COMBINING A DYNAMIC VERSION OF KOHONEN ALGORITHM AND A TWO-REGIME MARKOV SWITCHING MODEL: AN APPLICATION TO THE PERIODIZATION OF INTERNATIONAL BIMETALLISM (1821-1873)

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ABSTRACT
Before the gold standard spread over the world in the 1870s, the international monetary system relied on two metals, gold and silver. A European monetary system was then *de facto* in operation, connecting three monetary zones. Series of pertinent variables have been built on the basis of the information extracted from relevant newspapers twice a week. A dynamic interpretation of SOM is proposed to reveal the periodization of international bimetallism in the 19th century. The Kohonen algorithm is used to obtain a grouping of weekly observation into homogeneous classes of exchange rates and prices which are then grouped in a small number of super classes. Each super class represents a set of sub-periods during which the variables show close profiles. A Two Regime Markov Switching Model is estimated to reveal the main relations between the variables of interest.

Key words: international bimetallism, Kohonen algorithm, Markov Switching Model, hidden Markov chain

MSC.: 62P20

RESUMEN
Antes de expandirse el patrón oro por todo el mundo en los 1870s, el sistema monetario internacional reposaba en dos metales, oro y plata.. Un sistema monetario tenia en operación *de facto*, conexión de tres zonas monetarias. Series de variables pertinentes han sido construidos sobre la base de la información extraída de periódicos relevantes dos veces por semana. Una interpretación dinámica del SOM es propuesta para revelar la periodización de; bimetalismo international en el siglo 19. El algoritmo de Kohonen es usado para obtener un agrupamiento de las observaciones semanales en clases homogéneas del intercambio de cotizaciones y precios los que entonces son agrupados en un numero menor de súper clases. Cada súper clase representa un conjunto de sub-periodos durante los cuales las variables muestran perfiles cercanos. Un Modelo de Dos Regímenes de Intercambio de Markov es estimado para revelar las relaciones principales entre las variables de interés.

1. INTRODUCTION.

Before the gold standard spread over the world in the 1870s, the international monetary system relied on two metals, gold and silver. This offered the opportunity of numerous arbitrages, between the major national financial centers, about those precious metals and also alternative means of international payment, namely foreign exchange. Of particular interest is the study of payments between London, Paris and Hamburg, from 1821 (resumption of the convertibility of Bank of England notes in gold) to 1873 (adoption of the gold standard in Germany, France, and the United States). A European monetary system was then *de facto* in operation, connecting three monetary zones: the gold-standard one, centered on London; the double-standard one, centered on Paris; and the silver-standard one, centered on Hamburg. This system, which, because of the coexistence of two monetary standards, may be called "international bimetallism"[2], is the main historical precedent of what could be in the future an international system of payments based on two (instead of only one) key currencies, the US dollar and the Euro.

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[2] See Boyer-Xambeu et al. [2], [3] and [4]

The working of this international bimetallism evolved during its half-century of existence, under the influence of various factors: institutional changes (such as the first attempt at the monetary unification of Germany in 1838, the reform of the Bank of England in 1844, the change in the convertibility policy of the *Banque de France* in 1853, or the creation of the Latin Union in 1865) and monetary or real shocks (such as the crises of 1847, 1857, 1866, the American “gold rush” of 1850 or the Franco-German war of 1870). It is then interesting to inquire about the existence of distinct sub-periods from 1821 to 1873, on the basis of the market prices and ratios allowed to vary with the implementation of arbitrages between London, Paris, and Hamburg, namely: a) the three bilateral exchange rates between the pound sterling, the French franc, and the Hamburg mark; b) the two domestic prices of the precious metals which were free from any direct intervention of a central bank, i.e. the price of silver in London and the price of gold in Hamburg; c) the three domestic gold-silver price ratios, which played a central role in the working of international bimetallism.

Series of these variables have been built on the basis of the information extracted from relevant newspapers twice a week. This amounts to treating time series of more than 5000 observations, although the time continuity is sometimes jeopardized by known or unknown events having affected the availability of the data (absence of the desired information in a given newspaper issue or missing issues in archives; interruptions of quotation due to financial crises or political events). These series have already been exploited in several publications, and their theoretical interpretation is part of an active debate among economists about international payments in the age of metallic monetary standards. Throwing light on the periodization of international bimetallism in the 19^{th} century would be a useful contribution to that debate.

A dynamic interpretation of SOM is proposed to reveal this periodization. Kohonen maps (in one or two dimensions) are constructed using the weekly observations of exchange rates, “free” domestic prices of precious metals and domestic gold-silver price ratios.

A first step is aimed at constructing a periodization of the phenomenon identified here as “international bimetallism”.

The Kohonen algorithm is used to obtain a grouping of weekly observations into homogeneous classes of exchange rates and prices. These classes are then grouped using a hierarchical ascending classification in order to obtain a small number of super classes. Each super class consists in a set of weekly observations, which can be continuous or not; they represent a set of sub-periods during which the variables show close profiles.

The only data available within the period are those used for the classification; there are no other variables available on the same basis of frequency that could be used to describe the classes. So, in order to differentiate the classes, we use the means and variances of these same variables, and confront them to the main historical and institutional events of the period identified above.
A canonical discriminant analysis is then used to bring some light on the major influences explaining the classification.

There remains a technical problem: that of missing data. A convenient type of matching may be used to substitute a correct information. Of course this treatment is applied only to data missing for technical reasons, not when the observations are missing because they never existed (*e.g.* no quotation due to financial crisis or political events).

If the initial hypothesis of a unique international system of payments referred to as “international bimetallism” is confirmed, it is interesting, in a second step, to reveal the main relations between the variables of interest, relative prices of precious metals and rates of exchange between the three currencies. We estimate a two-regime Markov switching model of one of theses variables: a hidden Markov chain controls the passage between the two regimes. Intuitively there must be something common between the sub-periods and the changes from one regime to the other. The variables identifying each of these regimes represent the specific regulations of the markets during the sub-periods. Of course, due to the specificity of the data used, we consider here the short period regulation, that is to say the continuous arbitrages between the different places, involving the whole set of prices and rates. The long period regulation is another program of research.

