

ECONOMIC ORDER QUANTITY MODEL FOR DETERIORATING ITEMS WITH IMPERFECT QUALITY

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ABSTRACT

In an EOQ model it is tacitly assumed that the 100% items in a lot are of perfect quality. This is possible only under very idealistic situation but practically this assumption may not be true. Thus, the inspection of lot becomes indispensable in all public and private sector organizations. Moreover, it plays a very significant role when the products are of deteriorating in nature. In this paper an attempt has been made to formulate an inventory model for deteriorating items with imperfect quality. The inspection/screening rate is assumed to be more than the demand. This assumption helps one to fulfill the demand, out of the products which are found to be of perfect quality, along with the screening process. A hypothetical example is considered to validate the model and results are obtained using Matlab7.0.1. Comprehensive sensitivity analysis has also been presented.

KEYWORDS: Inventory; imperfect items; Deterioration; Inspection.

MSC : 90B05

RESUMEN

En un modelo EOQ (tamaño económico del lote) es asumido tácitamente que el 100% de los ítems en un lote tiene una calidad perfecta. Esto es posible solo bajo situaciones muy ideales, pues prácticamente esta asunción debe ser no cierta. Entonces, la inspección del lote es indispensable en todas las organizaciones tanto públicas como privadas. Más aun, esto juega un rol muy significativo si los productos son deteriorables. En este trabajo se ha hecho un intento de formular un modelo de inventario para ítems deteriorables con calidad imperfecta. La tasa de inspección se asume sea mayor que la demanda. Esta asunción ayuda a cumplimentar la demanda, fuera de los productos que fueron hallados con una calidad perfecta, junto con el proceso de valoración. Un ejemplo hipotético es considerado para validar el modelo y los resultados son obtenidos usando Matlab7.0.1. Un análisis comprensible de la sensibilidad es presentado también.

1. Introduction

The primal assumption of the classical Economic Order Quantity (EOQ) model is that the order quantity should be absolutely perfect. But due to so many unavoidable factors this assumption may not be valid always for most of the production environment. To address this very fact researcher developed various EOQ and EPQ models, which is having some defective items. Rosenblatt and Lee [5] proposed an EPQ model for a production system which contains defective production after a time point which is a random variable. On the other hand, due to unreliable supply process "Random Yield" models were developed for cases where usually the quantity received may not equal to quantity ordered. Yano and Lee [8] summarize the work done on the random yield models. Salameh and Jaber [7] extended the random yield modeling and developed an economic production quantity model for items with imperfect quality with a known probability distribution and raise the issue that imperfect quality items are sold as a single batch by the end of the screening process. Cardenas-Barron [1] corrected the optimal order quantity expression and Goyal and Cardenas-Barron [2] suggested a near approximation for order quantity. Most recently,

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Papachristos and Konstantaras [4] looked at the issue of non-shortages in model with proportional imperfect quality, when the proportion of the imperfects is a random variable. Maddah and Jaber [3] rectify a flaw in an economic order quantity (EOQ) model with unreliable supply, characterized by a random fraction of imperfect quality items and a screening process. They analyze the effect of screening speed and variability of the supply process on the order quantity, and show that the order quantity in our model is larger than that of the classical EOQ model when the variability of the yield rate is reasonably low.

In this paper, an inventory model for deteriorating items with imperfect quality has been developed. The screening rate is assumed to be larger than the demand rate. This assumption helps one to meet his demand parallel to the screening process, out of the items which are of perfect quality.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions are used in developing the model:

1. A constant fraction θ ($0 \leq \theta \leq 1$) of the on-hand inventory deteriorates per unit time.
2. The demand rate is known, constant, and continuous.
3. Shortages are not allowed.
4. The lead-time is known and constant.
5. The replenishment is instantaneous.
6. The screening process and demand proceeds simultaneously, but the screening rate (λ) is greater than demand rate (D), $\lambda > D$.
7. The defective items exist in lot size (Q). Also assume that percentage defective random variable (α) is uniformly

distributed with its pdf as $f(\alpha)$, $E(\alpha) = \int_a^b \alpha f(\alpha) d\alpha$, $0 < a < b < 1$.

