OPTIMAL ORDERING POLICIES UNDER CONDITIONS OF EXTENDED PAYMENT PRIVILEGES FOR DETERIORATING ITEMS

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ABSTRACT
This paper deals with optimal order quantities for firms, where units in an inventory are subject to deterioration at a constant rate, which are offered a one time opportunity to delay payment for an order of a commodity. Such delayed payment reduces purchase cost which is a function of the return available on alternative investments, the number of units ordered and the length of the extended period. Optimal order quantities are developed for extended payment privileges that occur at a reorder point. Four supplier's extended payment scenarios are evaluated. An analysis is conducted to determine the sensitivity of derived model to changes in the various input parameters.

Key words: Deterministic model, extended payment privilege, deterioration of units.

MSC: 90B05

RESUMEN
Este trabajo trata de las cantidades óptimas de las órdenes para empresas donde las unidades en un inventario están sujetas a deterioración con una razón proporción constante, la que se ofrece una oportunidad en un solo tiempo para retardar el pago de la orden de un artículo. Tales retrasos en el pago reducen el costo de la compra, que es una función del retorno disponible en las inversiones alternativas, del número de unidades pedidas y de la longitud del periodo extendido. Se desarrollan las cantidades del orden óptimas por privilegios en el pago extendido que ocurre en un punto de reorden (nuevo) pedido. Cuatro escenarios de pago extendido del proveedor se evalúan. Un análisis es desarrollado para determinar la sensibilidad de modelo derivado ante los cambios en varios parámetros de la entrada.

INTRODUCTION

In practice, a supplier offers their customers a one-time opportunity to delay settlement of payment for an order of a particular commodity. For example, a wholesaler of a variety of commodities like cosmetics, foodstuff, bakery items etc may find that their inventories, is demandless because of extraordinary high yields or substitutable products in the market. Allowing an extended payment privilege to the customer is an approach of reducing their excessive inventories. Such payment policies may be introduced as an alternative to price discounts because such policies are not meant to force competitors to reduce their prices. In these situations, the customers would like to order more units because paying later effectively reduces to purchase cost of the order. The amount of the reduction is a function of the return available on alternative investments by the customer, the number of units ordered that qualify for an extended payment privilege and the length of the extended payment period offered. The effect of reduction in costs on total operating cost should be studied.

Baker (1976) developed ordering policies when supplier discounted the price of their finished goods by a fixed percentage, which was to be applied to all units ordered on a single order. In it, a discount is applied to all units, even if only an extra unit is ordered above the reorder quantity, which seems to be unrealistic. Aggrawal (1979) dealt with the extended payment privilege policy as an alternative to price discounting. Davis and Gaither (1985) developed an optimal ordering policy for items that are offered extended payment privileges at a reorder point or between reorder points.

In this paper, we develop optimal ordering policies for firms, in which units in inventory are subject to deterioration; those are offered extended payment privileges. Four supplier's extended payment scenarios are developed and described for which optimal ordering policies are derived.

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The paper is organized into the following sections:

- Basic model formulation,
- Optimal ordering policies for the basic model,
- Four extended optimal payment scenarios,
- Model sensitivity, and
- Summary.

BASIC MODEL FORMULATION

Assume that the extended payment opportunity coincides with a replenishment cycle, two options are open to the customer. Option I denote the case where the customer chooses not to take advantage of the extension and continue to order Q (using Hwang and Sojn (1983)) units. Option II represents ordering an additional x – units to take advantage of the extension. The constant demand rate of R – units per annum is known.

The following assumptions are used in the formulation of the cost function.