2. THE PERIODIZATION: THE USE OF KOHONEN MAPS.

2.1. The Kohonen algorithm: major properties[3]

The algorithm presented and used later on real data is referred to as Self-Organizing Map (SOM), a model presented by T. Kohonen (1982). Its main characteristics are briefly presented.
An unsupervised algorithm. Among the great diversity of artificial neural networks the kind used here have this characteristic: the constitution of the groups is not guided by any rule or any pre-defined classification, and nothing is acting during the process to validate the class membership of each observation.

There are no a priori properties of the groups before the execution of the algorithm: they are identified using the qualitative and quantitative variables measured on the set of observations. One may say that they are revealed as a result of the process.
Similarly, the number of groups is not fixed: it is chosen by the operator, depending on the aim of the study and the size of the set.

A topology preserving algorithm. This means that observations that are neighbors in data space with p dimensions are grouped in the same class or in neighbor classes. From the input data space, a partition of this space leads to a node whose code vector represents all the input vectors of this region.

The nodes are the output of the process, they are organized according to a predetermined topology. Most of the time, the representation used is a grid. It should be noted that the lines traced between the nodes give different expressions of the neighborhood: in the simplest definition a node have four neighbors but during the execution of the algorithm this number may vary[4].

A competitive learning algorithm producing a self-organized map. The competitive property means that when an observation is presented to the network, one node is the winner of the competition among all the nodes according to the rule defined. This node and its neighbors move towards this observation: the code vector of each one tends to capture the characteristics of the input space.

2.2. The data

2.2.1. Time series of prices and rates of exchange.

Relative prices of gold in silver are computed from the price of each metal observed in each of the three financial places, Paris, London and Hamburg (respectively, *poa*, *lgs*, and *hoa*), twice a week from the beginning of 1821 until the end of 1873. The same type of data is available for the exchange rates (Pound in Francs, Pound in Marks, Mark in Francs: respectively, *lpv*, *hlv*, and *phv*).
An observation is a set of twelve values, two quotations (Tuesday and Friday) for each of the six variables[5].
A computed variable has been added to emphasize the relation between the relative price of metals in Hamburg and the average level in Paris and London of this value (*hpl*).
Most of the time the quotations show rather small differences within a given week, but periods with important troubles, Paris in the late 1840s for instance, may be well separated from the more classical ones.

2.2.2. Choice of the structure and the problem of missing data.

A grid of 25 nodes is chosen: that means 25 initial classes of weeks of quotations which will be characterized, at the end of the iterative process of classification, by their code vector.
This number of classes is selected to satisfy two opposite constraints; the first one is to constitute groups with a sufficient number of observations to obtain pertinent characteristics of their distribution

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[3] Theoretical developments and major demonstrated results may be found in Kohonen [9], Cottrell et al. [5]
[4] See later the contents of the algorithm
[5] Actually the price of each precious metal is observed on its market and the relative price of gold in silver is then computed.

(about 2500 observations grouped in 25 classes gives an average frequency of 100 observations per class).

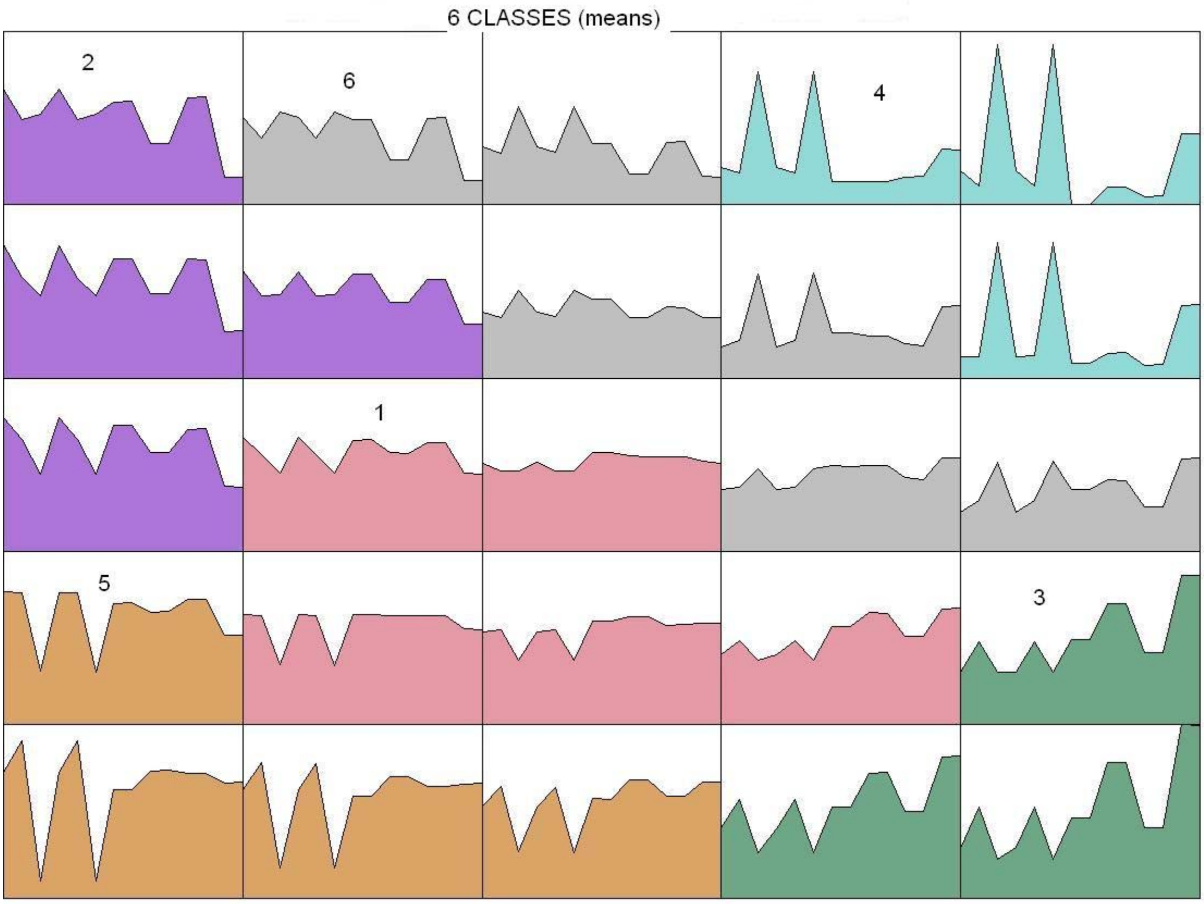
After the Kohonen classification, a hierarchical ascending classification is used to produce a small number of macro classes, in this case 6 macro classes, corresponding to the main sub-periods. This latter classification is constructed with the code vectors obtained from the first process.
We have considered as missing data only the observations that are missing among the actual periods of quotation. So there are some long periods without quotations (since the beginning of the war between Germany and France) or with one set of prices completely unknown (the year 1861 for Hamburg quotations[6]), that we have not tried to "re-build" because it would mean to construct data that never existed. This is why we have reduced the period of this study to 1821-1860.

