AN EOQ MODEL FOR DETERIORATING ITEMS UNDER PERMISSIBLE DELAY IN PAYMENTS WHEN SUPPLY IS RANDOM

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ABSTRACT
In classical EOQ model, it is assumed that the quantity requisitioned is same as the quantity ordered, payment of the goods is made as soon as it is received by the system and units in inventory are not subject to deterioration. In the present article, an attempt is made to develop an inventory model when retailer announces delay in payments, units in inventory are subject to constant rate of deterioration under random input. A developed model is supported by a hypothetical numerical to study interdependence of parameters on the decision variables and objective function.

Key words: EOQ, deterioration, delay in payments, random supply.

RESUMEN
En el modelo clásico EOQ, es supuesto que la cantidad requisada es la misma que la cantidad pedida, el pago de las mercancias es hecho en cuanto se reciba por el sistema y las unidades en el inventario no están sujetas a deterioración. En el artículo presente, se trata de desarrollar un modelo de inventario que cuando el minorista anuncia el retraso en los pagos, las unidades en el inventario están sujetas a una proporción constante de deterioro bajo entradas aleatorias azar. Un modelo es desarrollado y este se apoya en un estudio numérico hipotético para estudiar la interdependencia de parámetros en las variables de decisión y en la función del objetivo.

1. INTRODUCTION

In the classical EOQ model, it is implicitly assumed that the quantity received matches with the quantity requisitioned and there is no damage or deterioration of the units while in inventory and also that the supplier must be paid for the goods procured as soon as they are received by the inventory system. However, in practice, it happens that the quantity received may be different from the quantity ordered. Also it is normal practice in business that the supplier allows certain fixed credit period for settling the accounts and no interest charges are payable if the account is settled within the prescribed period.

Silver (1976) has developed an EOQ model when the quantity received is uncertain and is a random variable with specified mean and variance. Kalro and Gohil (1982) have extended the above model by allowing shortages. Noori and Keller (1986) developed a stochastic model when quantity received is uncertain. Ghare and Schrader (1963) developed an EOQ model for exponentially decaying inventories. This model has been generalized by Covert and Philip (1973) and the by Philip (1974) by using weibull distribution to describe time to deterioration of an inventory. Goyal (1985) has developed an EOQ model when supplier allows fixed credit period for settling the accounts. Mandal and Phaujdar (1989) generalized the work of Goyal (1985) by taking into consideration a variety of realistic situations.

In this article, we analyze an EOQ model for deteriorating items when the supply is random and supplier allows a certain fixed credit period for settling the accounts. During the time account is not settled, it is assumed that the cost of unit sold is deposited in an interest bearing accounts and the profit margin is used to meet the further operational expenses of the system.

2. ASSUMPTIONS AND NOTATIONS

- The demand rate of R units per time unit is known and constant.
- Shortages are not allowed. Lead time is zero.

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- The replenishment rate is infinite. The replenishment size Q is the decision variable. The order size is Q - units per order, however, the actual quantity received (say) Y is a normal decision variable with

$$E(Y) = bQ$$

$$V(Y) = \sigma_0^2 + \sigma_1^2 Q^2 \quad (1)$$

where $b \geq 0$ is the bias factor, σ_0^2 and σ_1^2 are the positive known constants.

- At time t of a cycle, the constant fraction θ $(0 \leq \theta \leq 1)$ of the on hand inventory deteriorates per unit of time.
- There is no repair or replacement of the deteriorated inventory during the period under consideration.
- Supplier gives a specified credit period of M - time units to settle the accounts.
- The unit cost C does not depend upon the quantity ordered or received.

The other notations are as follows :

- h denotes the unit holding cost exclusive of interest charges.
- I_c denotes the interest charges per rupee investment per time unit.
- I_e denotes the interest earned per rupee investment per time unit.
- A denoted the ordering cost.
- AEC(Q) denotes the average expected cost per unit per time when Q is the quantity requisitioned.