- The extended payment privilege is a one-time opportunity for a single commodity.
- Replenishment rate is infinite. Lead-time is zero.
- Shortages are not allowed.
- Regular purchase cost per unit is C.
- Return rate for alternative uses of funds for the given extended period is α and $0 < \alpha < 1$.
- Effective purchase cost per unit with extended payment privilege is $(1 - \alpha)C$. The regular purchase cost C is reduced by the quantity αC because the funds are available for other uses during the extended payment period that earn a return rate of α, and, thus, effectively reduces the purchase cost.
- Carrying charge fraction I per annum is known and constant.
- Cost of placing an order is A.
- The units in inventory are subject to deteriorate at a constant rate θ $(0 < \theta < 1)$. There is no repair or replacement of the deteriorated units during the cycle time under review.
- Relevant cost includes purchasing, deteriorating, ordering and carrying costs.

Total cost functions TC(I) and TC(II) will be developed for options I and II respectively. To isolate the impact on costs from ordering an additional x – units during T_1 – time units (a decision variable), a cost difference function $D(T_1)$ will be developed as the difference between TC(I) and TC(II). To find optimum value of T_1 (and, hence, x), we maximize the difference $D(T_1)$, indicating the optimal response to extended payment opportunity.

Option I: When the decision maker chooses to take advantage of the extended payment privilege, following Shah (1977)

$$TC(I) = \frac{CR}{\theta}\left\{e^{\theta T_0} - 1\right\} + \frac{C(\theta + I)R}{\theta^2}\left\{e^{\theta T_0} - \theta T_0 - 1\right\} + A \tag{1}$$

$$T_0 = \sqrt{\frac{2AR}{C(2\theta + I)}} \tag{2}$$

where T_0 can be evaluated using Hwang and Sojn (1983) and, hence

$$Q_0 = RT_0 \tag{3}$$

Here Q_0 denotes the optimum purchase quantity.

Option II: When the decision maker takes advantage of the extended payment privilege, the total cost expression is derived as follows:

Let $Q_x(t)$ denotes the on hand inventory at any instant of time t when additional x – units are to be ordered. The differential equation that governs the situation at any instant t is given by

$$\frac{dQ_x(t)}{dt} + \theta Q_x(t) = -R, \quad 0 \le t \le T_0 + T_1$$

Under boundary condition $Q_x(T_0+T_1) = 0$, the solution of above differential equation is given by

$$Q_x(t) = \frac{R}{\theta}\left\{e^{\theta(T_0+T_1-t)} - 1\right\}$$

and number of additional units to be purchased

$$x = \frac{R}{\theta} e^{\theta T_0}\left\{e^{\theta T1} - 1\right\} \tag{4}$$

1. Number of a unit, which deteriorate during $(0, T_0 + T_1)$, is

Thus, total cost due to deterioration is

$$DC = \frac{C(1-\alpha)R}{\theta}\left\{e^{\theta(T_0+T_1)} - \theta(T_0+T_1) - 1\right\} + \frac{CR}{\theta}\left\{e^{\theta T_0} - \theta T_0 - 1\right\}\left\{\frac{1-T_0-T_1}{T_0}\right\}$$

2. Inventory holding cost is

$$IC = \frac{C(1-\alpha)RI}{\theta}\left\{e^{\theta(T_0+T_1)}\theta(T_0+T_1) - 1\right\} + \frac{CIR}{\theta}\left\{e^{\theta T_0} - \theta T_0 - 1\right\}\left\{\frac{1-T_0-T_1}{T_0}\right\}$$

3. Purchase cost is

$$PC = \frac{C(1-\alpha)R}{\theta}\left\{e^{\theta(T_0+T_1)} - 1\right\} + \frac{CR}{\theta T_0}\left\{e^{\theta T_0} - 1 - T_0(e^{\theta(T_0+T_1)} - 1)\right\}$$

4. Ordering cost is

$$OC = A + \frac{A(1-T_0-T_1)}{T_0}$$

Therefore, for Option II, the total annual cost is

$$TC(II) = PC + DC + OC + IC \tag{5}$$

The difference cost function $D(T_1)$ is obtained as

$$D(T_1) = TC(I) - TC(II) \tag{6}$$

The objective of this paper is to establish relationship between TC(I), TC(II) and $D(T_1)$ to find the optimal behavior of the customer with reference to the optimal number of additional x – units to be ordered when units in inventory are subject to deterioration at a constant rate.