For less severe lacks of data, typically one quotation missing while the other variables are observed, the matching process which constitutes the basis of the algorithm uses the available information to find the closest class. At the end of the process the missing value is replaced by the corresponding element of the code vector computed for this class.

3. The grid obtained: Kohonen classes and macro classes[7].

The classification produces classes constituted with large sequences of consecutive weeks and a relatively small number of sparse weeks.

Graph 1
The Kohonen Map



[6] At the time of this first study.
[7] The program used to construct this kind of Kohonen map has been developed by P. Letrémy, CES-Samos and may be downloaded from http://samos.univ-paris1.fr

This first observation is confirmed with the construction of the macro classes: going from the top to the bottom of the map or from the bottom-left corner to the top-right one[8], the code vectors appear well contrasted, illustrating the sub-periods.

3.1. General observations.

3.1.1. Two dimensions are appearing

- From the top left to the bottom right the relative price in Hamburg varies from low to high levels (in its distribution)
- From the bottom left to the upper right, high values of most of the variables used are declining to low levels and conversely, rates of exchange have opposite features (when comparing the mark to the two other currencies)

3.1.2. Large sequences of contiguous weeks are grouped in the macro-classes, however a few years are fragmented in short periods situated in different classes

- Class 1 (pink), center of the map, is constituted with 3 groups of years 1829-30, 1834-38, 1848-49 and a lot of fragments of years
- Class 2 (violet) is more simple to describe with 3 intervals 1832-33, 1842-43 and 1846-47 and some sparse weeks from the 1830s

It seems that they represent a central position contrasting to the well identified other classes:

- Class 3 (green): 2 sets constituted by years 1824-25 and 1827-28, with almost no missing weeks in these intervals, indicating that this sub-period is very homogeneous
- Class 4 (blue): the end of year 1853 and the whole period 1854-60; again only a small number of weeks are missing for this continuous sub-period of more than seven years
- Class 5 (brown): 1821-24 and 1826-beginning 1827 plus small parts of 1830 and 1832
- Class 6 (grey): two sets 1839-41 and 1851-53

The mean of the variables used to obtain the classification may be represented to illustrate the great differences appearing between the sub-periods. Changing hierarchies between the relative prices is the characteristic identifying the four last macro-classes.

3.2. The major influences producing the classes: results of a canonical discriminant analysis.

3.2.1. A Canonical Discriminant Analysis

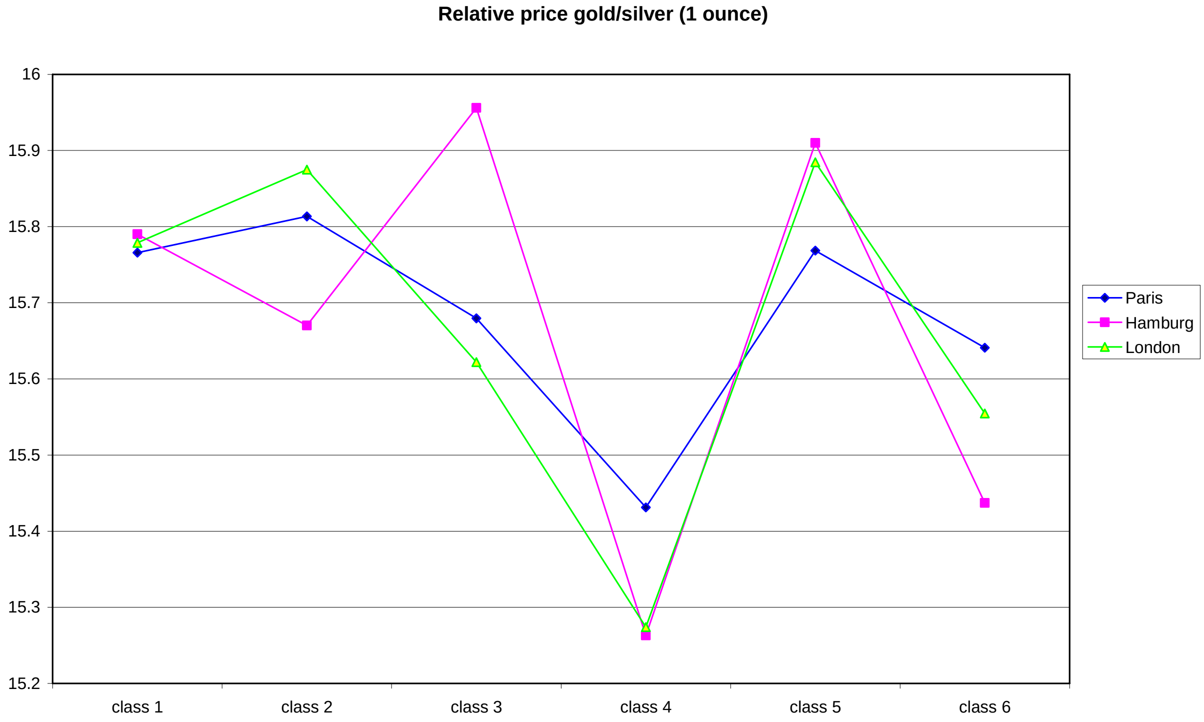
- produces combinations of quantitative variables correlated with the class division, ordered according to the decreasing multiple correlation with the classes
- each canonical component is uncorrelated with another one
- it permits to measure the weight of each variable in these new variables

