

A SIMPLE CONTINUOUS MEASURE OF CREDIT RISK

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ABSTRACT

In this paper it is introduced a simple continuous measure of credit risk, associating to each firm a risk parameter related to the firm's risk neutral default intensity, which may be computed from quoted bond prices, and allows assignment of credit ratings much more adjusted than those offered by rating agencies. Moody's Investor Services, Standard and Poor's Corporation and Fitch are commercial high grade rating companies for assessable default risks (high grade means low credit risks or, conversely high probability of future payments). These gradings are accessible through the distance to default measures (Merton Model, 1974). Traditional ratings are of a discrete nature, not usually updated and not implying probability characteristics. The new measure, together with the other ones, are complementary, showing a good rank correlation between the continuous measure and Moody's ratings. The new parameters are obtained from quoted bond prices and can be considered as the new risk intensity of immediate default. Instantaneous short rates (forward rates, in general) determine this risk measure. The approach of this paper can be used to extract the entire intertemporal default intensity distribution of firms, useful to calculate the expected time to default, to value and manage the prices and risks of credit derivatives.

Key words: Continuous measures of credit risk, risk neutral default intensity of credit risk, to credit firms ratings, pricing and risk management of credit derivatives.

RESUMEN

En esta ponencia, introducimos una medida continua y simple del riesgo de crédito, que asocia a cada empresa un parámetro de riesgo relacionado con la intensidad del incumplimiento neutral respecto al riesgo por parte de la empresa. Se pueden calcular estos parámetros por medio de los precios de las obligaciones cotizadas y que permiten asignar tarificaciones del crédito mucho más ajustadas que las ofrecidas por diferentes agencias de tarificación. Estimamos las medidas de riesgo en una base diaria, para una determinada muestra de empresas y las comparamos con las tarificaciones correspondientes ofrecidas por Moody's y la distancia para medidas de incumplimiento calculadas usando el modelo de Merton (1974). Las tres medidas agrupan la muestra de empresas en varias clases de riesgo, de un modo similar, pero lejos de ser idénticas, posiblemente reflejan los diferentes horizontes de predicción de los modelos. Entre las tres medidas, la correlación de más alto rango se encuentra entre nuestra medida continua y las tarificaciones de Moody's. Se pueden utilizar las técnicas que proponemos aquí para obtener la distribución total de las intensidades del incumplimiento neutrales respecto al riesgo e inter-temporales, que son útiles para las estimaciones del tiempo de incumplimiento, así como para valorar derivados de crédito.

1. INTRODUCTION

In this paper we face the main goal of modelling credit problems, for measuring portfolio risk and for pricing defaultable securities, credit derivatives and other assets subject to credit risk.

We have in mind three complementary audiences:

- First, we target those whose responsibilities are the measurement and control of financial market. Very important is the risk associated with large portfolios of over the counter (OTC) derivatives, financial contracts and investment portfolios or broker-dealer inventories of securities.
- In a second place, we consider approaches to pricing credit risk for those whose responsibilities are trading or marketing products involving credit risk.
- Finally, we take in mind professionals whose aim is to determine the possibility of default.

There are reasons to track credit risk, by counterparty. The information systems necessary to quantify most forms of credit risk differ from those appropriate for more traditional forms of market risk, such as changes in the market prices or rates. A very common attitude among broker-dealers is that the market values of open positions should be remarked each day.

There are differences between credit risk and market price risk, that can be determined in terms of time horizons and liquidity.

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The recent important increased attention to credit risk is due, in part, to the concerns of regulatory agencies and investors, preoccupied for exposures of financial institutions because of their large positions in OTC (Over the Counter) derivatives.

We may consider that market risk is the risk of changes in the market value of a firm's portfolio of positions and includes the risk of default or fluctuation in the credit quality of one's counterparties (a credit risk is a source of market risk).

Risks faced by financial institutions can be classified in the following broad categories:

- Market risk: risk of unexpected changes in prices or rates.
- Credit risk: risk of changes in value, associated with unexpected changes in credit quality.
- Liquidity risk: risk that the costs of adjusting positions will increase substantially, or that a firm will lose access to financing.
- Operational risk: risk of fraud, system failures, trading errors and many other internal organizational risks.
- Systemic risk: risk of breakdowns in market wide liquidity or chain reaction default.