OPTIMAL ORDERING POLICIES FOR THE BASIC MODEL

Since TC(I) is independent of T_1 and TC(II) is convex with respect to T_1, the two functions will have equal values for at most one value of T_1. At these point, the difference function $D(T_1)$ is zero. Using classical optimization technique on the difference function $D(T_1)$ the location of the global x_0 can be found by solving $D'(T_1)$ equal to zero for T_1.

FOUR EXTENDED OPTIMAL PAYMENT SCENARIOS:

The terms of extended payment plans offered to a firm may differ, and accordingly, the customer's response to such terms will differ. Four different scenarios will be discussed in this article. They are

1. Extended payment privilege allowed on all units, when $(Q_0 + x)$ – units are ordered, if $x > 0$ at a reorder point.

2. Extended payment privilege allowed on all units, when $(Q_0 + x)$ – units are ordered, if $x > x_{min}$ where x_{min} is some stated quantity at a reorder point.

3. Extended payment privilege allowed on additional x – units only, when $(Q_0 + x)$ – units are ordered, if $x > 0$ at a reorder point.

4. Extended payment privilege allowed on additional x – units are ordered, if $x > x_{min}$ where x_{min} is some stated quantity at a reorder point.

Scenario 1. The first situation where extension is allowed on all units when $(Q_0 + x)$ – units are ordered, if $x > 0$ is the basic model developed in section 3.

Scenario 2. If a minimum quantity, x_{min} , above the reorder quantity Q_0 is specified by the supplier to qualify for the offer of extended payment privilege, the customer's reponse depends upon the magnitude of x_{min} . The minimum additional quantity x_{min} can be established as follows:

As defined earlier T_0 represents cycle time for optimum purchase quantity Q_0 and T_1 denotes cycle time for the purchase of additional x – units (say x_0) then $x_0 > x_{min}$ implies

$$x_{min} = \frac{F \pm \sqrt{F^2 - 4EG}}{2E} \tag{8}$$

where $E = \theta$, $F = 2(1 - \theta T_0)$ and $G = 2RT_1$. Once x_{min} is established, two possibilities must be considered to determine the optimum policy:

1. $x_{min} < x_0$, x_0 – additional units should be ordered.

2. $x_{min} > x_0$, x_{min} – additional units should be ordered.

Scenario 3: When the supplier will only allow extended payment privilege on additional units ordered beyond Q_0 at a reorder point, the impact is only on the various components involved in TC(II). The various cost component are as follows:

- Purchase cost is

- Ordering cost is

$$PC = C(1 - \alpha)x + C(R - x)$$

$$OC = A + \frac{A(1 - T_0 - T_1)}{T_0}$$

$$= \frac{C(1-\alpha)Re^{\theta T_0}}{\theta}\left\{e^{\theta T_1} - 1\right\} + \frac{CR}{\theta T_0}\left\{e^{\theta T_0} - 1 - T_0 e^{\theta T_0}(e^{\theta T_1} - 1)\right\} \tag{9}$$

- Inventory holding cost

For inventory holding cost, we will have to consider inventory in following three intervals.

0 $\quad$ T_0 $\quad$ T_0+T_1 $\quad$ T

We have differential equation as

$$\frac{dQ_x(t)}{dt} + \theta Q_x(t) = \begin{cases} 0, & 0 \le t \le T_0 \\ -R, & T_0 \le t \le T_0 + T_1 \\ -R, & T_0 \le T_1 \le t \le T \end{cases}$$

Hence

$$I_1(T_1) = \begin{cases} \frac{R}{\theta^2}(e^{\theta T_0} - 1)(e^{\theta T_1} - 1), & 0 \le t \le T_0 \\ \frac{R}{\theta^2}((e^{\theta T_0} + e^{\theta T_1} - 1)(1 - e^{\theta T_0}) - \theta T_1 e^{\theta T_0}), & T_0 \le t \le T_0 + T_1 \\ \frac{R}{\theta^2}(e^{\theta T_0} - \theta T_0 - 1)\left(\frac{1 - T_1}{T_0}\right), & T_0 + T_1 \le t \le T \end{cases}$$

