SOLVING THE PRODUCTION-TRANSPORTATION PROBLEM IN THE PETROLEUM INDUSTRY

Zrinka Lukač *[1] , Dubravko Hunjet** , Luka Neralić*
* Faculty of Economics and Business Zagreb, Trg J. F. Kennedy 6, 10000 Zagreb, Croatia
** SRCE University Computing Center, Josipa Marohnića 5, 10000 Zagreb, Croatia

ABSTRACT
In this paper we formulate two new models of the production-transportation problem which can be described as follows. Let us suppose that there are several plants at different locations producing certain number of products and large number of customers of their products. Each plant can operate in several modes characterized by different quantities of products and variable production costs. The customers' demand for each product during the considered time period is known. We consider the problem of finding the production program for each plant as well as the transportation of products to customers for which the sum of the production and transportation costs is minimized given the condition that each customer can satisfy its demand for a given type of product from one plant only. We also formulate the problem as a bilevel mixed-integer programming problem. We solve the models for the available data from a petroleum industry and compare the results.

Key words: production-transportation problem, petroleum industry, linear programming, mixed-integer programming, bilevel programming

MSC 90C08, 90C11, 90B50

RESUMEN
En este trabajo formulamos dos nuevos modelos para el modelo de producción-transporte el que puede describir como sigue. Supongamos que hay varias plantas en diferentes lugares que producen cierto número de productos y de un número grande de clientes de sus productos. Cada planta puede operar de varios modos caracterizados por diferentes cantidades y costos variables de producción. Las demandas de los clientes para los cuales cada producto durante el periodo de tiempo considerado es conocido. Nosotros consideramos el problema de hallar el programa de producción para cada planta así como el transporte de productos a los clientes para los cuales la suma de los costos de producción y transporte son minimizadas dadas las condiciones de que cada cliente puede satisfacer sus demandas para un tipo dado de producto de una sola planta. Nosotros formulamos el problema como un problema de programación binivel entera mixta. Nosotros resolvemos los modelos para los datos existentes para la industria del petróleo y compramos los resultados

1. INTRODUCTION

The production transportation problem (PTP) is one of the very important problems in the continuous production industries such as petroleum industry. It deals with the problem of how to plan production and transportation in such an industry given several plants at different locations and large number of customers of their products. This problem has been addressed previously in the literature (Aronofsky et al. (1978), Grobatenko-. Suvorov (1967), Ivshin, (1972), Hunjet et al. (2003), Neralić (1979)) and can be formulated as a linear programming problem.

In this paper we formulate two new models of the general production-transportation problem. The first one considers the production-transportation problem in which each customer satisfies its demand for a given type of product from one plant only. It is formulated as a mixed-integer programming problem. The second one is obtained by introducing the hierarchy of decision making into the first model, thus obtaining a bilevel mixed-integer programming problem.

The paper is organized as follows. In *Section 2* we describe the production-transportation problem (PTP).

[1] {zlukac,lneralic}@efzg.hr

In *Section 3* we describe the bilevel production-transportation problem (B-PTP). The discrete production-transportation problem (D-PTP) is introduced in *Section 4.* In *Section 5* we introduce the bilevel discrete production-transportation problem (BD-PTP). The solutions and the comparison of the results for the different models presented in this paper on the available data from a petroleum industry are given in *Section 6.* The last section contains some conclusions and gives some suggestions for further research.

2. THE PRODUCTION-TRANSPORTATION PROBLEM

The production-transportation problem can be described as follows (Aronofsky et al. (1978) , Grobatenko-Suvorov (1967), Ivshin, (1972), Hunjet et al. (2003), Neralić (1979)): there are several plants placed at different locations (refineries for example) and large number of customers of their products. Each producer can operate in several modes of production, where each mode is characterized by different quantities of products as well as variable production costs and by maximum possible production of a certain type of product. We assume that the customer's demand for each product in a short time planning period as well as the unit transportation costs from plants to customers are known. Each customer can satisfy its demand for a certain type of product from any combination of plants. Now the production-transportation problem consists of determining producers' production program as well as the transport of products to the customers with the minimal production and transportation costs which meet the customers' demand in a satisfactory way.