3. MATHEMATICAL MODEL

Let T(Y) denotes the cycle time and Q(t/Y) denotes the on hand inventory of the system at any instant of time t of a cycle. Then the differential equation that describes the instantaneous states of Q(t/Y) is given by

$$\frac{dQ(t/Y)}{dt} + \quad Q(t/Y) = -R, 0 \leq t \leq T(Y) \quad (2)$$

T(Y) is a function of random variable Y. Noting that Q(0 / Y) = Y, the solution of (2) is

$$Q(t/Y) = e^{- \quad t}Y - \frac{R}{}(1 - e^{- \quad t}), 0 \leq t \leq T(Y) \quad (3)$$

Since,

$$Q(T / Y) = 0, \text{ we get } T(Y) = \frac{1}{}\log\left(1 + \frac{Y}{R}\right) \quad (4)$$

Now, the total number of units carried into the inventory during the cycle time is

$$I_1(Y) = \int_0^{T(Y)} Q(t/Y)dt = \frac{Y - RT(Y)}{} \quad (5)$$

and the number of units that deteriorate during the cycle time is

$$D(Y) = Y - RT(Y) \quad (6)$$

For obtaining the total expected cost per time unit we assume that θ and M^2 are very small as compared to other quantities, we use series form of expansions and retain the terms up to θ and M^2 only. We analyze one cycle. Two cases may arise.

Case 1: $Y \geq RM$ i.e in this case account is to be settled before the end of the cycle.

$$\text{Interest earned} = \frac{CI_eRM^2}{2} \tag{7}$$

$$\text{Interest charged} = \frac{CI_cY}{\theta}(e^{-\theta M} - e^{-\theta T(Y)}) - \frac{CI_cR}{2}(e^{-\theta T(Y)} + \theta T(Y)) - (e^{-\theta M} + \theta M)) \tag{8}$$

The total cost of the system is given by

$$K_1(Y/Q) = \frac{(h + CI_c + \theta C)Y^2}{2R} - \frac{\theta(h + CI_c)Y^3}{3R^2} + \frac{(\theta M - 2)CI_cMY}{2} + \frac{C(I_c - I_e)RM^2}{2} + A \tag{9}$$

The expected total cost of the system during the random cycle time is

$$E(K_1(Y/Q)) = \frac{(h + CI_c + \theta C)(\sigma_0{}^2 + (\sigma_1{}^2 + b^2)Q^2)}{2R} - \frac{\theta(h + CI_c)(b^3Q^3 + 3bQ(\sigma_0{}^2 + \sigma_1{}^2Q^2)}{3R^2} +$$

$$+ \frac{(\theta M - 2)CI_cMbQ}{2} + \frac{C(I_c - I_e)RM^2}{2} + A \tag{10}$$

Following Silver (1976), the average expected cost of the system per time unit is

$$AEC(Q_1) = E(K_1(Y/Q_1)) / E(T(Y)) \tag{11}$$

$$AEC(Q_1) = \left\{ \frac{(h + CI_c + \theta C)(\sigma_0{}^2 + (\sigma_1{}^2 + b^2)Q_1{}^2)}{2R} - \frac{\theta(h + CI_c)(b^3Q_1{}^3 + 3bQ_1(\sigma_0{}^2 + \sigma_1{}^2Q_1{}^2)}{3R^2} + \right.$$

$$\left. + \frac{(\theta M - 2)CI_cMbQ_1}{2} + \frac{C(I_c - I_e)RM^2}{2} + A \right\} * \left\{ \frac{R}{bQ_1} + \frac{\theta((\sigma_0{}^2 + (\sigma_1{}^2 + b^2)Q_1{}^2)}{2b^2Q_1{}^2} \right\} \tag{12}$$

For optimum value of Q_1, equating partial derivative of $AEC(Q_1)$ to zero, we get,

$$E_1Q_1^4 + E_2Q_1^3 + E_3Q_1 + E_4 = 0 \tag{13}$$

where

$$E_1 = \theta(h + Ci_c)\,[4b^2(b^2 + 3\sigma_1^2) - 3(\sigma_1^2 + b^2)]$$

$$E_2 = 3bR(\sigma_1^2 + b^2)\,[\theta\, CI_cM - (h + CI_c + \theta\, C)]$$

$$E_3 = 3bR[(h + CI_c + \theta\, C)\,\sigma_0^2 - \theta\, CI_cM\sigma_0{}^2 + C(I_c - I_e)R^2M^2 + 2AR]$$

$$E_4 = 3\theta\, \sigma_0{}^2[(h + CI_c)\,\sigma_0^2 + C(I_c - I_e)R + 2AR] \tag{14}$$

For obtaining optimum value of Q_1, solve the above equation (14) by any suitable numerical method. For solving (14) by Newton - Raphson method, we can take the initial iterate of $Q_1 = Q_{10}$ given by

$$Q_{10} = \left[\frac{(h+CI_c)\sigma_0^2 + 2AR + C(I_c - I_e)R^2M^2}{(h+CI_c)(\sigma_1^2 + b^2)}\right]^{1/2} \quad (15)$$

Case 1.1: Keeping variance fixed i.e. $\sigma_1 = 0$ and $\sigma_0 \geq 0$.