3.2.2. The results

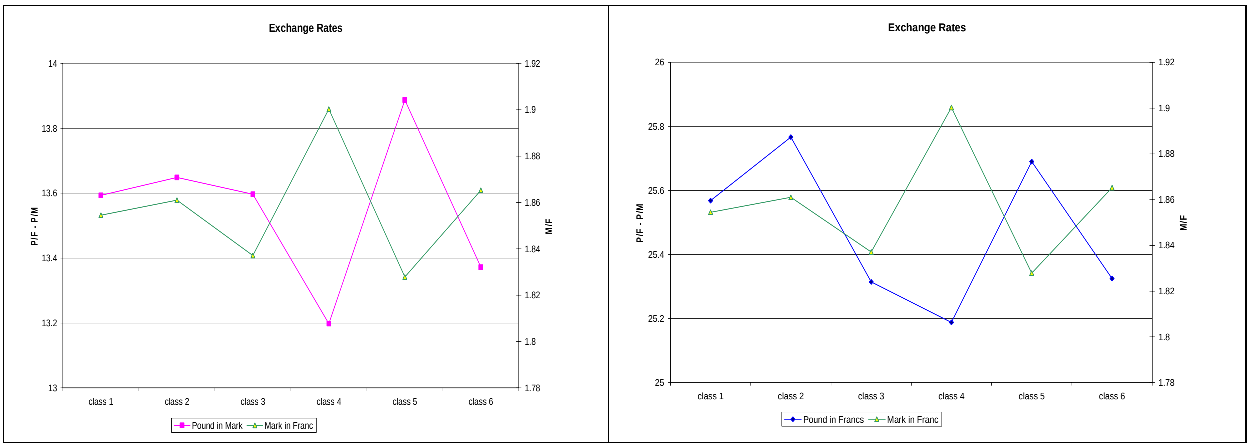
Variable	Can1	Can2
poa1	0.605	-0.813
hoa1	-0.016	-0.189
lgs1	1.235	0.005
hpl1	0.637	1.399
lpv1	0.284	-0.540
hlv1	0.380	0.609
phv1	-0.851	-0.167

[8] The colored form have no particular meaning: the variables are standardized, so one may just compare one point in a class (the value of the element representing a variable in the code vector of the class) and the same point in another class, if they are very different in level that means that these classes are grouping observations very distant in the distribution of this variable.

Graph 2
Relative prices of gold in silver



Graph 3
Exchange Rates



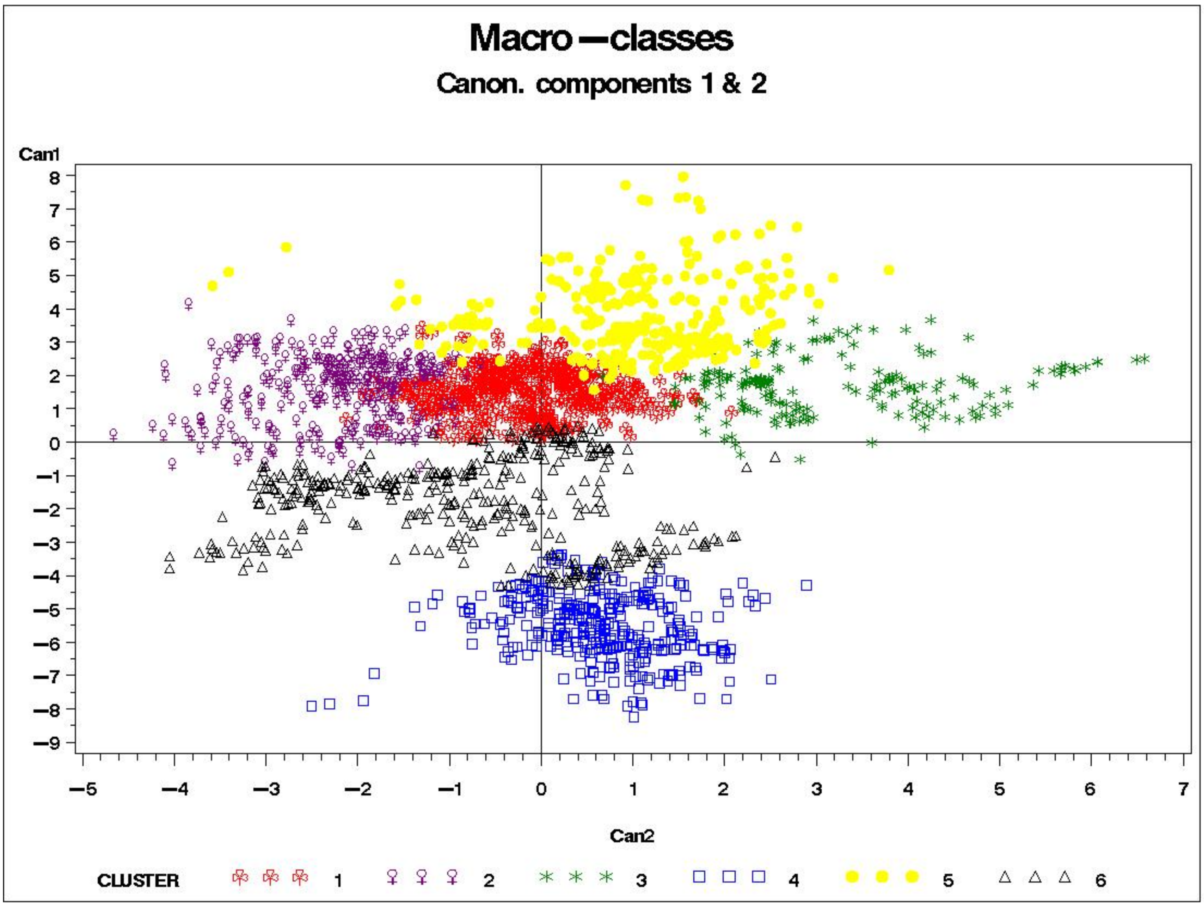
- The first 2 components have, respectively, correlations of .951 and .822
- Component 1 is constructed with, mainly, relative prices in London and Paris (*lgs* and *poa*) and negatively the rate of exchange between the mark and the franc (*phv*)
- Component 2: difference between relative prices Hamburg/Paris+London (*hpl*) and negatively relative price in Paris (*poa*)

The principal observations are:

- The classes are well separated in this representation, using the two first components, which are various combinations of the variables used
- Classes 4, 5 and 6 are clearly differentiated by component (*lgs* and *poa – phv*)
- Classes 1, 2 and 3 by component 2 (*hpl – poa*)
- Over the whole period what happens in each of the three places is significant and contributes to a typical set of relations.