Market price risk includes the risk that the volatility of market prices and of daily profits and losses will change over time. Prices of option-embedded securities and the probability of a portfolio loss of a given amount will be affected by the degree of volatility of market prices.

Credit risk is the risk of default or of reductions in market value due to changes in the credit quality of issuers or counterparties.

Liquidity risk includes the possibility that bid-ask spreads will widen in a short period of time, or that the quantities that counterparties are willing to trade at given bid ask spreads will decline substantially reducing consequently the ability to restructure the portfolio at moments of financial distress.

The recent losses arising from the string of recent corporate collapses, include:

- In 1990, the Bank of New England faced insolvency, in part because of potential losses and severe illiquidity on its foreign exchanges and interest- rate derivatives.
- In 1991, Salomon Brothers faced and largely averted, a liquidity crisis, stemming from its Treasury bond "scandal". Access to both credit and customers was severely threatened; careful public relations and efficient balance-sheet reductions were important for survival.
- In 1998, a decline in liquidity associated with the financial crises in Asia and Russia led (along with certain other causes) to the collapse in values of several prominent hedge funds, including long term capital management, and sizable losses at many major financial institutions.

Changes in liquidity can be considered as a component of market risk, too. A very well known example of it was the greater liquidity of Japanese Bank debt, compared to Japanese Government bonds.

Systemic risk implies the collapse of financial markets, through multiple defaults, "domino style", or through widespread disappearance of liquidity.

Some of the major losses by financial institutions we read in the financial press, over the passed decade, were the result of operational problems of the sort. The ample evidence provided by Enron, Worldcom and Kmart are examples of this sort.

The risk of default, by its very nature, is very difficult to quantify, with the traditional credit ratings offered by agencies, such as "Standard and Poor's Corporation, Moody's Investor Service and Fitch, being not what it is desirable. We have to indicate that the discrete nature of these ratings give way to groups of firms being assigned the same ratings, although there are differences in their credit worthiness and the changes in the credit worthiness of firms are not immediately reflected in their ratings.

Another drawback of those ratings, due to agencies, is that they rely on past studies and historical stock volatilities.

We bring to attention some very important models: the structural Models of Default Probability, which asserts that firms default when they cannot, or choose not to, meet their financial obligations. Most of these models are based on a balance-sheet notion of solvency, in that default occurs when assets are too small, relative to liabilities.

2. MODERN CREDIT RISK PRICING. APPROACHES TO A CONTINUOUS TIME FRAMEWORK

The usual literature on credit risk refers to traditional actuarial methods of credit risk. Although the methods are usually used in banks, they bear some difficulties. The usual thing is to estimate the probability of default (probability to fail payment of debt) and to value the contract at possible default times.

Rating agencies are sources for study of default probabilities. The techniques used to forecast default probabilities for individual firms are described by Altman (1996a).

Modern credit risk analysis is in the line of the continuous development of financial research on the integration of uncertainty. We may say that the investor faces risks that have been categorized as market risks, credit risk, country risk and operational risk, among others. Modern appraisal of credit risk follows directly from the advances made for the management of credit risks.

Market risks have been a preoccupation in modern finance and we introduce some chronology about market risks, their developments and the needs that have increased with respect to them.

Market risk integrates interest rate risk, exchange rate risk and stock market risk. With the Bretton Woods Agreements, exchange rates were allowed to float leading to volatilities in interest rates. From the last year at the end of 1970's decade, many economic studies were undertaken giving rise to financial theory.

On the mathematical side, it was very interesting the introduction of stochastic calculus.

Stochastic calculus allows the refining of the time space into infinitesimal points as a limit of the discrete time approach.

The continuous-time framework is very useful because it enables much more easily closed-form solutions to specific financial problems to be obtained, while the discrete approach is of great help to visualize the choices to market through time, choices made on specific dates, while they are made continuously in continuous time. As noted in Merton's articles, two basic assumptions have to be made to justify the use of continuous-time approaches in the portfolio selection problems of modern finance.