The above equation (15) becomes

$$Q_{10} = \left[\frac{(h+CI_c)\sigma_0^2 + 2AR + C(I_c - I_e)R^2M^2}{(h+CI_c)b^2}\right]^{1/2} \quad (16)$$

Then the average expected cost $AEC(Q_1 = Q_{10})$ is

$$\left\{\frac{(h+CI_c+\theta C)(\sigma_0^2 + b^2Q_1^2)}{2R} - \frac{\theta(h+CI_c)(b^3Q_1^3 + 3bQ\sigma_0^2)}{3R^2} + \right. \quad (17)$$

$$\left. + \frac{(\theta M - 2)CI_cMbQ_1}{2} + \frac{C(I_c - I_e)RM^2}{2} + A\right\} * \left\{\frac{R}{bQ_1} + \frac{\theta((\sigma_0^2 + b^2Q_1^2)}{2b^2Q_1^2}\right\}$$

For obtaining optimum value of Q_1, we have to solve the equation (14) whose constants are given by

$$E_1 = 4\theta\,(h + Ci_c)\,b^4$$

$$E_2 = 3R\,b^3\,[\theta\,CI_cM - (h + CI_c + \theta C)]$$

$$E_3 = 3bR\,[(h + CI_c + \theta C)\,\sigma_0^2 - \theta CI_cM\sigma_0^2 + C(I_c - I_e)R^2M^2 + 2AR]$$

$$E_4 = 3\theta\,\sigma_0^2\;\;[(h + CI_c)\sigma_0^2 + C(I_c - I_e)R + 2AR\,] \quad (18)$$

Case 1.2: The variance is proportional to the ordered quantity, i.e. i.e. $\sigma_0 = 0$ and $\sigma_1 > 0$. The above equation (15) becomes

$$Q_{10} = \left[\frac{2AR + C(I_c - I_e)R^2M^2}{(h+CI_c)(\sigma_1^2 + b^2)}\right]^{1/2} \quad (19)$$

and the average expected total cost is given by

$$AEC(Q_1) = \left\{\frac{(h+CI_c+\theta C)(\sigma_1^2 + b^2)Q_1^2}{2R} - \frac{\theta(h+CI_c)(b^3Q_1^3 + 3b\sigma_1^2Q_1^3)}{3R^2} + \right.$$

$$\left. + \frac{(\theta M - 2)CI_cMbQ_1}{2} + \frac{C(I_c - I_e)RM^2}{2} + A\right\} * \left\{\frac{R}{bQ_1} + \frac{\theta(\sigma_1^2 + b^2)Q_1^2}{2b^2Q_1^2}\right\} \quad (20)$$

For optimum value of Q_1, we solve the equation,

$$E_1Q_1^3 + E_2Q_1^2 + E_3 = 0 \quad (21)$$

where

$$E_1 = \theta(h + CI_c)\,[4b^2\,(b^2 + 3\sigma_1^2) - 3(\sigma_1^2 + b^2)]$$

$$E_2 = 3bR\,(\sigma_1^2 + b^2)[\theta\,CI_cM - (h + CI_c + \theta C)\,]$$

$$E_3 = 3bR[C)I_c - I_e)R^2M^2 + 2AR] \quad (22)$$

Case 2: $Y < RM$, in this case, no interest charges are payable for the items kept in inventory and the interest earned is

$$CI_eMY - \frac{CI_e(1+\theta M)Y^2}{2R} + \frac{\theta CI_eY^3}{2R^2} \tag{23}$$

The total cost of the system during the random cycle is

$$K_2(Y/Q) = \frac{(h+(1+\theta M)CI_e + \theta C)Y^2}{2R} - \frac{\theta(2h+3CI_e)Y^3}{6R^2} - CI_eMY + A \tag{24}$$