This is a first evidence of the existence of the "international bimetallism".

Graph 4
Canonical Discriminant Analysis



4. SOME HISTORICAL REMARKS ON THE CLASSES.

When put together, the description of the resulting classes shown in graphs 1 to 3 and historical evidence provided by research suggests the following interpretation:

4.1. On the structure of the European system of international payments

The study confirms that the working of the system did not only rely on the two main financial centers (London and Paris) and currencies (the British pound and the French franc) of the time, but also on a third center (Hamburg) and currency (the Hamburg mark), related to the payments of Northern and Eastern Europe. Graph 2 on gold/silver prices shows that the differences between the various classes as to the position of Hamburg are more contrasted than the ones of Paris and London. Graph 4 on canonical components shows that the mark/franc exchange rate and the ratio of the Hamburg gold/silver price to the Paris and London ones appear as relevant variables in both significant components.

4.2. On the periodization of the 40-year time-span

Rearranging the various classes according to calendar time allows distinguishing between three sub-periods: a) the 1820s (classes 5 and 3, covering 1821 to 1828); b) the 1830s and 1840s (classes 1 and 2, covering 1829 to 1849); c) the 1850s (classes 6 and 4, covering 1851 to 1860).

Only the years 1839-41 resist to that rearrangement, since they belong to class 6, while they should appear in classes 1 and 2 relative to the 1830s and 1840s.

Graph 2 illustrates the differences between these sub-periods: the 1820s are characterized by a much higher gold/silver price in Hamburg than in London and Paris, the 1850s by a much lower one, and the 1830s and 1840s by a close one. This division in three sub-periods is by no means artificial: historical events stand out as landmarks to separate them, such as the return of the Bank of England note to convertibility in 1821, the commitment of the same bank to purchase gold at a fixed price in 1827, the stabilization of the price of silver in Hamburg from 1829 on, the reform of the Bank of Hamburg in 1847, the "gold rush" of 1850, the change in the policy of the Banque de France relative to precious metals in 1853. Moreover, each sub-period may be historically distinguished by idiosyncratic aspects: the 1820s differ from the following years in the direction of bullion flows with Hamburg, which ran from Hamburg to London and Paris, while they ran the opposite way afterwards; the two freely-moving market prices of the precious metals – the price of gold in Hamburg and the price of silver in London (the price of gold was maintained fixed in London by the Bank of England, so as the price of silver in Hamburg by the Bank of Hamburg; both prices in Paris were administered by the Banque de France, which prevented them from varying too much, i.e. less than the price of gold in Hamburg and the price of silver in London) - were far more stable in the 1830s and 1840s than in the 1820s and the 1850s.

4.3. On the characteristics of particular sub-periods

The two classes significantly determined by canonical component 1 in Graph 3, one positively (class 5), the other negatively (class 4) are sharply contrasted, as shown by graphs 1 and 2: the years 1821-23 and 1826 (class 5) are marked by a low mark/franc exchange rate and by a high gold/silver price in all three financial centers, the Hamburg one being higher than the Paris one; the years 1854-60 (class 4) are marked by a high mark/franc exchange rate and by a low gold/silver price in all three centers, the Hamburg one being below the Paris one. Again, these observations, which also apply respectively to the rest of the 1820s (class 3) and to the rest of the 1850s (class 6) are consistent with historical analysis: while the Hamburg mark was always anchored to silver, the French franc was during the 1820s and 1850s anchored to gold (in contrast with the 1830s and 1840s when it was anchored to silver); it is then normal that the mark depreciated against the franc when silver depreciated against gold, and more in Hamburg than in Paris (as in class 5 and 3), and that the mark appreciated against the franc when silver appreciated against gold, and more in Hamburg than in Paris (as in class 4 and 6).

5. A TWO-REGIME MARKOV SWITCHING MODEL.

We choose one of the variables used, the price of gold in silver computed from the quotations of the two metals in Paris (it could be done with any of the variables) and we estimate a two-regime model to explain this variable by its past values and the other variables.

5.1. A Few Words on Autoregressive Switching Markov Models

Introduced by Hamilton (1989, [8]), switching Markov models allowed assessing the impact of the business cycle on the post-war United-States Gross National Product (GNP). Using these models put into evidence the existence of two different behaviors of the GNP series according to the recession or the normal periods of the economy.

This example made switching Markov models very popular in the econometric world, although their statistical properties were not studied until recently. What makes them very appealing is the possibility of modeling nonlinear and non-stationary time series by piecewise linear and stationary models.

The key assumption is that the time series to be modeled follows a different pattern or a different model according to some unobserved, finite valued process. Usually, the unobserved process is a Markov chain whose states are called "regimes", while the observed series follows a linear autoregressive model whose coefficients depend on the current regime.

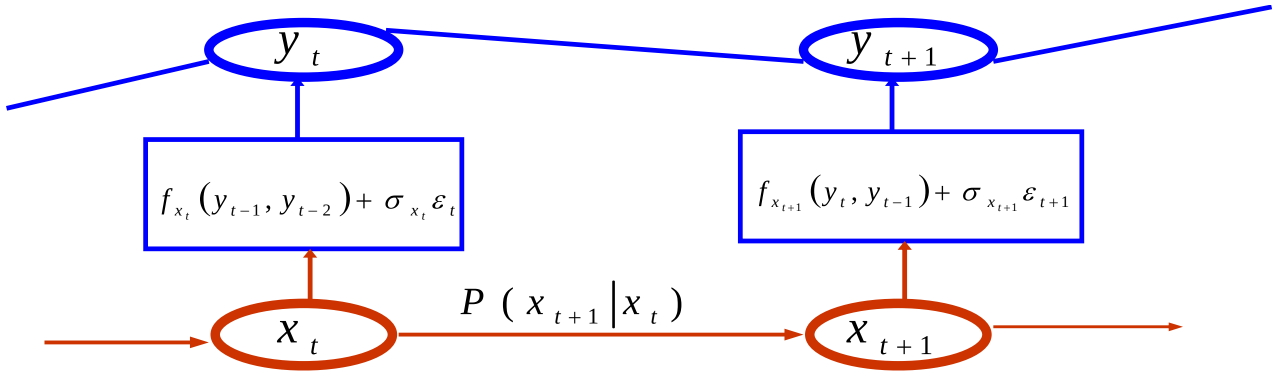
Let us put this in a mathematical language. Suppose that $(y_t)_{t\in Z}$ is the observed time series and that the unobserved process $(x_t)_{t\in Z}$ is a two-stated Markov chain with transition matrix

$$A = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}, \text{ where } p,q \in]0,1[.$$

