investigated a production model with linear stock-dependent demand. Urban (2005) provided a comprehensive overview on inventory models with inventory level dependent demand, and distinguished between the initial inventory dependent models and instantaneous inventory level dependent models. Urban (2005) dealt with a periodic review inventory model under the assumption that demand is serially correlated and dependent on the initial inventory level. Although these models offer very good insights into the literature of the lot sizing problem and many of its aspects, they do not take into account the continuous model with deteriorated seasonal products in finite planning horizon. Incorporating deteriorated seasonal products and stock and price sensitive demand explicitly in the model may reveal new insights about the setting and ordering of price relationship. Tripathy and Pattnaik (2008) investigated a fuzzy entropic order quantity model for perishable items where pre and post deterioration discounts are allowed with two component demand. Tripathy and Pattnaik (2011) extended this work with modification of the model with constant demand and instant deterioration discount for perishable items in fuzzy decision space. Finally, most of these models dealt with single-level lot sizing problems. Incorporating multiple price changes when the demand is price and stock dependent for deteriorated seasonal products adds accuracy to the model and new insight about the relationship between the number of price change, selling price and lot sizing.

To survive and thrive in the highly competitive retail world, retailers must become more attentive and meticulous with their pricing. The financial success of companies selling retail goods depends on their pricing strategy. In single-price strategy customers have to pay the same price for all items in entire planning horizon. With such a policy the variety of offerings is often limited. The strength is being able to avoid employee error and facilitate the speed of transactions. Despite the increased role of non-price factors in the modern marketing process, price remains an important element in case of marketing decisions. Historically, price has been the major factors affecting buyer's choice. Price is also one of the most flexible elements of the marketing mix as it can be increased or decreased according to need. At the same time pricing is one of the major problems faced by many marketing executives or decision makers as product move through their life cycle. This led many researchers to investigate inventory models with price dependent demand. Inventory management plays an important role in business since it can help the companies reach the goal of ensuring the prompt delivery, avoiding shortages, helping sales at competitive prices and so forth. Since a firm may use a pricing strategy to spur demand for its seasonal goods, the inventory problems with price and stock dependent demand cannot be ignored. Urban and Baker (1997) investigated a deterministic inventory problem in which the demand is a multivariate function of price, time and inventory level. Their basic model with a single price is extended to a model with a single price markdown.

It is noted that the literature herein rarely considers the cases with multiple price changes when the demand is price and stock dependent. Since a firm may reset its selling price to spur demand when consumer's purchasing behavior are price and stock dependent, this paper studies single replenishment inventory model to deal with this problem. The purpose of this paper is to develop the solution procedure for determining the optimal order size and optimal selling prices for a deteriorated seasonal item.

The remainder of the paper is organized as follows. In section 2 assumptions and notations are provided for the development of the model. The mathematical formulation is developed in section 3. In section 4, inventory model without price change is derived and the existence of the solution is verified. The inventory model with a single price change and the solution procedure are given in section 5. The inventory model with two price changes is formulated and the solution procedure is given in section 6. In section 7, the numerical examples are presented to illustrate the development of the three different cases. The sensitivity analysis is carried out in section 8 to observe the changes in the optimal solution. Finally section 9 deals with the summary and the concluding remarks.

Table. 1Summary of the Related Researches

Author	Demand factors	Demand patterns	Deterioration	Planning Horizon	Changing Price	Structure of the model
Bhunia et al. (1998)	Stock and time	Linear	Yes	Finite	No	Crisp
Giri et al. (1998)	Stock	Power form	Yes	Finite	No	Crisp
Mandal et al. (1999)	Stock	Power form	Yes	Finite	No	Crisp
Balkhi etal (2004)	Stock and time	Power form	Yes	Finite	No	Crisp
You et al. (2007)	Stock and price	Linear (sensitive)	No	Finite	Yes	Crisp
Tripathy et al. (2008)	Stock	Linear	Yes (Heaviside)	Finite	No	Fuzzy
Tripathy et al. (2010)	Stock	Linear	Yes (Heaviside)	Finite	No	Fuzzy
Present paper (2011)	Stock and price	Linear (sensitive)	Yes (constant)	Finite	Yes	Crisp

2. ASSUMPTIONS AND NOTATION

Suppose a firm purchases Q units of a deteriorated seasonal item and sells them over a finite time horizon L and θ is the constant deterioration rate, where $0 \leq \theta \leq 1$. Demand for the item is assume to be price and stock dependent. The firm previously divides the planning horizon L into $n \leq \bar{n}$ equal time periods, each with T = L / n time units. The firm set an initial selling price at the start of period 1. At the start of subsequent periods, the firm resets the selling price. The selling price set during period is j denoted by p_j.