In order to formulate the problem we use the following set of indexes:

- i - index for production plants; $i = 1,2,...,I$
- j - index for customers; $j = 1,2,...,J$
- t - index for type of products; $t = 1,2,...,T$
- s - index for modes of production; $s = 1,2,...,S_i$; $i = 1,2,...,I$

The parameters of the problem are as follows:

- pc_{is} - unit variable production cost for mode s at plant i
- tc_{ijt} - unit transportation cost for product t from plant i to customer j
- a_{its} - quantity of product t produced at plant i by mode s
- b_{jt} - demand for product t by customer j

The decision variables of the problem are:

- z_{is} - the intensity of using mode s by plant i being part of the planning period, which is taken to be 1
- u_{ijt} - the amount of product t shipped from plant i to customer j

The PTP can now be formulated as the following linear programming problem:

$$\min \ f(\mathbf{z},\mathbf{u}) = \sum_{i=1}^{I}\sum_{s=1}^{S_i} pc_{is} \cdot z_{is} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{i=1}^{I} tc_{ijt} \cdot u_{ijt} \tag{1}$$

subject to

$$\sum_{i=1}^{I} u_{ijt} = b_{jt} \qquad \forall j, \forall t \tag{2}$$

$$\sum_{j=1}^{J} u_{ijt} \le \sum_{s=1}^{S_i} a_{its} \cdot z_{is} \qquad \forall i, \forall t \tag{3}$$

$$\sum_{s=1}^{S_i} z_{is} \le 1 \qquad \forall i \tag{4}$$

$$z_{is} \ge 0 \qquad \forall i, \forall s \tag{5}$$

$$u_{ijt} \ge 0 \qquad \forall i, \forall j, \forall t \tag{6}$$

The objective function (1) implies that all variable production costs as well as the transportation costs should be minimized. According to the constraints (2) each customer's demand for each product will be satisfied. Constraints (3) specify that for each type of product and for each plant the amount of the product shipped from the plant to all the customers cannot be greater than the quantity produced in that plant. Because of the inequality sign in constraints (4) the producers are not forced to operate in the whole planning period. Constraints (5) and (6) are nonnegativity constraints on all the variables.

3. BILEVEL PRODUCTION-TRANSPORTATION PROBLEM

In this section we obtain the bilevel production-transportation problem (B-PTP) by introducing the hierarchy of decision making into the PTP. The bilevel programming is motivated by the fact that many planning problems contain a hierarchical decision structure where each level has independent and often conflicting objective ([2]). Bilevel programming is the simplest case of such a situation where there are only two independent decision makers located at different levels of decision making. The one on the upper level is called the leader, and the one on the subordinate level is called the follower. Furthermore, the situation of perfect information is assumed, meaning that each player is familiar with the objective as well as the choices available to the other. Each player is entitled to one move only. The leader acts first. When making the decision, he must anticipate all possible responses of the follower. Given the decision of the leader, the follower reacts in a way that is personally optimal for him. Therefore, the sets of feasible choices available to each of the players are interdependent. Moreover, the leader's decision affects both the follower's objective and available decisions and vice versa.

The B-PTP is obtained by introducing the hierarchy of decision making into the PTP ([3]), which is as follows. The decision maker at the upper level - the leader - has to organize the production so as to meet the demand at minimal production costs. His decision concerns production modes of the plants. Once he has made his decision, the decision maker at the lower level - the follower - has to organize the transportation of the products to the customers so as to achieve minimal transportation costs. The bilevel formulation of the B-PTP is now as follows:

$$\min \quad f(\mathbf{z}) = \sum_{i=1}^{I} \sum_{s=1}^{S_i} pc_{is} \cdot z_{is} \tag{7}$$

subject to

$$\min \; g(\mathbf{u}) = \sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{i=1}^{I} tc_{ijt} \cdot u_{ijt} \tag{8}$$