Then, assuming that y_t depends on the first *l* lags of time, we have the following equation of the model:

$$y_t = a_0^{x_t} + a_1^{x_t} y_{t-1} + ... + a_l^{x_t} y_{t-l} + \sigma^{x_t}\varepsilon_t,$$

where $a_i^{x_t} \in \{a_i^1, a_i^2\} \in R^2$ for every $i \in \{0,1,...,l\}$, $\sigma^{x_t} \in \{\sigma^1, \sigma^2\} \in (R_+^*)^2$ and ε_t is a standard Gaussian noise.



The parameters of the model are then $\{a_0^1, a_1^1, ..., a_l^1, a_0^2, a_1^2, ..., a_l^2, \sigma^1, \sigma^2, p, q\}$ and they are usually estimated by maximizing the log-likelihood function via an EM (Expectation – Maximization) algorithm.

Our characteristic of interest will be the "a posteriori" computed conditional probabilities of belonging to the first or to the second regime. Indeed, as our goal is to derive a periodization of the international bimetallism, the "a posteriori" computed states of the unobserved Markov chain will provide a natural one.

Although the results obtained with a switching Markov model are usually satisfying in terms of prediction and the periodizations are interesting and easily interpretable, the critique one could make is how does one choose the number of regimes? In the absence of a complete theoretical answer, the criteria for selecting the "right" number of regimes are quite subjective from a statistical point of view. In this paper we chose to use a two-regime model since we expected to exhibit one regime in which arbitrage opportunities between the three financial centres were numerous, leading to significant international bullion flows, and one regime in which they were not.

5.2. Provisional results[9]

Two different models have been estimated for the selected variable, the relative price of gold in silver in Paris (*poa*):

- *poa* explained by the current other relative prices and its past (raw series)
- *poa* explained by the current other relative prices and the current rates of exchange between currencies (series in first differences).

[9] The program used to estimate the models has been developed by J. Rynkiewicz, CES-Samos and may be downloaded from http://samos.univ-paris1.fr

A lot of models have been tested each time, with various orders of lags, the best one, among those giving stable regimes, is kept according to likelihood function.

5.2.1. Raw series, rates of exchange not in the model:

- **Estimated equation** :

poa2(t)=intercept+hpl2(t)+lgs2(t)+hoa2(t)+poa2(t-2)+poa2(t-1)

- **Régime 1**

Constant	0.00339494
hpl2	0.00667254
lgs2	-0.00137691
hoa2	3.66131
poa2(t-2)	-1.63327
poa2(t-1)	-2.38106

The significant variables are the last three ones: relative price in Hamburg (hoa2) and the two lags of the endogenous variable (two preceding quotations)

  - Residual Variance 0.000122619

- **Régime 2**

constant	0.0089681
hpl2	0.830952
lgs2	-0.0140078
hoa2	0.136328
poa2(t-2)	0.0908092
poa2(t-1)	-0.0955607

In this regime the significant variables are the difference between Hamburg and the average of quotations in Paris and London (hpl2) and the relative price in Hamburg (hoa2)

  - Residual Variance 0.0309001

- **Transition Matrix** : it shows a good stability of the two regimes

0.889798	0.110202
0.187657	0.812343

It may be seen on this graph that the long periods identified by regime 2 correspond to class 3 (1823-25 and 1827-28) and parts of class 4 (1854 and after).
A first enquiry on the periods governed by regime 2 gives some evidence of a systematic correspondence between this type of regulation and some shock observed in London: the relative price in London is never the major variable, but when something happens in London the relative price in Hamburg and its specificity compared to the quotations in Paris and London seem to lead the markets.

5.2.2. A model with rates of exchange and relative prices together (on first differences)

- **Regime 1**

const	0.000184738
hoa	2.1206
lgs	-1.05941
hpl	-33.2228
lpv	-0.0137164
hlv	0.00147314
phv	0.0420996

  - variance1 3.76589e-05

- **Regime 2**

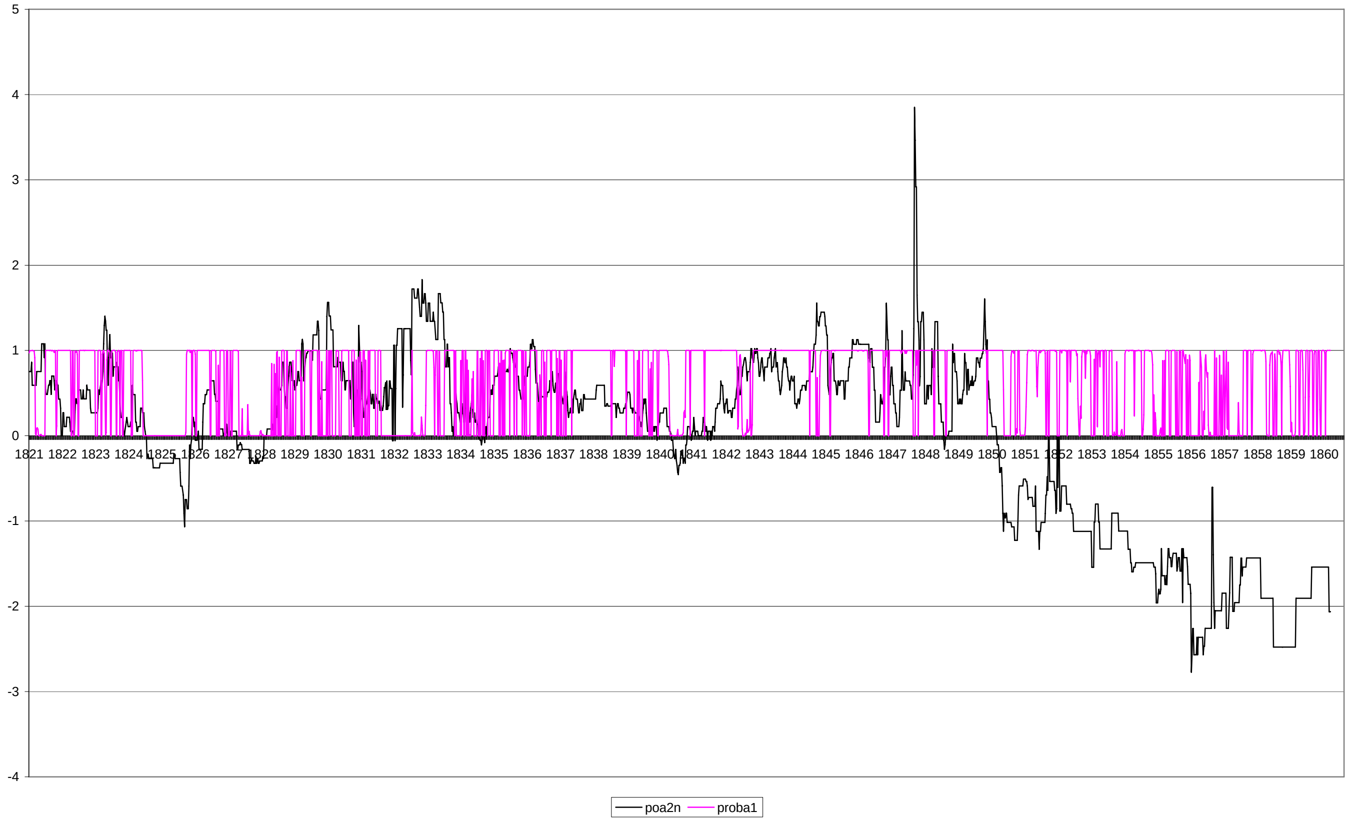
const	0.000150906
hoa	-0.0107028
lgs	0.00416001
hpl	0.120433