The demand rate at t of period j is assumed to follow the form of $\lambda_j\left(p_{j,}t\right) = \alpha - \beta p_j + \eta I_j(t)$ where α is the intersection of the demand curve, the values of β and η are constants, and $I_j(t)$ represents the inventory level with time t of period j.

It is assumed that the unit time holding cost per unit is h and the unit purchasing cost is c. Changing price may involve some costs, such as changing price lists, tags and catalogues, changing product label, advertising price changes as well as communicating the logic behind the list price changes to different firms. It assumes that there is an ordering cost K associated with each price setting. The price adjustment cost can be estimated by the sum of all the component costs. The firm aims to maximize its profit by simultaneously determining (1) the order quantity Q and (2) the selling prices $p_{1,}p_{2,} \ldots\ldots, p_n$. The notation is summarized in the following.

Notation

Q: Order Quantity,

L: Planning time interval,

n: The total number of periods (n-1 also represents the number of price changes),

T: Length of a period, T = L/n,

t: Period index, period j refers to the time interval [(j-1) T, jT),

p: Selling price set during period j,

λ(p,t): Demand rate at time t of period j when the initial selling price is set at p,

c: Unit purchasing cost,

h: Unit inventory holding cost per unit time,

K: Pricing setting cost, and

θ : The deterioration rate.

3. FORMULATION

Now the model of the problem is developed. Suppose the firm divides the sales season into n periods. In addition, assume that the firm sets the order quantity and the selling price at Q and P = (p_1, p_2,….p_n), respectively. Then since the demand rate at time t of period j is λ_j (t,p_j) = α – βp_j +ηI_j(t), for period j.

$$\frac{dI_j(t)}{dt} = -\left(\alpha - \beta p_j + \eta I_j(t)\right) - \theta I_j(t), \; 0 \le t \le T \tag{1}$$

Where, time points 0 and T respectively denote the starting and ending times of a period. Let q_j be the inventory level at the start of period j. Then we have I_j (0) = q_j from which

$$I_j(t) = e^{-(\eta+\theta)t} q_j - \frac{\left(1-e^{-(\eta+\theta)t}\right)(\alpha-\beta p_j)}{\eta+\theta} \tag{2}$$

Since I_{j-1} (T) = I_j(0), it follows that, I_{j-1} (T) =q_j from which

$$q_j = x^{-1} q_{j-1} - \frac{\left(1-x^{-1}\right)(\alpha-\beta p_{j-1})}{\eta+\theta} \tag{3}$$

Where $x = e^{-(\eta+\theta)T}$. To reduce the unknown term q_j in (2), q_j is expressed in terms of Q and P. Since the initial inventory level is Q, we have q_1 = Q. Thus, $q_2 = x^{-1} Q - \frac{(1-x^{-1})(\alpha-\beta p_1)}{\eta+\theta}$. Let $q_j = x^{-(j-1)} Q - \frac{\alpha-(1-x^{-(j-1)})}{\eta+\theta} +$

$(1-x^{-1})\left(\frac{\sum_{k=2}^{j} x^{-(j-k)}\beta p_{k-1}}{\eta+\theta}\right)$. Then, $q_2 = \hat{q}_2$. Assume that $q_{j-1} = \hat{q}_{j-1}$. Then by induction, since it is shown that $q_j = \hat{q}_j$, q_j can be re-expressed as follows.

$$q_j = x^{-(j-1)}Q - \frac{\alpha\left(1-x^{-(j-1)}\right)}{\eta+\theta} + \frac{(1-x^{-1})\left(\sum_{k=2}^{j} x^{-(j-k)}\beta p_{k-1}\right)}{\eta+\theta} \tag{4}$$

Now the profit function is developed, which is comprised of sales revenues, inventory holding cost, purchasing cost and pricing setting cost. Substituting q_j in (4) into $I_j(t)$ in (2)

$$I_j(t) = e^{-(\eta+\theta)t}\left(x^{-(j-1)}Q - \frac{\alpha\left(1-x^{-(j-1)}\right)}{\eta+\theta} + (1-x^{-1})\left(\frac{\sum_{k=2}^{j} x^{-(j-k)}\beta p_{k-1}}{\eta+\theta}\right)\right) - \frac{\left(1-e^{-(\eta+\theta)t}\right)(\alpha-\beta p_j)}{\eta+\theta} \tag{5}$$

Sales Revenues

Let Δq_j denote the sales amount during period j. Then, $\Delta q_j = q_j - q_{j+1}$

$$= \frac{x^{-j}(x-1)(\alpha+(\eta+\theta)Q)}{\eta+\theta} + \frac{\beta\left(1-x^{-1}\right)}{\eta+\theta}\left(\sum_{k=2}^{j} x^{-(j-k)} p_{k-1}(1-x^{-1})\right) - \frac{(1-x^{-1})\beta}{\eta+\theta} p_j$$

$$= \frac{x^{-j}(x-1)(\alpha+(\eta+\theta)Q)}{\eta+\theta} + \frac{\beta\left(1-x^{-1}\right)^2}{\eta+\theta}\left(\sum_{k=1}^{j-1} x^{-(j-k-1)} p_k\right) - \frac{\beta\left(1-x^{-1}\right)p_j}{\eta+\theta} \tag{6}$$