subject to

$$\sum_{i=1}^{I} u_{ijt} = b_{jt} \qquad \forall j, \forall t \tag{9}$$

$$\sum_{j=1}^{J} u_{ijt} \le \sum_{s=1}^{S_i} a_{its} \cdot z_{is} \qquad \forall i, \forall t \tag{10}$$

$$\sum_{s=1}^{S_i} z_{is} \le 1 \qquad \forall i \tag{11}$$

$$z_{is} \ge 0 \qquad \forall i, \forall s \tag{12}$$

$$u_{ijt} \ge 0 \qquad \forall i, \forall j, \forall t \tag{13}$$

The leader wishes to organize the production so as to achieve minimal production costs (7) subject to the constraints (8)-(13) which make another optimization problem - the one of the follower. Given the decision of the leader, the follower wishes to achieve minimal transportation costs (8) subject to the constraints (9)-(13). These constraints correspond to the constraints (2)-(6) of the PTP.

4. DISCRETE PRODUCTION-TRANSPORTATION PROBLEM

Let us now introduce the discrete production-transportation problem (D-PTP). This problem is obtained form the PTP by imposing the restriction that each customer can satisfy it's demand for a given type of product from one plant only. The index set as well as the parameters of the model remain the same as in the PTP, but now we have the following set of decision variables:

- z_{is} - intensity of using mode s by plant i
- $y_{ijt} = \begin{cases} 1; & \text{if the customer } j\text{'s demand for product } t \text{ is satisfied from plant } i \\ 0; & \text{otherwise} \end{cases}$

Because of the assumption that each customer satisfies its demand for a certain type of product from one plant only, in D-PTP we know that if customer j satisfy it's demand for product t from plant i $(y_{ijt} = 1)$ then the amount of product t shipped from plant i to customer j is exactly equal to b_{jt} . Therefore, in order to know the amount of product t shipped from plant i to customer j , it is sufficient to keep track of variable y_{ijt} only.

The D-PTP can now be formulated as the following linear mixed-integer programming problem:

$$\min \ f(\mathbf{z},\mathbf{y}) = \sum_{i=1}^{I}\sum_{s=1}^{S_i} pc_{is}\cdot z_{is} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{i=1}^{I} tc_{ijt}\cdot b_{jt}\cdot y_{ijt} \quad (14)$$

subject to

$$\sum_{i=1}^{I} y_{ijt} = 1 \qquad \forall j, \forall t \qquad (15)$$

$$\sum_{j=1}^{J} b_{jt}\cdot y_{ijt} \le \sum_{s=1}^{S_i} a_{its}\cdot z_{is} \qquad \forall i, \forall t \qquad (16)$$

$$\sum_{s=1}^{S_i} z_{is} \le 1 \qquad \forall i \qquad (17)$$

$$z_{is} \ge 0 \qquad \forall i, \forall s \qquad (18)$$

$$y_{ijt} \in \{0,1\} \qquad \forall i, \forall j, \forall t \qquad (19)$$

Again, the objective function (14) implies minimization of all variable production costs as well as the transportation costs. The difference between the objective function (1) and (14) is in the part considering the transportation costs. Constraints (15) say that each customer's demand for each product will be satisfied. Similar to (3), constraints (16) specify that for each type of product and each plant the amount of the product shipped from the plant to all the customers cannot be greater than the quantity produced in that plant. Again, because of the inequality sign in constraints (17) the producers are not forced to operate in the whole planning period. Constraints (18) are nonnegativity constraints on variables z_{is}, and constraints (19) say that variables y_{ijt} are binary.