lpv 0.00655353
hlv 0.00699334
phv 0.0294265
- variance2 3.76749e-05

- **Transition Matrix**

0.85663	0.14337
0.127121	0.872879

Graph 5
The two regimes according to the first model[10]



In theses two regimes the exchange rates seem to be negligible compared to the major influences of the current relative prices and the distance between Hamburg and the two other places (regime 1) or mainly this distance only (regime 2).

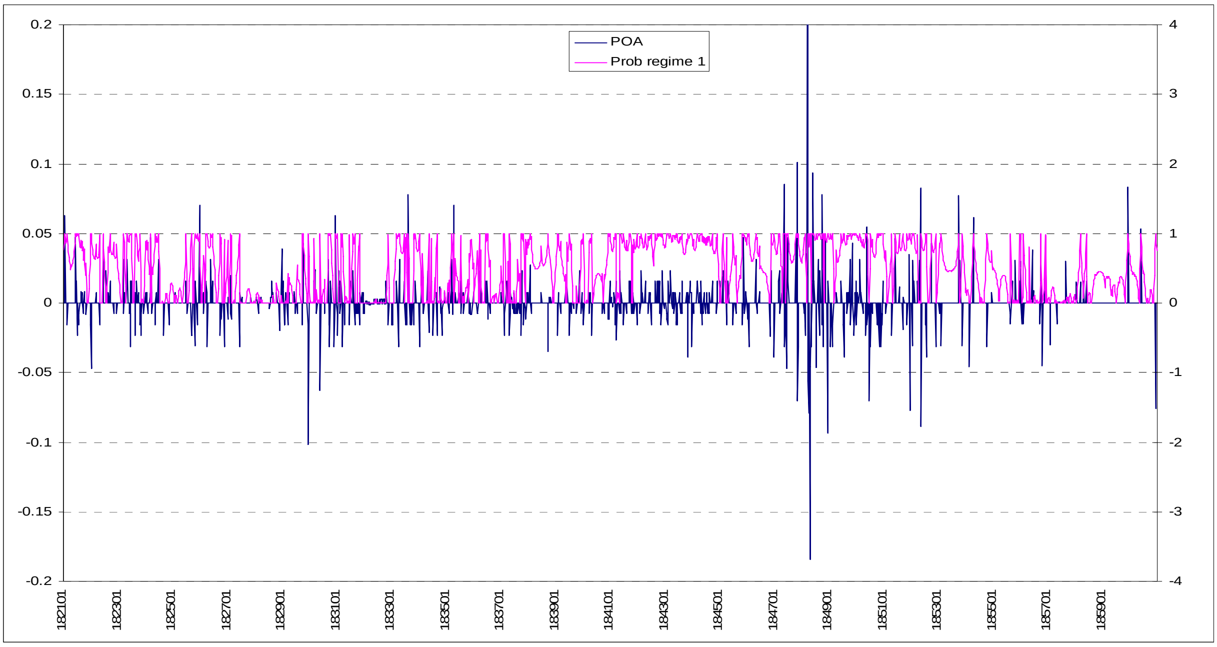
Again, this second model shows a good correspondence between regime 2 and the most homogenous sub-periods (middle of the 1820s and most of the 1850s).

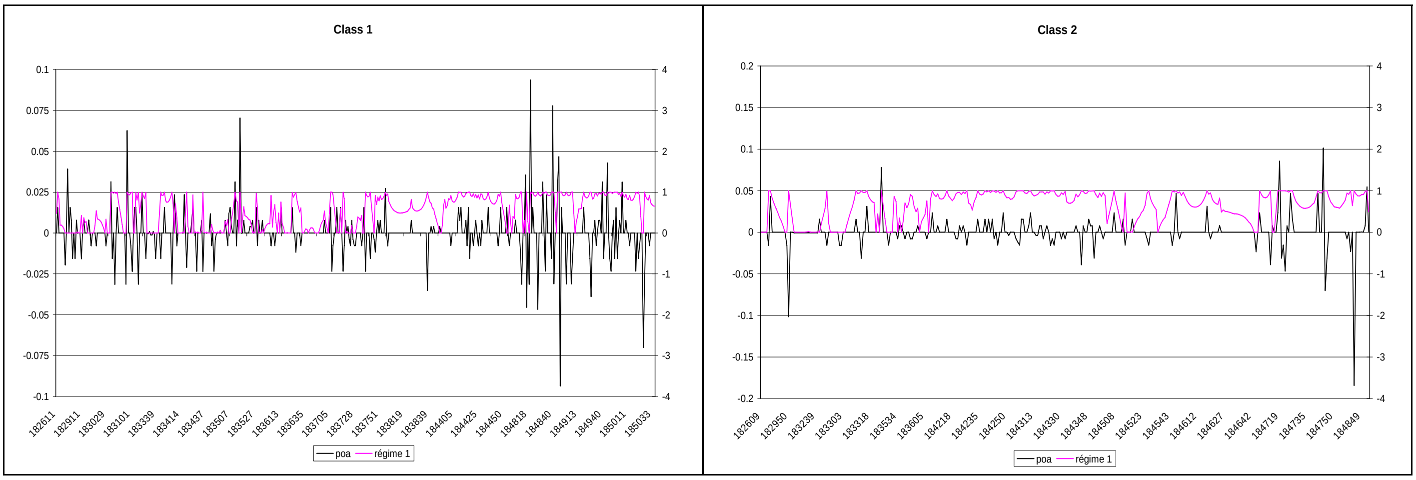
5.2.3. A representation of the two regimes (using the second model) crossed with the classification gives some idea of this correspondence