Let $\bar{R}(n)$ be the sales revenue when the firm divides the sales season into n periods. Then, $\bar{R}(n) = \sum_{j=1}^{n} \Delta q_j . p_j$

$$= \frac{(x-1)(\alpha+(\eta+\theta)Q)}{\eta+\theta} \sum_{j=1}^{n} x^{-j} p_j + \frac{\beta\left(1-x^{-1}\right)^2}{\eta+\theta} \sum_{j=2}^{n} \sum_{k=1}^{j-1} x^{-(j-k-1)} p_k p_j - \frac{\beta\left(1-x^{-1}\right)}{\eta+\theta} \sum_{j=1}^{n} p_j^2 \tag{7}$$

Inventory Carrying Cost

Let $\bar{H}_j(n)$ be the carrying cost of period j when the firm divides the sales season into n periods. Then, $\bar{H}_j(n) = \int_{t=0}^{T} I_j(t)\, hdt = (1-x^{-1})\left[(\eta+\theta)x^{-(j-1)}Q - \alpha\left(1-x^{-(j-1)}\right) + \beta(1-x^{-1})\sum_{k=2}^{j} x^{-(j-k)} p_{k-1}\right]\frac{h}{(\eta+\theta)^2}$

$$- \frac{\left(1-(\eta+\theta)T - x^{-1}\right)(\alpha-\beta p_j)}{(\eta+\theta)^2} h \tag{8}$$

From which $\bar{H}(n) = \sum_{j=1}^{n} \bar{H}_j(n)$

$$= \left[(1-x^{-n})(\alpha+(n+\theta)Q) + \beta(1-x^{-1})^2 \sum_{j=1}^{n}\sum_{k=2}^{j} x^{-(j-k)} p_{k-1}\right]\frac{h}{(\eta+\theta)^2} + \frac{\beta h}{(\eta+\theta)^2}\left(1-(\eta+\theta)T - x^{-1}\right)\sum_{j=1}^{n} p_j + \frac{\alpha n T h}{\eta+\theta} - \frac{2\alpha n h}{(\eta+\theta)^2}(1-x^{-1}) \tag{9}$$

Where, $\bar{H}(n)$ is the total carrying cost when the firm divides the sales season into n periods.

Total Profit Function

Let F(n,P,Q) be the total profit when the firm divides the sales season into n periods. Then,

$$F(n,P,Q) = \bar{R}(n) - \bar{H}(n) - Q(n)c - nK \tag{10}$$

Note that the inventory level at the ending time of period n is zero. Thus, $I_n(T) = 0$. Let Q(n) be the solution to the equation $I_n(T) = 0$. Then,

$$Q(n) = \frac{\alpha(x^n - 1) - \beta(1 - x^{-1})\sum_{k=1}^{n} x^k p_k}{\eta + \theta} \quad (11)$$

Substituting Q= Q(n) into $\overline{\mathrm{R}}(n)$ and $\overline{\mathrm{H}}(n)$ and letting F(n,P) be the result,

F(n,P) = R(n) – H(n) – Q(n)c – nK (12)

Where, $R(n) = (x-1)\left(\frac{\alpha(x^n-1)-\beta(1-x^{-1})\sum_{k=1}^{n} x^k p_k}{\eta+\theta} + \frac{\alpha}{\eta+\theta}\right) \quad \sum_{j=1}^{n} x^{-j} p_j + \frac{\beta(1-x^{-1})^2}{\eta+\theta} \sum_{j=2}^{n} \sum_{k=1}^{j-1} x^{-(j-k-1)} p_k p_j - \beta(1-x^{-1}) \quad \sum_{j=1}^{n} \frac{p_j^2}{\eta+\theta}$ (13)

$H(n) =$
$(1 - x^{-n})\left(\alpha + (\eta+\theta)\left(\frac{\alpha(x^n-1)-\beta(1-x^{-1})\sum_{k=1}^{n} x^k p_k}{\eta+\theta}\right)\right) + \frac{\beta(1-x^{-1})^2 \sum_{j=1}^{n} \sum_{k=2}^{j} x^{-(j-k)} p_{k-1}}{(\eta+\theta)^2} h - \frac{\beta(1-(\eta+\theta)T - x^{-1})\sum_{j=1}^{n} p_j}{(\eta+\theta)^2} h + \frac{nT\alpha}{\eta+\theta} h - \frac{2\alpha n h}{(\eta+\theta)^2}(1 - x^{-1})$ (14)