The following notation is used:

c	the purchasing cost per unit
h	the holding cost per order
A	the ordering cost per order
p	the selling price per unit
s	the salvage value per defective unit, $s < c$
β	the screening cost per unit

3. THE MATHEMATICAL MODEL

In the present inventory system, Q items are used at the beginning of the period with a purchasing cost c and ordering cost of A . Each lot is having α percent defective items with a known probability density function, $f(\alpha)$. The behavior of the inventory level is shown in Fig.1 where screening process is done for all the received quantity at the rate of λ units per unit time which is greater than demand rate (D) for the time period 0 to t_1 . During the screening process the demand occurs parallel to the screening process and is fulfilled from the goods which are found to be of perfect quality by the screening process is Dt_1 . The defective items, αQ are sold at time t_1 as a single batch at a discounted price s per unit. After the screening process at time t_1 the inventory level will be $(1 - \alpha)Q - Dt_1$ and at time T , inventory level will become zero mainly due to demand and partially due to deterioration. To avoid shortages within screening time (t_1), the defective percentage (α) is restricted to

$$(1 - \alpha)Q \geq Dt_1 \Rightarrow \alpha \leq 1 - \frac{D}{\lambda} \quad (1)$$

Where $t_1 = \frac{Q}{\lambda}$ and the random variable α is uniformly distributed in a range $[a, b]$, where $0 < a < b < 1$.

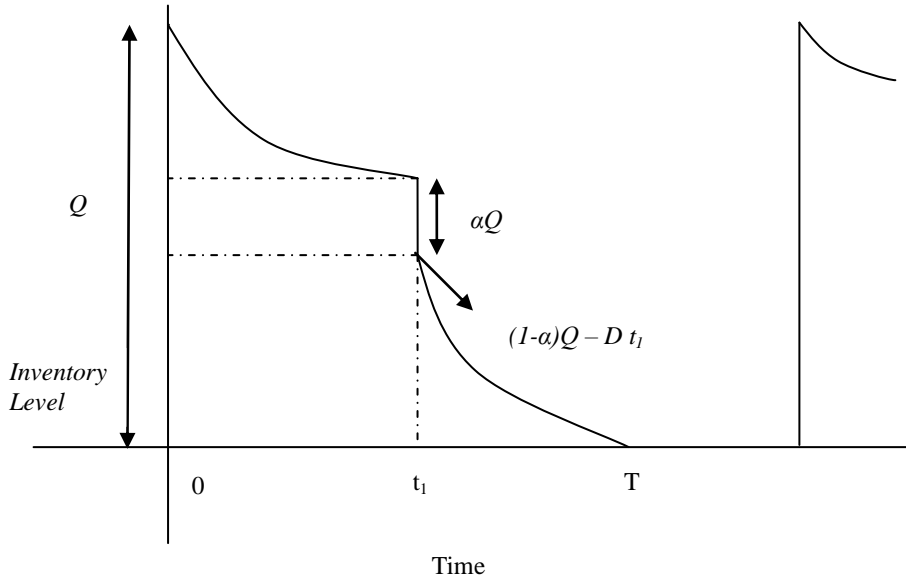


Figure1. Inventory system with inspection

Let $I(t)$ be the inventory level of the system at t ($0 \leq t \leq T$). The instantaneous states of $I(t)$ over the period $(0, T)$ is given by

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \quad 0 \leq t \leq t_1 \quad (2)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \quad t_1 \leq t \leq T \quad (3)$$

The solution of the above differential equations along with the boundary condition, $t=0, I(t)=Q$ and $t=t_1, I(t_1)=(1-\alpha)Q - Dt_1$ are

$$I(t) = Qe^{-\theta t} + \frac{D}{\theta} [e^{-\theta t} - 1], \quad 0 \leq t \leq t_1 \quad (4)$$

$$I(t) = e^{\theta(t-t_1)} \left[(1-\alpha)Q - Dt_1 + \frac{D}{\theta} [e^{\theta(t_1-t)} - 1] \right], \quad t_1 \leq t \leq T \quad (5)$$

At $t=T, I(T) = 0$, from equation (5),

$$T = t_1 - \frac{1}{\theta} \log \left[\frac{D}{Z} \right] \quad (6)$$

where, $t_1 = \frac{Q}{\lambda}$ and $Z = D + (1-\alpha)\theta Q - Dt_1\theta$

The retailer's profit, $T_p(Q)$ is the sum of revenue generated minus order cost, purchase cost, screening cost and holding cost during the time period $(0, T)$:

- Total revenue is the sum of revenue generated by the demand meet during the time period $(0, T)$ and sale of imperfect quality items is

$$T_R(Q) = pDT + s\alpha Q \quad (7)$$

- Order Cost, $A_1 = A$ (8)

- Purchase Cost, $P_1 = cQ$ (9)

- Screening Cost, $S_1 = \beta Q$ (10)

- Holding Cost during the time period 0 to t_1 and t_1 to T ,

$$H_1 = h \left[\int_0^{t_1} I(t) dt + \int_{t_1}^T I(t) dt \right]$$

$$H_1 = h \left[\frac{Q}{\theta} (1 - e^{-\theta t_1}) - \frac{D}{\theta^2} \{ \theta t_1 + e^{-\theta t_1} - 1 \} + \left\{ \frac{1}{\theta} (1 - e^{\theta(t_1-T)}) ((1-\alpha)Q - Dt_1) \right\} - \frac{D}{\theta^2} \{ e^{\theta(t_1-T)} + (T-t_1)\theta - 1 \} \right] \quad (11)$$