The expected cost of the system in this case is

$$E(K_2(Y/Q)) = \frac{(h+(1+\theta M)CI_e + \theta C)(\sigma_0^2 + (\sigma_1^2 + b^2)Q^2)}{2R} - \frac{\theta(2h+3CI_e)(b^3Q^3 + 3bQ(\sigma_0^2 + \sigma_1^2Q^2)}{6R^2} - CI_eMbQ + A \tag{25}$$

Following Silver (1976), the average expected cost of the system per time unit is

$$AEC(Q_2) = \left\{ \frac{(h+(1+\theta M)CI_e + \theta C)(\sigma_0^2 + (\sigma_1^2 + b^2)Q_2{}^2)}{2R} \right.$$

$$\left. - \frac{\theta(2h+3CI_e)(b^3Q_2^3 + 3bQ_2(\sigma_0^2 + \sigma_1^2Q_2^2)}{6R^2} - CI_eMbQ_2 + A \right\} * \left\{ \frac{R}{bQ_2} + \frac{\theta(\sigma_0^2 + (\sigma_1^2 + b^2)Q_2{}^2)}{2b^2Q_2^2} \right\} \tag{26}$$

For optimum value of Q_2, equating partial derivative of $AEC(Q_2)$ to zero, we get,

$$E_1Q_2^4 + E_2Q_2^3 + E_3Q_2 + E_4 = 0 \tag{27}$$

where

$$E_1 = \theta\,[2b^2(2h + 3CI_e)(b^2 + 3\sigma_1^2) - 3(h + CI_e)(\sigma_1^2 + b^2)^2]$$

$$E_2 = -3bR(\sigma_1^2 + b^2)(h + CI_e + \theta C)$$

$$E_3 = 3bR\ [(h + \theta C + CI_e)\sigma_0^2 + 2AR]$$

$$E_4 = 3\theta\sigma_0^2[(h + CI_e)\sigma_0^2 + 2AR] \tag{28}$$

For obtaining optimum value of Q_2, solve the above equation (27) by any suitable numerical method. For solving (27) by Newton - Raphson method, we can take the initial iterate of Q_{20} given by

$$Q_{20} = \left[\frac{(h+CI_e)\sigma_0^2 + 2AR}{(h+CI_e)(\sigma_1^2 + b^2)} \right]^{1/2} \tag{29}$$

Case 2.1: Keeping variance fixed i.e. $\sigma_1 = 0$ and $\sigma_0 \geq 0$.

The above equation (29) becomes

$$Q_{20} = \left[\frac{(h+CI_e)\sigma_0^2 + 2AR}{(h+CI_e)b^2} \right]^{1/2} \tag{30}$$

and the average expected total cost of the system is

$$AEC(Q_2 = Q_{20}) = \left\{ \frac{(h + (1+\theta M)CI_e + \theta C)(\sigma_0^2 + b^2 Q_2^2)}{2R} \right.$$

$$\left. - \frac{\theta(2h + 3CI_e)(b^3 Q_2^3 + 3bQ_2\sigma_0^2)}{6R^2} - CI_e MbQ_2 + A \right\} * \left\{ \frac{R}{bQ_2} + \frac{\theta(\sigma_0^2 + b^2 Q_2^2)}{2b^2 Q_2^2} \right\} \quad (31)$$

For optimum value of Q_2, we have to solve equation (27) whose constants are given by

$$E_1 = \theta[2b^4(2h + 3Ci_e) - 3(h+CI_e)b^4]$$

$$E_2 = - 3R\, b^3(h + CI_e + \theta C)$$

$$E_3 = 3bR[(h + \theta C + CI_e)\,\sigma_0^2 + 2AR]$$

$$E_4 = 3\theta\sigma_0^2[(h + CI_e)\,\sigma_0^2 + 2AR] \quad (32)$$

Case 2.2: The variance is proportional to the ordered quantity, i.e. i.e. $\sigma_0 = 0$ and $\sigma_1 > 0$. The above equation (29) becomes

$$Q_{20} = \left[\frac{2AR}{(h + CI_e)(\sigma_1^2 + b^2)} \right]^{1/2} \quad (33)$$