4. INVENTORY MODEL WITHOUT PRICE CHANGE

In this section, it is assumed that the firm sets its setting price at the start of the sales season and does not reset its selling price thereafter substituting n=1 into (12) then, $F(1, p_1) = \frac{-\beta(x-1)p_1^2}{\eta+\theta} + \frac{(x-1)p_1}{(\eta+\theta)^2} \times [\alpha(\eta+\theta) + \beta h + \beta c(\eta+\theta)] - \frac{\beta p_1 T h}{\eta+\theta} - \left[\frac{(1-x^{-1})\alpha h(x-2)}{(\eta+\theta)^2} + \frac{Th\alpha}{\eta+\theta} + \frac{\alpha(x-1)c}{\eta+\theta} + K\right]$ (15)

Taking the first and second derivatives of $F(1, p_1)$ with respect to p_1 gives

$$\frac{\partial F(1,p_1)}{\partial p_1} = \frac{-2p_1\beta(x-1)}{\eta+\theta} + \frac{(x-1)}{(\eta+\theta)^2}[(\eta+\theta)\alpha + \beta h + \beta(\eta+\theta)c] - \frac{\beta T h}{\eta+\theta} \quad (16)$$

$$\frac{d^2 F(1,p_1)}{dp_1^2} = \frac{-2\beta(x-1)}{\eta+\theta} < 0 \quad (17)$$

Let Pj(n) be the optimal setting price for period j when the firm divides the sales season into n periods. Since $\frac{d^2 F(1,p_1)}{dp_1^2} < 0$, objective function is concave in p_1. Accordingly, the optimal setting price is given by the solution in the first order condition Let (16) be equal to zero.

Then, $p_1(1) = \frac{\alpha(\eta+\theta) + \beta(c(\eta+\theta)+h)}{2\beta(\eta+\theta)} + \frac{Th}{2(1-x)}$ (18)

Substituting $p_1 = p_1(1)$ into (15), $Q(1) = \frac{\beta T h}{2(\eta+\theta)} + \frac{(x-1)[\alpha(\eta+\theta) - \beta(c(\eta+\theta)+h]}{2(\eta+\theta)^2}$ (19)

5. INVENTORY MODEL WITH A SINGLE PRICE CHANGE

In this section, it is assumed that the firm sets its selling price at the start of the sales season and resets its selling price at the time of L/2 substituting n=2 into (12) then,

$F(2,p_1,p_2) = \frac{-\beta(x-1)(p_1^2+p_2^2)}{\eta+\theta} - \frac{\beta(x-1)^2 p_1 p_2}{\eta+\theta} + \frac{(x-1)(x\alpha(\eta+\theta)+\beta h+\beta c(\eta+\theta))}{(\eta+\theta)^2} p_1 + \frac{(x-1)(\alpha(\eta+\theta)+x\beta h+x\beta c(\eta+\theta))p_2}{(\eta+\theta)^2} - \frac{\beta h T(p_1+p_2)}{\eta+\theta} - \frac{2T\alpha h}{\eta+\theta} - \frac{\alpha(x^2-1)(h+c(\eta+\theta))}{\eta+\theta} - 2K$ (20)

The Hessian matrix of F (2, p_1, p_2) is given by $H_2 = \begin{pmatrix} \frac{-2\beta(x-1)}{\eta+\theta} & \frac{-\beta(x-1)^2}{\eta+\theta} \\ \frac{-\beta(x-1)^2}{\eta+\theta} & \frac{-2\beta(x-1)}{\eta+\theta} \end{pmatrix}$ (21)

Note that $\frac{\partial^2 \mathrm{F(2,p1,p2)}}{\partial p_1^2} < 0$ and $|H_2| = \frac{-\beta^2}{(\eta+\theta)} \times (x-1)^2(x-3)(x+1) > 0$ for x < 3. Thus, F is concave function of selling prices when $x = e^{(\eta+\theta)T} = e^{0.5(\eta+\theta)L} < 3$. Accordingly, for x< 3, the optimal setting prices are given by the solutions to the first order conditions

$$\frac{\partial F(2,p_1,p_2)}{\partial p_1} = \frac{-2\beta(x-1)p_1}{\eta+\theta} - \frac{-\beta(x-1)^2 p_2}{\eta+\theta} + \frac{(x-1)(\alpha x(\eta+\theta)+\beta c(\eta+\theta)+\beta h)}{(\eta+\theta)^2} - \frac{\beta hT}{\eta+\theta} = 0 \quad (22)$$

$$\frac{\partial F(2,p_1,p_2)}{\partial p_2} = \frac{-\beta(x-1)^2 p_1}{\eta+\theta} - \frac{-2\beta(x-1)p_2}{\eta+\theta} + \frac{(x-1)(\alpha(\eta+\theta)+\beta xc(\eta+\theta)+x\beta h)}{(\eta+\theta)^2} - \frac{\beta hT}{\eta+\theta} = 0 \quad (23)$$