Using equations (7)-(11), total profit, $T_P(Q)$ is

$$= p(1-\alpha)Q + s\alpha Q - \left[A + cQ + \beta Q + h \left\{ \frac{Q}{\theta} (1 - e^{-\theta t_1}) - \frac{D}{\theta^2} \{ \theta t_1 + e^{-\theta t_1} - 1 \} + \frac{1}{\theta} (1 - e^{\theta(t_1-T)}) ((1-\alpha)Q - Dt_1) \right\} - \frac{D}{\theta^2} \{ e^{\theta(t_1-T)} + (T-t_1)\theta - 1 \} \right] \quad (12)$$

$$\text{Total profit per cycle, } T_{PC}(Q) = \frac{T_P(Q)}{T}$$

Since α is a random variable with known probability density function, $f(\alpha)$ then the expected total profit per unit time after applying renewal-reward theorem Ross(1996) is

$$E[T_{PC}(Q)] = E \left[\frac{T_P(Q)}{T} \right] = \frac{E[T_P(Q)]}{E[T]} \quad (13)$$

Using equation (6) and (12), equation (13) reduces to

$$= \frac{E[T_{PC}(Q)]}{E[T]} = \frac{E \left[pDT + s\alpha Q - \left[A + cQ + \beta Q + h \left\{ \frac{Q}{\theta} (1 - e^{-\theta t_1}) - \frac{D}{\theta^2} \{ \theta t_1 + e^{-\theta t_1} - 1 \} + \frac{1}{\theta} (1 - e^{\theta(t_1-T)}) ((1-\alpha)Q - Dt_1) \right\} - \frac{D}{\theta^2} \{ e^{\theta(t_1-T)} + (T-t_1)\theta - 1 \} \right] \right]}{E[T]}$$

$$\text{where } t_1 = \frac{Q}{\lambda} \text{ and } E[T] = t_1 - \frac{1}{\theta} \log \left[\frac{D}{D + (1 - E[\alpha])\theta Q - Dt_1\theta} \right] \quad (14)$$

4. SOLUTION PROCEDURE

Our objective is to find optimal value of lot size, Q , which maximizes the expected total profit function per unit time, $E[T_{PC}(Q)]$. Since, the expression of $E[T_{PC}(Q)]$ is complicated and it is very difficult to prove concavity mathematically. Alternatively we have shown graphically (Figure 2) that $E[T_{PC}(Q)]$ is concave in nature. Solving

the $E[T_{PC}(Q)]$ using pattern search tool method of Matlab 7.0.1, we get the optimum value of lot size, Q^* and expected total profit, $E[T_{PC}(Q^*)]$.