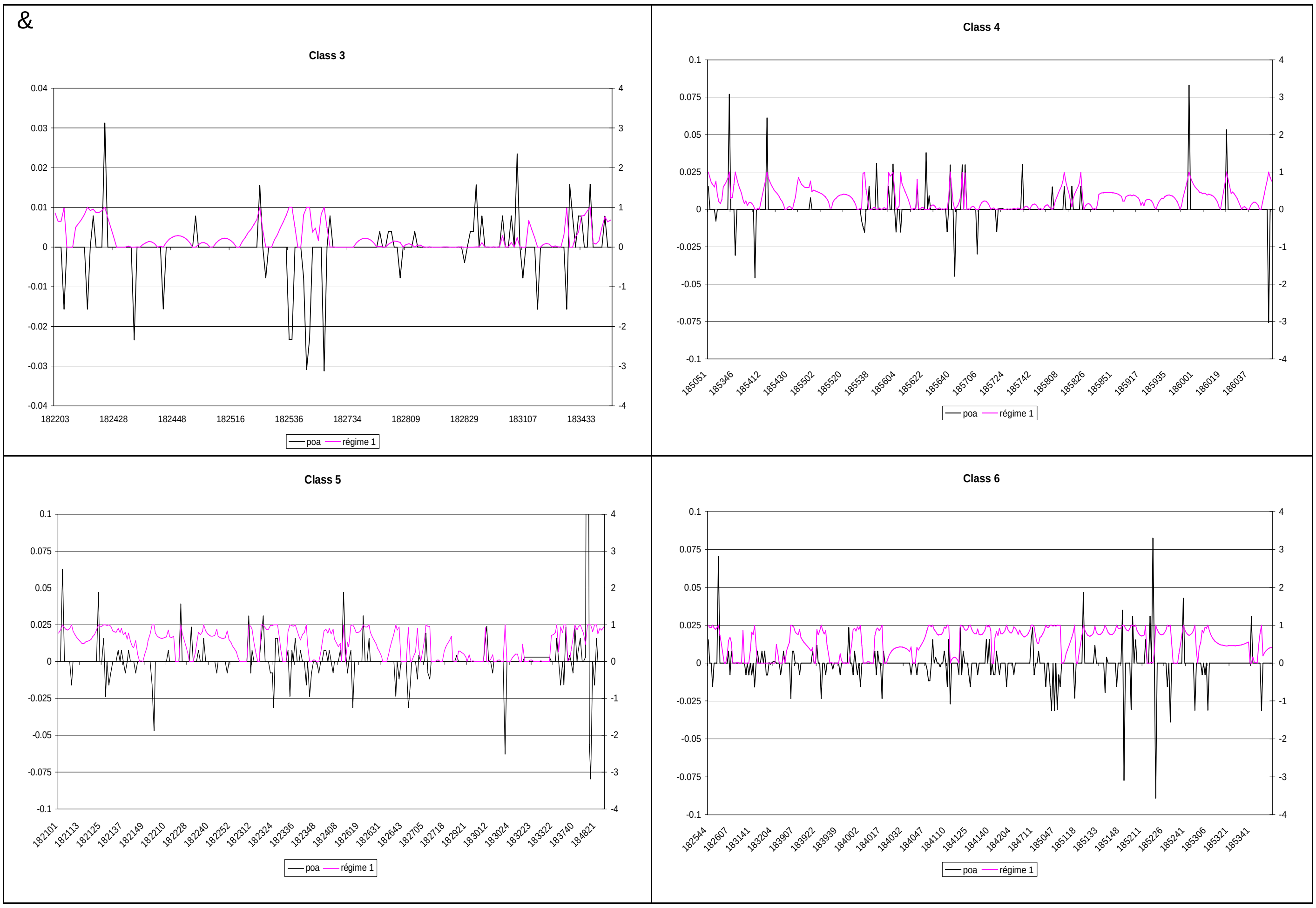
Classes 3 and 4 are mainly explained by regime 2 model (80 % of the total duration) contrasting with class 2 explained most of the time by regime 1, the three other ones being under regime 1 for a little less than 2/3 of their respective periods.

[10] The line in pink represents the probability of regime 1

Graph 6
The two regimes according to the second model







6. CONCLUDING REMARKS.

Provisional conclusions may be proposed but a lot of work remains to be done to elucidate the short-period regulation of international bimetallism.

Three main sub-periods are observed as a result of the combination of the neural classification and the Markov switching model:

- The first group of sub-periods is identified by a higher price of gold in silver in Hamburg and a low mark/franc rate of exchange
- The last group is symmetrical respectively to this two variables
- The middle one shows very close rates and ratios and a greater stability than the two others.

The most important influences are the relative price of gold in silver in Hamburg and its distance to the average level of the same relative prices in Paris and London. The idea of a dominance of the financial place of London is rejected over the whole period, but shocks affecting this place seem to emphasize the role of Hamburg.

Of course, a lot of work has to be done now:

- To try a more complete model (relative prices, rates of exchange and lags)
- Explain the periods with a great stability within a regime and those with numerous switches
- Explicit the role of shocks and their diffusion.

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