Solving the above equation system gives

$$p_1(2) = \frac{(c(\eta+\theta)+h)(x-2)}{(\eta+\theta)(x-3)} - \frac{hT}{(x-1)(x+1)} - \frac{\alpha}{\beta(x-3)} \quad (24)$$

$$p_2(2) = \frac{\alpha(x-2)}{\beta(x-3)} - \frac{h+c(\eta+\theta)}{(\eta+\theta)(x-3)} - \frac{Th}{(x-1)(x+1)} \quad (25)$$

Substituting $p_1 = p_1(2)$ and $p_2 = p_2(2)$ into (11), then

$$Q(2) = \frac{2(x-1)(\beta(h+c(\eta+\theta))-\alpha(\eta+\theta))}{(\eta+\theta)^2(x-3)} + \frac{\beta hT}{\eta+\theta} \quad (26)$$

6. INVENTORY MODEL WITH TWO PRICE CHANGES

In this section, it is assumed that the firm sets its selling price at the start of the sales season and resets its selling prices at the times of L/3 and (2/3)L, respectively. Substituting n=3 into (20) then,

$$F(3,p_1p_2,p_3) = -\frac{\beta(x-1)(p_1^2+p_2^2+p_3^2)}{\eta+\theta} - \frac{\beta(x-1)^2(p_1p_2+p_2p_3+xp_1p_3)}{\eta+\theta} + \Big(\alpha(\eta+\theta)(x^3-x^2)+\beta\big(h+c(\eta+\theta)\big)x - \beta(h+Th(\eta+\theta)+c(\eta+\theta))\Big)\frac{p_1}{(\eta+\theta)^2} + \big((\alpha(\eta+\theta)+\beta c(\eta+\theta)+\beta h)(x^2-x)-\beta Th(\eta+\theta)\big)\frac{p_2}{(\eta+\theta)^2} + \Big(\big(\beta(h+c(\eta+\theta))\big)(x^3-x^2)+\alpha(\eta+\theta)x-\beta Th(\eta+\theta)-\alpha(\eta+\theta)\Big)\frac{p_3}{(\eta+\theta)^2} + \frac{6\alpha h(x-1)}{(\eta+\theta)^2} - \frac{3T\alpha h}{\eta+\theta} - \frac{\alpha(x^3-1)c}{\eta+\theta} - \frac{\alpha(x-1)(x^2+x+1)}{(\eta+\theta)^2}h - 3K \quad (27)$$

The Hessian matrix of $F(3,p_1p_2,p_3)$ is given by $H_3 = \begin{pmatrix} \frac{-2\beta(x-1)}{\eta+\theta} & \frac{-\beta(x-1)^2}{\eta+\theta} & \frac{-\beta(x-1)^2x}{\eta+\theta} \\ \frac{-\beta(x-1)^2}{\eta+\theta} & \frac{-2\beta(x-1)}{\eta+\theta} & \frac{-\beta(x-1)^2}{\eta+\theta} \\ \frac{-\beta(x-1)^2x}{\eta+\theta} & \frac{-\beta(x-1)^2}{\eta+\theta} & \frac{-2\beta(x-1)}{\eta+\theta} \end{pmatrix}$ (28)

Note that $\frac{\partial^2 \mathrm{F(3,p1,p2p3)}}{\partial p_1^2} < 0$ and $|H_2| = \frac{-\beta^2(x-1)^2}{(\eta+\theta)^2}(x-3)(x+1) > 0$ for x < 3 and $|H_3| = \frac{2\beta^3}{(\eta+\theta)^3}(x-1)^3(x-1)^2(x-2) < 0$ for x < 2. Thus, $F(3,p_1p_2,p_3)$ is concave function of selling prices when x < 2. Accordingly, for $x = e^{(\eta+\theta)T} = e^{(\eta+\theta)L/3} < 2$ the optimal setting prices are given by the solutions to the first order conditions. The first order conditions are

$$\frac{\partial F(3,p_1,p_2,p_3)}{\partial p_1} = \frac{-\beta(x-1)}{\eta+\theta}\big(2p_1+(x-1)(p_2+xp_3)\big) + \frac{\alpha(\eta+\theta)(x^3-x^2)+\beta(c(\eta+\theta)+h)x-\beta(h+(\eta+\theta)hT+c(\eta+\theta))}{(\eta+\theta)^2} = 0 \quad (29)$$