and average expected total cost of the system per time unit is

$$AEC(Q_2 = Q_{20}) = \left\{ \frac{(h + (1+\theta M)CI_e + \theta C)(\sigma_1^2 + b^2)Q_2^2}{2R} \right.$$

$$\left. - \frac{\theta(2h + 3CI_e)(b^3 Q_2^3 + 3bQ_2\sigma_1^2 Q_2^2}{6R^2} - CI_e MbQ_2 + A \right\} *$$

$$\left\{ \frac{R}{bQ_2} + \frac{\theta(\sigma_1^2 + b^2)Q_2^2}{2b^2 Q_2^2} \right\}$$

(34)

and the optimum value of $Q_2 = Q_{20}$ is the solution of

$$E_1 Q_2^3 + E_2 Q_2^2 + E_3 = 0 \quad (35)$$

where

$$E_1 = \theta[2b^2(2h + 3CI_e)\,(b^2 + 3\sigma_1^2) - 3(h+CI_e)\,(\sigma_1^2 + b^2)^2]$$

$$E_2 = - 3bR\,(\sigma_1^2 + b^2)(h + CI_e + \theta C)$$

$$E_3 = 6bR^2A \quad (36)$$

4. NUMERICAL ILLUSTRATION

Consider an inventory system with following parameters (in proper units):

$$[R, h, C, I_c, I_e, A] = [1000, 2, 20, 0.20, 0.12, 250]$$

Table 1. Variations in Q and AEC with changes in $^{2}_{0}$ and $^{2}_{1}$

b = 0.75, M = 0.083, θ = 0.01

$^{2}_{1}$ \ $^{2}_{0}$		**5.00**	**10.00**	**15.00**
0.10	Q	353.40	353.44	353.45
	AEC	1928.07	1928.13	1928.19
0.15	Q	340.84	340.85	340.86
	AEC	1999.58	1999.64	1999.70
0.20	Q	329.50	329.51	329.52
	AEC	2068.64	2068.70	2068.77

Table 2. Variations in Q and AEC with changes in $^{2}_{0}$ and $^{2}_{1}$

b = 0.75, M = 0.083, θ = 0.02

$^{2}_{1}$ \ $^{2}_{0}$		**5.00**	**10.00**	**15.00**
0.10	Q	348.49	348.40	348.52
	AEC	1955.37	1955.43	1955.50
0.15	Q	336.11	336.12	336.14
	AEC	2027.97	2028.03	2028.10
0.20	Q	324.96	324.97	324.98
	AEC	2098.11	2098.17	2098.29

Table 3. Variations in Q and AEC with changes in $^{2}_{0}$ and $^{2}_{1}$

b = 0.75, M = 0.083, θ = 0.03

$^{2}_{1}$ \ $^{2}_{0}$		**5.00**	**10.00**	**15.00**
0.10	Q	343.74	343.75	343.76
	AEC	1982.30	1982.36	1982.43
0.15	Q	331.56	331.57	331.58
	AEC	2055. 98	2056.05	2056.11
0.20	Q	320.60	320.59	320.58
	AEC	2127.32	2127.25	2127.18

Table 4. Variations in Q and AEC with changes in $^{2}_{0}$ and M

b = 0.75, σ_1^2 = 0.20, θ = 0.01

M \ $^{2}_{0}$		**5.00**	**10.00**	**15.00**
0.0417	Q	326.83	326.84	326.85
	AEC	2053.51	2053.58	2053.64
0.0833	Q	329.53	329.54	329.55
	AEC	2068.79	2068.85	2068.92
0.1250	Q	333.99	334.00	334.01
	AEC	2095.10	2095.16	2095.22

Table 5 .Variations in Q and AEC with changes in $^{2}_{0}$ and M

b = 0.75, σ_1^2 = 0.20, θ = 0.02

M \ $^{2}_{0}$		**5.00**	**10.00**	**15.00**
0.0417	Q	322.31	322.32	322.33
	AEC	2084.37	2084.44	2084.51
0.0833	Q	324.98	324.99	325.01
	AEC	2098.25	2098.31	2098.38
0.1250	Q	329.39	329.40	329.41
	AEC	2123.33	2123.39	2123.45