5. BILEVEL DISCRETE PRODUCTION-TRANSPORTATION PROBLEM

In this section we obtain the bilevel production-transportation problem (BD-PTP) by introducing the hierarchy of decision making into the D-PTP. Given the D-PTP in the context of bilevel programming we consider two decision-makers placed at different hierarchical levels. The one on the upper level - the leader - has to organize the production so as to meet the demand at minimal production costs. His decision concerns production modes of the plants. Once he has made his decision, the decision maker at the lower level - the follower - has to organize the transportation of the products to the customers so as to achieve minimal transportation costs. The bilevel formulation of the BD-PTP is now as follows:

$$\min \ f(\mathbf{z}) = \sum_{i=1}^{I}\sum_{s=1}^{S_i} pc_{is}\cdot z_{is} \qquad (20)$$

subject to

$$\min \ g(\mathbf{y}) = \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{i=1}^{I} tc_{ijt}\cdot b_{jt}\cdot y_{ijt} \qquad (21)$$

subject to

$$\sum_{i=1}^{I} y_{ijt} = 1 \qquad \forall j, \forall t \tag{22}$$

$$\sum_{j=1}^{J} b_{jt} \cdot y_{ijt} \le \sum_{s=1}^{S_i} a_{its} \cdot z_{is} \qquad \forall i, \forall t \tag{23}$$

$$\sum_{s=1}^{S_i} z_{is} \le 1 \qquad \forall i \tag{24}$$

$$z_{is} \ge 0 \qquad \forall i, \forall s \tag{25}$$

$$y_{ijt} \in \{0,1\} \qquad \forall i, \forall j, \forall t \tag{26}$$

The leader wishes to organize the production so as to achieve minimal production costs (20) subject to the constraints (21)-(26) which make another optimization problem - the one of the follower. Given the decision of the leader, the follower wishes to achieve minimal transportation costs (21) subject to the constraints (22)-(26). These constraints correspond to the constraints (15)-(19) of the D-PTP.

6. NUMERICAL RESULTS

In this section we present the solutions for the models on two sets of available data from a petroleum industry and compare the results. The first data set involves 2 plants, 5 customers and 4 types of products (Neralić (1979), (1980)), while the second data set involves 3 plants, 66 customers and 4 types of products (Neralić (1979)). Using these two data sets we solve and compare the results for four different models: PTP, B-PTP, D-PTP and BD-PTP.

The problems are solved using CPLEX 9.0 programming package (*ILOG AMPL CPLEX (*2003)) and AMPL mathematical programming language (Fourer et al (2000)). Furthermore, the two of the bilevel programming problems B-PTP and BD-PTP were solved using the following simple algorithm:

Step 1. Solve the leader's problem (20),(22)-(26) for BD-PTP (i.e. problem (7),(9)-(13) for B-PTP).

Step 2. Fix leader's decision $\mathbf{z}$.

Step 3. Solve the follower's problem (21)-(26) for BD-PTP (i.e. problem (8)-(13) for B-PTP).

Since in both problems BD-PTP and B-PTP the leader's objective function value is independent of the follower's decision, and vise versa, such an algorithm will produce optimal decisions and values of the corresponding bilevel programming problems BD-PTP and B-PTP. In Step 1 we obtain the feasible solution of the corresponding bilevel problem which at the same time gives the optimal decision of the leader $\mathbf{z}$. If $\mathbf{z}$ was not the optimal leader's decision, there would exist another decision $\mathbf{z'}$ having better value of the leader's objective (20), which contradicts Step 1. Therefore $\mathbf{z}$ is the optimal decision of the leader. Given this decision, in Step 3 we obtain the optimal decision of the follower.

Table 1 shows the comparison of the results for the data set 1 in terms of different objective function values appearing in four models compared. For each model we compare the optimal value of production costs (leader's objective in B-PTP and DB-PTP), transportation costs (follower's objective in B-PTP and DB-

PTP), as well as the overall costs (objective in PTP and D-PTP).

Table1. Comparison of the results for the data set 1

	PTP	**B-PTP**	**D-PTP**	**BD-PTP**
Production costs	5658830	5658510	5670670	5665420
Transportation costs	925706	926351	1028640	1035710
Overall costs	6584530	6584860	6699310	6701130

As expected, table 1 shows that PTP achieves better objective function value than D-PTP in all three cases. The same is true for B-PTP and BD-PTP due to the discretization of the D-PTP and BD-PTP models.

Table 2 compares the difference (in %) between the objective function values of the B-PTP compared to the PTP model, as well as the BD-PTP compared to the D-PTP.