$$\frac{\partial F(3,p_1,p_2,p_3)}{\partial p_2} = \frac{-\beta(x-1)}{\eta+\theta}\big(2p_2+(x-1)(p_1+p_3)\big) + \frac{(\alpha(\eta+\theta)+\beta c(\eta+\theta)+\beta h)(x^2-x)-\beta Th(\eta+\theta)}{(\eta+\theta)^2} = 0 \quad (30)$$

$$\frac{\partial F(3,p_1,p_2,p_3)}{\partial p_3} = \frac{-\beta(x-1)}{\eta+\theta}\big(2p_3+(x-1)(xp_1+p_2)\big) + \frac{\beta(c(\eta+\theta)+h)(x^3-x^2)+(\alpha(\eta+\theta)x-\beta hT(\eta+\theta)-\alpha(\eta+\theta))}{(\eta+\theta)^2} = 0 \quad (31)$$

Using the above equation system yields

$$p_1(3) = \frac{(2x-3)(h+c(\eta+\theta))}{2(\eta+\theta)(x-2)} + \frac{(x-3)Th}{2(x-1)(x+1)} - \frac{\alpha}{2\beta(x-2)} \quad (32)$$

$$p_2(3) = \frac{\alpha(\eta+\theta)+\beta(c(\eta+\theta)+h)}{2\beta(\eta+\theta)} - \frac{(x^2+4-3x)Th}{2(x-1)(x+1)} \quad (33)$$

$$p_3(3) = \frac{\alpha(2x-3)}{2\beta(x-2)} + \frac{(x-3)Th}{2(x-1)(x+1)} - \frac{h+c(\eta+\theta)}{2(\eta+\theta)(x-2)} \quad (34)$$

Substituting $p_1 = p_1(3)$, $p_2 = p_2(3)$ and $p_3 = p_3(3)$ into (11),

$$Q(3) = \frac{3(x-1)[\beta(c(\eta+\theta)+h)-\alpha(\eta+\theta)]}{2(x-2)(\eta+\theta)^2} \quad (35)$$

7. NUMERICAL EXAMPLES

Example 1

Suppose L=120, h=0.005, α = 50, β=1.5, η = 0.01, c = 20 and K = 500, θ = 0.002. It is noted that $x = e^{(\eta+\theta)T} = 4.220695817$. Then, from (18) and (19) the optimal values are obtained in Table 2.

Table-2 Optimal Values for n=1(T=L)

Crisp Decision Variables	$p_1(1)$	$Q_1(1)$	$F(1, p_1(1))$
Optimal Values	26.78185243	2637.540894	14429.35518

Example 2

Suppose the parameters are the same as Example-1. It is noted $x = e^{(\eta+\theta)T} = 2.054433211 < 3$. Thus the optimal setting prices are given by the solutions to the first order conditions. From (24) – (26) the optimal values are obtained in Table 3.

Table-3 Optimal Values for n = 2 (T=L/2)

Crisp Decision Variables	$p_1(2)$	$p_2(2)$	$Q(2)$	$F(2, p_1(2), p_2(2))$
Optimal Values	33.98375634	19.57994852	3648.451955	17744.01597

Example 3

Suppose the parameters are the same as Example-1. It is noted $x = e^{(\eta+\theta)T} = 1.616074402 < 2$. Thus the optimal setting prices are given by the solutions to the first order conditions. From (32) – (35) we have obtained the optimal values in Table 4.

Table-4 Optimal Values for n = 3 (T=L/3)

Crisp Decision Variables	$p_1(3)$	$p_2(3)$	$p_3(3)$	$Q(2)$	$F(3, p_1(3), p_2(3), p_3(3))$
Optimal Values	37.15263474	26.76558279	16.42562999	3923.81339	25504.43834

8. SENSITIVITY ANALYSIS

We refer to the data set used in the above example as the basis data set, W, where W = {L = 120, h = 0.005, α = 50, β = 1.5, η = 0.01, c = 20, k = 500, θ = 0.002}. This study investigates the changes in the optimal decision values of

$p_1(3)$, $p_2(3)$, $p_3(3)$, Q(3) and F(3, $p_1(3)$, $p_2(3)$, $p_3(3)$) when only one parameter in the set W changes and other remain unchanged the computational results are described in Table 5.

In Example 3, the firm has two chances to adjust its selling prices. Table 5 shows that the initial selling price $p_1(3)$ and secondary selling price $p_2(3)$ increase in to the length of L, and the final selling price $p_3(3)$ decreases in L. This phenomenon can be explained as follows. First, we consider the situation in which the firm has a longer selling season as situation-1 and the situation in which the firm has a shorter selling season as situation-2. Note that the firm would expect to obtain higher unit profit. Compared with situation-2, situation-1 gives the firm longer time to achieve this goal. Thus it can be conjectured that the firm in situation-A may set higher initial and secondary selling prices to achieve higher unit profit. It can also be conjectured that the remaining inventory in situation-1 may be larger than that of situation-2 when the final selling price is to be set. To, sell out its inventory, the firm in situation-1 would set a lower final selling price to sell its items. Hence, $p_1(3)$and $p_2(3)$ increase in length L and $p_3(3)$decreases in L. Finally, it is observed from the Table 5 that only $p_2(3)$ is insensitive to changes in parameter L whereas other optimum decision variables are highly sensitive to changes in the parameter L.