Table 6. Variations in Q and AEC with changes in $^{2}_{0}$ and M

b = 0.75, σ_1^2 = 0.20, θ = 0.03

M \ $^{2}_{0}$		**5.00**	**10.00**	**15.00**
0.0417	Q	317.96	317.97	317.98
	AEC	2114.84	2114.91	2114.98
0.0833	Q	320.60	320.61	320.62
	AEC	2127.31	2127.38	2127.45
0.1250	Q	324.96	324.97	324.98
	AEC	2151.15	2151.22	2151.29

Table 7. Variations in Q and AEC with changes in $^{2}_{0}$ and b σ_0^2

σ_1^2 = 0.20, M = 0.083, θ = 0.01

$^{2}_{0}$ \ b		**0.75**	**0.80**	**0.85**
5.00	Q	329.53	313.95	299.57
	AEC	2068.79	2035.62	2007. 73
10.00	Q	329.54	313.96	299.58
	AEC	2068.85	2035.69	2007.79
15.00	Q	329.55	313.97	299.59
	AEC	2068.91	2035.75	2007.85

Table 8. Variations in Q and AEC with changes in σ_0^2 and b

σ_1^2 = 0.20, M = 0.083, θ = 0.02

$^{2}_{0}$ \ b		**0.75**	**0.80**	**0.85**
5.00	Q	324.98	309.61	295.42
	AEC	2098.25	2064.56	2036.23
10.00	Q	324.99	309.61	295.42
	AEC	2098.31	2064.56	2036.30
15.00	Q	325.00	309.63	295.44
	AEC	2098.38	2064.69	2036.36

Table 9. Variations in Q and AEC with changes in σ_0^2 and b

σ_1^2 = 0.20, M = 0.083, θ = 0.03

$_0^2$ \ b		0.75	0.80	0.85
5.00	Q	320.60	305.42	291.42
	AEC	2127.31	2093.11	2064.35
10.00	Q	320.61	305.43	291.43
	AEC	2127.38	2093.18	2064.42
15.00	Q	320.62	305.44	291.44
	AEC	2127.45	2093.25	2064.49

Table 10. Variations in Q and AEC with changes in σ_0^2 and b

σ_1^2 = 0.20, M = 0.0417, θ = 0.01

$_0^2$ \ b		0.75	0.80	0.85
5.00	Q	326.83	311.37	297.11
	AEC	2053.51	2020.62	1992.95
10.00	Q	326.84	311.38	297.12
	AEC	2053.58	2020.68	1993.01
15.00	Q	326.85	311.39	297.13
	AEC	2053.67	2020.75	1993.07

Table 11. Variations in Q and AEC with changes in σ_0^2 and b

σ_1^2 = 0.20, M = 0.083, θ = 0.01

$_0^2$ \ b		0.75	0.80	0.85
5.00	Q	329.53	313.95	299.57
	AEC	2068.79	2035.62	2007.73
10.00	Q	329.54	313.96	299.57
	AEC	2068.85	2035.69	2007.79
15.00	Q	329.55	313.98	299.58
	AEC	2068.64	2035.75	2007.85

Table 12. Variations in Q and AEC with changes in σ_0^2 and b

σ_1^2 = 0.20, M = 0.125, θ = 0.01

$_0^2$ \ b		0.75	0.80	0.85
5.00	Q	333.99	318.20	303.62
	AEC	2095.10	2061.48	2033.21
10.00	Q	334.00	318.21	303.64
	AEC	2095.16	2061.54	2033.27
15.00	Q	334.01	318.21	303.64
	AEC	2095.22	2061.61	2033.33

Table 13. Variations in Q and AEC with changes in σ_1^2 and b

σ_0^2 = 5.00, M = 0.083, θ = 0.01

$^2_1$ \ b		**0.75**	**0.80**	**0.85**
0.10	Q	353.46	334.43	317.20
	AEC	1928.21	1910.49	1895.68
0.15	Q	340.87	323.70	308.01
	AEC	1999.72	1974.04	1952.50
0.20	Q	329.53	313.95	299.57
	AEC	2068.79	2035.62	2007.73