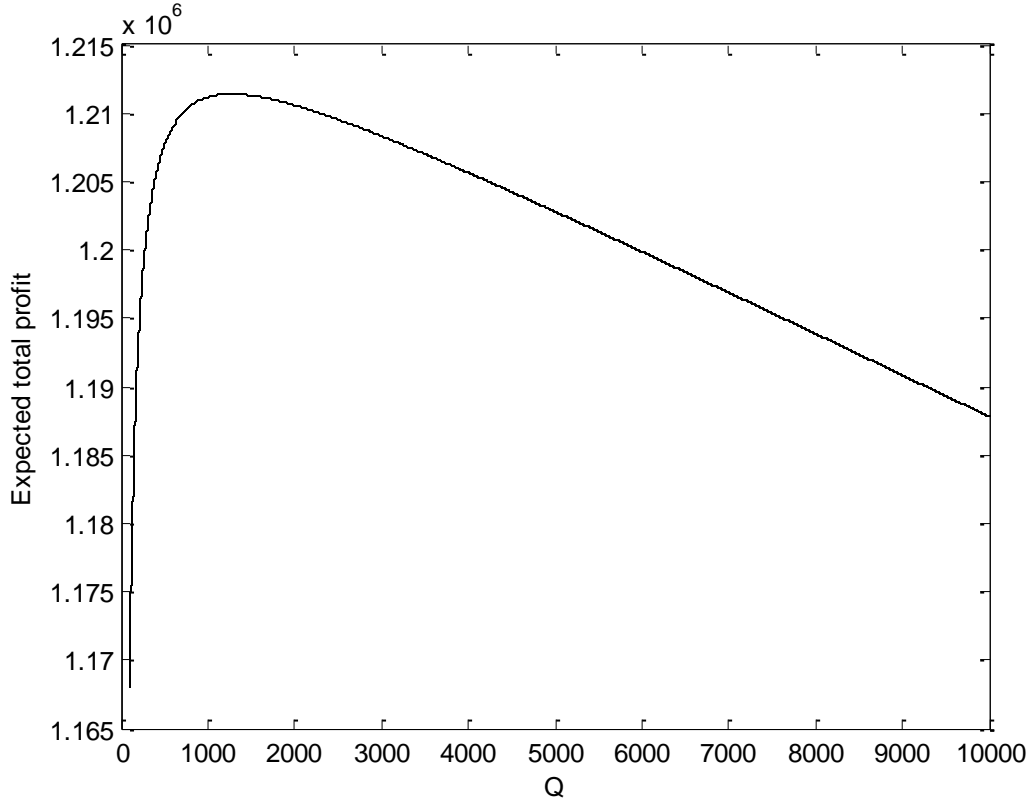


Figure 2: Concavity of expected total profit function, $E[T_{PC}(Q)]$

5. A NUMERICAL EXAMPLE

The model has been validated with the following data, $D = 50000$ units/year, $A = 100$ per cycle, $h = \$5$ /unit/year, $\lambda = 1$ unit/min, $c = \$25$ /unit, $p = \$50$ /unit, $s = \$20$ /unit, $\beta = 0.25$ /unit and $\theta = 0.1$ and percentage defective random

variable α with its p.d.f., $f(\alpha) = \begin{cases} 25, & 0 \leq \alpha \leq 0.04 \\ 0, & \text{otherwise} \end{cases}$, $E[\alpha] = 0.02$.

Testing the condition for avoiding the shortages with in the screening period on the above parameters from equation (1),

$$\alpha \leq 1 - \frac{D}{\lambda} \Rightarrow \alpha \leq 1 - (50000 / 175200) \Rightarrow \alpha \leq 0.715 \Rightarrow \alpha \leq 0.715$$

After solving the equation (14) with the help of pattern search tool method of Matlab7.0.1, the optimal value of $Q^* = 1283$ units. Substituting the optimal value Q^* in equation (6), the screening time, $t_l^* = 0.0073$ and expected cycle length, $E[T^*] = 0.0251$ yr. Finally, the optimal retailer's expected total profit per unit time, $E[T_{PC}(Q^*)] = \$1224183$ /yr.

6. SENSITIVITY ANALYSIS

Sensitivity analysis has been performed to study the impact of deterioration (θ) and expected number of imperfect quality items ($E[a]$) on the effective lot size (Q), expected cycle length ($E[T]$) and the retailer's expected profit ($E[T_{PC}(Q)]$). Results are summarized in *Table 1* and *Table 2* respectively.

Table 1: Impact of θ on the optimal replenishment policy

θ	0.20	0.15	0.10	0.05
Q	1171	1223	1283	1352
t_1	0.0067	0.0070	0.0073	0.0077
$E[T]$	0.0229	0.0239	0.0251	0.0265
$E[T_{PC}(Q)]$	1223418	1223792	1224183	1224595

Table 2: Impact of $E[a]$ on the optimal replenishment policy

$E[a]$	0.050	0.030	0.020	0.015	0.010
Q	1315	1293	1283	1277	1272
t_1	0.0075	0.0074	0.0073	0.0073	0.0073
$E[T]$	0.0250	0.0251	0.0251	0.0251	0.0252
$E[T_{PC}(Q)]$	1215678	1221407	1224183	1225550	1226903

The results are very much consistent with reality, i.e.:

- From Table 1, as deterioration rate (θ) decreases then there is significant increase in expected cycle length ($E[T]$), lot size (Q) and the retailer's expected profit ($E[T_{PC}(Q)]$). Further, it suggests that the highly deteriorating products should be procured more frequently, which eventually helps one to minimize his loss due to deterioration.
- Table 2 shows that as the expected number of imperfect quality items ($E[a]$) decreases, the optimal order quantity (Q) decreases and the expected cycle length ($E[T]$) remains unchanged, but the retailer's expected profit ($E[T_{PC}(Q)]$) increases significantly as the decrease in the number of defective items directly effects the revenue.

7. CONCLUSIONS

This paper investigates the effect of deterioration on the retailer's economic order quantity when the items are of imperfect quality. Screening rate is assumed to be more than the demand rate. This assumption enables the retailer to fulfill the demand, out of the products which are found to be of perfect quality, along with the screening process. Findings clearly help the retailer to determine his ordering policy, for example in the case of highly deteriorating products he should order more frequently to reduce his loss due to deterioration. On the other hand when the

defective items increases, his expected profit decreases, in such a situation retailer needs to look into source of supply and requires to take the corrective measure in order to get good quality products or change his source of supply. Future this study could be extended for shortages and permissible delay in payments.

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