It shows that $p_1(3)$decreases in the value of h, and $p_2(3)$and $p_3(3)$increase in the value of h. this phenomenon could be explained as follows. First, we consider the situation in which the firm has a higher inventory carrying cost as situation-3 and the situation in which the firm has a lower inventory cost as situation-4. Note that the firm would shorten its holding cost and the inventory holding cost has a tight relationship with inventory level, inventory holding time and unit holding cost. Compared with situation-4, situation-3 gives the firm more incentives to reduce its inventory when the time to sell its items is still long. It can be conjectured that the firm in situation-3 may set a lower initial selling price to reduce its inventory. It can also be conjectured that the remaining inventory in situation-3 may be less than that in situation-4, when the secondary selling price is to be set. To obtain a higher unit profit, the firm then would set higher secondary and final selling prices to sell its items. According to this reason, $p_1(3)$decreases in h, and $p_2(3)$ and $p_3(3)$increase in h. Hence it is seen all the decision variables are very low sensitive or insensitive to changes in the parameter h.

It shows that $p_1(3)$, $p_2(3)$ and $p_3(3)$ decrease in the value of α. This phenomenon could be explained as follows. First, increasing the value of α moves the demand curve up. Thus, compared with an inventory system with a smaller value of α, the firm in situation-3 with a higher value of α may set higher selling prices to improve its unit profit. According to this reason, $p_1(3)$, $p_2(3)$ and $p_3(3)$ decrease in α. The optimum $p_1(3)$, ordering quantity Q (3) and the total profit F (3, $p_1(2)$, $p_2(2)$, $p_3(3)$) respectively are highly sensitive whereas other decision variables are moderately sensitive to changes in the parameter α.

It shows that $p_1(3)$increases in the value of η, and $p_2(3)$ and $p_3(3)$decrease in the value of η. This phenomenon could be explained as follows. First we consider the situation in which the firm has a higher value of η as situation-5 and the situation in which the firm has a lower value of η as situation-6. Note that the term of η I (t) has positive impact on demand. Initially the inventory level for an inventory system is high. Under the same selling price, the demand rate in situation-5 is higher than that in situation-6. Thus, it can be conjectured that the firm in situation-5 may have more incentives to set a higher selling price to obtain higher unit profit. Once the secondary selling price is set, the firm in situation-5 may have more stock on hand. To reduce its inventory, it can be conjectured that the firm would set a lower secondary and final selling prices to reduce its inventory in situation-5. According to this reason, $p_1(3)$ increases in h, and $p_2(3)$ and $p_3(3)$ decrease in h. It shows that $p_2(3)$ is insensitive to changes in parameter η whereas other decision variables are highly sensitive to changes in parameter η.

From Table 5, it is observed that $p_1(3)$, $p_2(3)$ and $p_3(3)$ increase in the value of c. This phenomenon could be explained as follows. First, note that increasing the value of c reduces the unit profit. Thus, compared with an inventory system with a smaller value of c, the firm with higher value of c may set higher selling prices to cover its unit cost and improve its unit profit. According to this reason, $p_1(3)$, $p_2(3)$ and $p_3(3)$ increase in α. Hence all the decision variables are moderately sensitive to changes in parameter c whereas the optimal total profit F(3) is highly sensitive to changes in parameter c.

It shows that $p_1(3)$ and $p_2(3)$ increase in θ, and $p_3(3)$ decreases in θ. This phenomenon could be explained as follows. First, we consider the situation in which the firm has a longer selling season as situation-1 and the situation in which the firm has a shorter selling season as situation-2. Note that the firm would expect to obtain higher unit profit. Compared with situation-2, situation-1 gives the firm longer time to achieve this goal. Thus it can be conjectured that the firm in situation-1 may set higher initial and secondary selling prices to achieve higher unit profit. It can also be conjectured that the remaining inventory in situation-1 may be larger than that of situation-2 when the final selling price is to be set. To, sell out its inventory, the firm in situation-1 would set a lower final selling price to sell its items. According to this reason, $p_1(3)$ and $p_2(3)$ increase in θ and $p_3(3)$ decreases in θ. Finally, it is observed from the Table 5 that only $p_2(3)$ is insensitive to changes in parameter θ whereas other optimum decision variables are highly sensitive to changes in the parameter θ.