Table 2. Difference (in %) between the objective function values of the bilevel models compared to appropriate single level models

	B-PTP	**BD-PTP**
Production costs	-0,01%	-0,09%
Transportation costs	+0.07%	+0,69%
Overall costs	+0,01%	+0,03%

One can see that both B-PTP and BD-PTP bilevel programming models have achieved an improvement in production costs (leader's objective), and deterioration in transportation costs (follower's objective). In both cases the overall costs (production and transportation costs) were slightly greater than in PTP and D-PTP models.

Table 3 shows the comparison of the results for the data set 2 in terms of different objective function values appearing in four models compared. As for data set 1, for each model we compare the optimal value of production costs (leader's objective in B-PTP and DB-PTP), transportation costs (follower's objective in B-PTP and DB-PTP), as well as the overall costs (objective in PTP and D-PTP).

Table3. Comparison of the results for the data set 2

	PTP	**B-PTP**	**D-PTP**	**BD-PTP**
Production costs	6233560	6233560	6233750	6233750
Transportation costs	842375	842375	844352	844352
Overall costs	7075940	7075940	7078100	7078100

Using data set 2, we have obtained the same objective function values for both PTP and B-PTP models, as well as for D-PTP and BD-PTP models.

7. CONCLUSIONS

In this paper we have formulated two new production-transportation models: the model of discrete production-transportation problem and the model of bilevel discrete transportation problem. We have solved the models on two sets of available real-life data from a petroleum industry and we have compared the results.

In our future work we plan to consider some further extensions of the models proposed such as discrete production-transportation problem with several modes of transportation, discrete production-transshipment transportation problem with the warehouses between the producers and the customers, and multimodal discrete production-transportation problem with capacity constraints. Furthermore, it would be interesting to formulate the D-PTP and BD-PTP as network problem due to the existence of many efficient network algorithms.

REFERENCES

[1] ARONOFSKY, J.S., J.M. DUTTON and M.T. TOYYABKHAN (1978.): **Managerial Planning with Linear Programming: In Process Industry Operations**. John Wiley & Sons, New York.

[2] BARD, J. F. (1998): **Practical Bilevel Optimization Algorithms and Applications,.** Kluwer Academic Publishers. Dordrecht.

[3] CORTEZ , R. T. (2000): **Stable Convex Parametric Programming and Applications**, Ph. D. Thesis, McGill University, Montreal, Canada,

[4] FOURER, R., D.M. GAY and B.W KERNIGHAN (2002): **AMPL: A Modeling Language for Mathematical Programming,** Brooks/Cole Publishing Company, N. York.

[5] GROBATENKO, N.G. and B.P. SUVOROV (1967): Model for short-time planning of production and transportation of petroleum industry, In: **B. P. Suvorov (Editor), Mathematical Methods and Models in Petroleum Industry Planning**, Nauka, Moscow.(In Russian)

[7] IVSHIN, Yu. F.(1972): Production-transportation model of refinery group. **Control and Economy of Material-Technical Supply**, No. 2, Moscow, (In Russian)

[8] HUNJET, D.,M. MILINOVIĆ, L. NERALIĆ and L. SZIROVICZA (2003): Production-transportation problem and its extensions, In: **K. Šorić, T. Hunjak and R. Scitovski (Eds.), Proceedings of the 9th International Conference on Operational Research, Croatian Operational Research Society and Department of Mathematics,** University of Osijek, Osijek, 73-81.

[9] NERALIĆ, L. (1979): **Modified Lagrangians and optimization of costs**, Ph. D. Thesis, Faculty of Economics, University of Zagreb, Zagreb. (In Croatian)

[10] NERALIĆ, L. (1980): An extended production-transportation model for oil products, **Nafta** 31, 267-271

[11] ILOG AMPL CPLEX (2003): System Version 9.0 User’s Guide, available online at http://www.rpi.edu/dept/math/math-programming/cplex90/amplcplex90userguide.pdf,

Received March 2007
Revised December 2007