$\therefore$ Total inventory holding cost is

$$IC = \frac{C(1-\alpha)IR}{\theta^2}(e^{\theta T_0} - 1)(e^{\theta T_1} - 1) + \frac{C(1-\alpha)IR}{\theta^2}((e^{\theta T_0} + e^{\theta T_1} - 1)(1 - e^{-\theta T_0}) - \theta T_1 e^{\theta T_0})$$

$$+ \frac{C(1-\alpha)IR}{\theta^2}(e^{\theta T_0} - \theta T_0 - 1)\left(\frac{1 - T_1}{T_0}\right) \qquad (11)$$

- Cost due to deterioration

$$DC = \frac{C(1-\alpha)R}{\theta^2}(e^{\theta T_0} - 1)(e^{\theta T_1} - 1) + \frac{C(1-\alpha)R}{\theta^2}((e^{\theta T_0} + e^{\theta T_1} - 1)(1 - e^{-\theta T_0}) - \theta T_1 e^{\theta T_0})$$

$$+ \frac{C(1-\alpha)R}{\theta^2}(e^{\theta T_0} - \theta T_0 - 1)\left(\frac{1 - T_1}{T_0}\right) \qquad (12)$$

Hence,

$$TC(II) = PC + DC + OC + IC \qquad (13)$$

Subtracting equation (13) from equation (1), the new difference function can be obtained. Knowing $Q = Q_0$ from (3), classical method of optimization is used to find the additional x – units to be purchased.

Scenario 4: Once a minimum quantity ($Q_0 + x_{min}$) (using (3) and (4)) is established to qualify for the delayed payment, the customer's response is similar to that defined in scenario 2.

MODEL SENSITIVITY

First, we see how sensitive total costs TC(I), TC(II), reorder quantity and additional units are to changes in ordering cost, demand, carrying charge fraction and return rate for alternate use of funds. Particular interest is the sensitivity of additional units to changes in ordering cost and carrying charge fraction.

Table 1.
[C, I, R, α] = [50, 0.15, 1000, 0.4]

A	θ	0.01	0.02	0.03
150	Q	4140.39	3302.89	2828.43
	x	71.78	69.35	63.31
	TC(I)	380204.80	354586.10	344770.30
	TC(II)	32617.65	46213.26	55770.23
	D	347587.10	308372.90	289000.10
200	Q	4780.91	3813. 85	3265.99
	x	96.91	94.13	91.88
	TC(I)	475323.40	453130.80	448548.00
	TC(II)	41402.19	55844.51	65988.65
	D	433921.20	397286.30	382559.40
250	Q	5345.23	4264.01	3651.48
	x	123.89	120.38	118.37
	TC(I)	568704.80	552670.60	555769.50
	TC(II)	52167.05	67288.62	77868.34
	D	516537.70	485382.30	477901.10

Table 2.
[C, I, A, α] = [50, 0.15, 150, 0.4]

R	θ	0.01	0.02	0.03
1000	Q	4140.39	3302.89	2828.43
	x	71.78	69.35	63.31
	TC(I)	380204.80	354586.10	344770.30
	TC(II)	32617.65	46213.26	55770.23
	D	347587.10	308372.90	289000.10
2000	Q	8280.79	6605.78	5656.85
	x	71.42	69.23	63.12
	TC(I)	760259.50	709022.30	689390.60
	TC(II)	65195.40	92377.88	111484.90
	D	695064.10	616644.40	577905.80
3000	Q	12421.18	9908.67	8485.28
	x	71.42	69.23	63.21
	TC(I)	1140314.00	1063458	1034011
	TC(II)	97773.15	138542.50	167199.60
	D	1042541.85	924915.9	866811.40

Table 3
[C, R, A, α] = [50, 1000, 150, 0.4]