We also see that the ordering quantity and the profit increase with L, α, η and θ decreases in h and c. it could be explained as follows. As L and θ increase, the firm has more time to sell its items. Thus the firm orders more and gets more profit. Since the parameters of α and η have positive effect on demand, the ordering quantity and the profit increase as α and η increase. Since the parameters of h and c have negative effect on demand, the ordering quantity and the profit decrease when h and c increase. The characteristics of the sensitivity analysis are summarized as follows. (1) $p_1(3)$increases with L, α, η, c and θ while it decreases with h; (2) $p_2(3)$ increases with L, h, α and c while it decreases with η; (3) $p_3(3)$increases with h, α and c while it decreases with L, η and θ; (4) Q (3) increases with L, α ,η and θ while it decreases with h and c; and (5) F (3, $p_1(3)$, $p_2(3)$, $p_3(3)$) increases L, α , η and θ while it decreases with h and c.

Table-5 Sensitivity Analysis for Parameters L, h, α, η, c and θ

Parameter		$\tilde{p}_1(3)$	$\tilde{p}_2(3)$	$\tilde{p}_3(3)$	$\tilde{Q}(3)$	$\tilde{F}(3)$
L	130	40.6496015	26.7693958	12.9442875	5235.37138	34265.5332
	140	46.2490906	26.7725726	7.35973243	7335.50466	48147.4108
	150	56.6602098	26.7751256	-3.03713029	11240.1262	73712.5987
	160	82.7478328	26.7770684	-29.1111635	21023.5612	137275.280
	170	267.601259	26.7784148	-213.951645	90344.2946	585387.529
h	0.007	37.06789841	26.80514924	16.60833889	3888.667411	25502.1973
	0.008	37.02553025	26.82493247	16.69969333	3871.094421	25499.3958
	0.009	36.98316208	26.84471569	16.79104778	3853.521432	25495.4739
	0.01	36.94079391	26.86449892	16.88240222	3835.948442	25490.4313
	0.011	36.89842575	26.88428214	16.97375667	3818.375453	25484.2682
α	55	41.4937363	28.43224946	15.4178444	4926.73297	40446.206
	60	45.8348725	30.09891613	14.4100588	5929.65255	48615.800
	65	50.1759914	31.76558279	13.4022733	6932.57214	80068.003
	70	54.5171103	33.43224946	12.3944877	7935.49172	104802.81
	75	58.8582292	35.09891613	11.3867022	8938.41131	132820.23
η	0.005	29.81251908	26.78651789	23.78833518	2000.630537	12863.5970
	0.006	30.64536955	26.78227484	22.95090983	2220.316798	14315.1164
	0.007	31.681408	26.77805695	21.90919345	2492.771478	16111.1623
	0.008	33.00381427	26.77386712	20.58339504	2839.594326	18392.6846
	0.009	34.74833953	26.76970816	18.8348041	3296.037093	21389.5996
c	21	36.85029907	27.26558279	17.72796566	3622.937515	21731.0630
	22	36.54796341	27.76558279	19.03030133	3322.061639	18258.5634
	23	36.24562774	28.26558279	20.332637	3021.185764	15009.9304
	24	35.94329207	28.76558279	21.63497266	2720.309889	12216.1918
	25	35.6409564	29.26558279	22.93730833	2419.434013	9646.3199
θ	0.003	40.6739540	26.76149363	12.8998923	4841.66452	31511.689
	0.004	46.3190196	26.75744319	7.25044778	6311.02047	41116.400
	0.005	56.8238072	26.91323278	-3.25867698	9042.37264	58950.929
	0.006	83.1609598	26.89636533	-29.6001224	15885.1521	103604.55
	0.007	269.839154	26.88190326	-216.282563	64366.1097	419839.31

9. CONCLUSIONS

Incorporation of allowing multiple price changes and enabling control of the number of price changes into the analysis can be really challenging but they are more realistic and therefore can justifiably be used in actual situations. This paper studied the pricing and ordering problem for an inventory system for deteriorating items under the condition that demand is price and stock dependent. Numerous inventory models have addressed this problem. However, these models have rarely considered a situation in which the sales price can be adjusted during the selling period and the number of price changes can be controlled. Taking this situation into account, this paper developed a continuous time model for finding optimal ordering quantity and pricing setting/changing strategy. Given the inherent complexities of the real world, lend themselves to the search for solutions, which are relatively robust over wide range situations rather than optimal for a narrow set of circumstances. So, this paper sets the stage to incorporate pricing setting/changing strategy. Finally, numerical examples have been given to illustrate the theoretical results, with consequent managerial implications.

We found that the optimal decision can be expressed in closed form for the case without price setting. In addition, we found some sufficient conditions for obtaining optimal decisions for the cases with one and two price changes. For these cases, the optimal decisions are also developed in close form solutions.

In future research, we would like to extend this study to allow for shortages with unequal time price changes and other applications.

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