Table 14. Variations in Q and AEC with changes in σ_1^2 and b

σ_0^2 = 5.00, M = 0.083, θ = 0.02

$^2_1$ \ b		**0.75**	**0.80**	**0.85**
0.10	Q	348.52	329.75	312.76
	AEC	1955.50	1937.52	1922.49
0.15	Q	336.14	319.21	303.72
	AEC	2028.10	2002.03	1980.16
0.20	Q	324.98	309.61	295.42
	AEC	2098.25	2064.56	2036.23

Table 15. Variations in Q and AEC with changes in σ_1^2 and b

σ_0^2 = 5.00, M = 0.083, θ = 0.03

$^2_1$ \ b		**0.75**	**0.80**	**0.85**
0.10	Q	343.77	325.24	308.47
	AEC	1982.42	1964.17	1948.92
0.15	Q	331.59	314.87	291.42
	AEC	2056.10	2029.64	2007.44
0.20	Q	320.61	305.43	291.42
	AEC	2127.31	2093.11	2064.36

Table 16. Variations in Q and AEC with changes in σ_1^2 and b

σ_0^2 = 5.00, M = 0.0417, θ = 0.01

$^2_1$ \ b		**0.75**	**0.80**	**0.85**
0.10	Q	350.56	331.69	314.60
	AEC	1914.08	1896.51	1881.82
0.15	Q	338.07	321.05	305.49
	AEC	1985.01	1959.54	1938.18
0.20	Q	326.83	311.37	297.12
	AEC	2053.51	2020.62	1992.95

Table 17. Variations in Q and AEC with changes in σ_1^2 and b

$\sigma_0^2 = 5.00$, M = 0.083, $\theta = 0.02$

$^2_1$ \ b		0.75	0.80	0.85
0.10	Q	353.46	334.43	317.20
	AEC	1928.21	1910.49	1895.68
0.15	Q	340.85	323.70	308.01
	AEC	1999.72	1974.04	1952.50
0.20	Q	329.53	313.95	299.57
	AEC	2068.79	2035.62	2007.73

Table 18. Variations in Q and AEC with changes in σ_1^2 and b

$\sigma_0^2 = 5.00$, M = 0.083, $\theta = 0.03$

$^2_1$ \ b		0.75	0.80	0.85
0.10	Q	358.24	338.95	321.49
	AEC	1952.62	1934.66	1919.66
0.15	Q	345.48	328.08	312.18
	AEC	2025.10	1999.07	1977.24
0.20	Q	333.99	318.20	303.62
	AEC	2095.10	2061.48	2033.21

5. INTERPRETATIONS

- In Tables 1 - 3, we study the effects of σ_0^2 and σ_1^2 keeping b constant and varying deterioration rate θ of the units in inventory. It is found that as σ_0^2 increases, the optimum purchase quantity and average expected total cost of the system increases whereas increase in σ_1^2 results decrease in optimum procurement quantity and increase average expected total cost of the system. With increase in deterioration of units, optimum purchase quantity decreases and cost of an inventory system increases.

- In Tables 4 - 6, the effects of variations in σ_0^2 and delay period M have been studied keeping σ_1^2 and b constant and varying deterioration rate of the units in inventory. Increase in σ_0^2 results increase in both, optimum purchase quantity and total expected cost. Same pattern is observed when delay in payment period increases.

- In Tables 7 - 8, the effects of b and σ_0^2 when σ_1^2 and delay period are fixed and deterioration rate of units increases. It is found that increase in b results decrease in optimum procurement quantity and total expected cost of an inventory system. As σ_0^2 increases, optimum purchase quantity and total expected cost increases. While as deterioration rate increases, number of units to be procured decrease and total cost of the system increases. When delay period increases, there is increase in procurement units and expected total cost.

- The effects of variations in deterioration, b and σ_1^2 have been studied on optimum purchase quantity and total expected cost in tables 13 - 15. It is observed that increase in b results decrease in optimum purchase units and total expected cost, whereas increase in σ_1^2 results decrease in purchase quantity and increase in total expected cost. Also, when deterioration of units in inventory increase optimum procurement quantity decreases and total expected cost increases.

- In Tables 16 - 18, we study variations of delay period, bias factor b and σ_1^2 have been studied on optimum purchase quantity and total expected cost. It is observed that increase in b results decrease in optimum purchase units and total expected cost, whereas increase in σ_1^2 results decrease in purchase quantity and increase in total expected cost. When delay period increases, optimum purchase quantity and total expected cost of an inventory system increases.

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