I	θ	0.01	0.02	0.03
0.20	Q	3872.98	3162.28	2738.61
	x	50.04	55.87	53.85
	TC(I)	365194.20	345592.30	338198.50
	TC(II)	42681.85	53090.61	60894.53
	D	322512.60	292501.70	277303.9
0.25	Q	3651.48	3038.22	2656.85
	x	35.89	45.85	46.38
	TC(I)	352777.80	337681.40	332241.40
	TC(II)	50646.84	58897.97	65382.34
	D	302131	278783.50	266859
0.30	Q	3464.10	2927.70	2581.99
	x	25.69	38.21	40.35
	TC(I)	342285.7	330653	326809.5
	TC(II)	57129.42	63871.95	69347.32
	D	285156.30	266781.10	257462.20

Table 4.
[R, I, A, α] = [1000, 0.15, 150, 0.4]

C	θ	0.01	0.02	0.03
50	Q	4140.39	3302.89	2828.43
	x	71.78	69.35	63.31
	TC(I)	380204.80	354586.10	344770.30
	TC(II)	32617.65	46213.26	55770.23
	D	347587.10	308372.90	289000.10
60	Q	3779.65	3015.11	2581.99
	x	49.92	47.27	42.24
	TC(I)	397756.40	366361.80	352629.40
	TC(II)	35034.99	50684.32	61673.15
	D	362721.40	315677.50	290956.20
70	Q	3499.27	2791.45	2390.45
	x	36.88	34.37	30.28
	TC(I)	414239.80	377841.30	360812
	TC(II)	38170.10	55749.15	68074.31
	D	376069.70	322092.20	292737.60

Table 5.
[C, I, A, R] = [50, 0.15, 150, 1000]

α	θ	0.01	0.02	0.03
0.35	x	71.78	69.35	63.31
	TC(II)	32617.65	46213.26	55770.23
	D	347587.10	308372.90	289000.10
0.40	x	71.42	69.23	63.12
	TC(II)	65195.40	92377.88	111484.90
	D	695064.10	616644.40	577905.80
0.45	x	71.42	69.23	63.21
	TC(II)	97773.15	138542.50	167199.60
	D	1042541.85	924915.9	866811.40

Interpretations:

- From Table 1, it is observed that keeping ordering cost fixed, increase in the deterioration of units, decreases additional units to be purchased and significant decrease in the difference cost function. While for fixed rate of deterioration, increase in the ordering cost results increase in additional units to be purchased and hence significant increase in the difference cost function.

- Table 2 suggests that for fixed demand, increase in the deterioration rate of units reduces optimum number of units and additional to be purchased and hence difference cost function decreases. While increase in the demand for fixed rate of deterioration, optimum purchase quantity to be procured increase and hence difference cost function increase by a good margin. It has very little effect on purchase of additional units.

- Table 3 suggests that increase in the inventory carrying charge fraction for fixed rate of deterioration reduces number of additional units to be procured and hence decrease in the difference cost function. For fixed carrying charge fraction, increase in deterioration rate reduces optimum number of units to be purchased and hence decreases the difference cost function while additional number of units to be purchased increases.

- From Table 4, it is observed that increase in purchase cost, for fixed rate of deterioration reduces optimum number of units and additional number of units to be purchased but increases the difference cost function. Similar observations are for fixed purchase cost and increase in the deterioration rate of units.

- Table 5 suggests that for fixed vale of return rate, increase in the deterioration rate reduces additional units to be procured and difference cost function significantly. Increase in return rate for alternative uses of fund increase difference cost function significantly while additional units to be procured are least affected.

SUMMARY

In this paper, we have developed optimal ordering policies for firms those are offered extended payment privileges by their suppliers for particular commodities which are subject to deterioration. Optimal ordering policies are developed for four different supplier's extended payment privileges when units in inventory are subject to deterioration.

Because of the simplicity of calculations of these decision rules, their use should be as straightforward as EOQ model and other commonly used inventory calculations. Therefore their application would be expected to be high.

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