Assumption 1: Capital markets are open at all times meaning that agents can trade continuously.

Assumption 2: The stochastic processes that generate the state variables can be described by diffusion processes with continuous sample paths.

The contribution of Merton is based is his capacity to relate financial theory and continuous time approach introducing the continuous time finance.

Contingent claims analysis (option pricing) can be used to value the component parts of a firm's liability mix. The value of each component depends upon the stochastic variables which determine the evolution of the firm's asset value, the evolution of the interest rate, the payouts (dividends, coupons, etc) to the various claimants, and the division of the firm at a point of reorganization. Merton starts with a simplified model that yields useful insights and shows the way to more complete valuation.

2.1. Stochastic Interest Rates and Credit Risk: Merton's Contribution

Merton showed that credit risk could be related to the financial economics of the firm and that it was possible to appraise it with the same kind of technology already developed for market risks, but it became clear that it could no longer be priced independently from market risks, from interest rate risk, in particular. It is no longer possible to treat the credit risk problem isolated from the risk of the term structure of interest rates.

2.2. Merton's approach: The intuition Behind structural Methods.

An important difficulty of actuarial methods is the complete dependence on historical data that are based in fitting expectations of default, to default data of the past. A rational theory of credit risk based on financial economics was developed in 1974 as an application of contingent claim analysis.

2.2.1. Structural Models and reduced-form models

Structural models rely upon the balance sheet of the borrower and the bankruptcy code to derive the probability of default and the credit spreads based on non-arbitrage arguments, making some assumptions on the recovery and the model for default-free interest rates.

Reduced-form approaches view the credit event as a perfectly unpredictable event. The probability of this event is called intensity rate since it represents the frequency of defaults that may occur in a given time interval.

2.3. Jarrow and Turnbull (1995) discrete approach

Jarrow and Turnbull's model (for a reduced form model) is consistent with the existence of a zero-coupon government bond term structure and a zero-coupon corporate bond term structure for a given standing class.

2.3.1. Jarrow, Lando and Turnbull's Markov model

This is a Markov Model for the term structure of credit spreads, with some specificities. The main assumption of their approach is that ratings are an accepted indicator of the credit worthiness. Here default is an exogenous process that does not depend on the underlying assets of the firm. The advantage of such methods against the structural ones is that the calculus of the available observable may be restricted, with no economic requirement.

2.4. Credit risk default ratings

There exist different ways to gauge defaulting credit rates:

1. Credit rating agencies like Standard and Poor's Corporation, Moody's Investor Service and Fitch, being the most important, although you find more everyday. As credit risk is very difficult to quantify, and as agencies use their own economic means to get information, their findings belong to the agencies and they don't usually actualize their values (only once or twice a year). The agencies 'ratings haven't got an economic meaning and they can't be transformed in a probability measure. Another draw-back is that their search is retrospective, backward-looking.
2. Models using balance-sheet and historical stock volatility, traditional scoring models or structural models like the Merton (1974) model. Those ratings are backward-looking, they rely on balance sheet information that is presented with a lag (one or two years) and sometimes with manipulation in figures on information, in general (remind the Enron's problems.)

Both rates (agencies' and structural) have a discrete nature and not a continuous one, and for that reason they result in groups of firms being assigned the same rating despite the differences in their credit worthiness, and the changes in their credit worthiness are not reflected in their ratings.

2.4.1. Review of Merton's model (a modified version using Black-Scholes' formula)

A classic model is Black- Scholes' (1973) application to Merton's model (1974), under which default occurs at the maturity date of debt in the event that the issuer's assets have less value than the face value of the debt.

KNV Corporation has developed a successful empirical estimator of default probabilities, called the "estimated default frequency" (EDF) based in the model.

We consider a second structural approach based on this assumption that default occurs as soon as the asset value of a firm falls below a certain boundary often taken to be the face value of its liabilities.

As it has been commented, a drawback for Merton's model is that it is not prospective and it refers to periods of default of one or two years.

We may say that both Merton's and private agencies' ratings are not probabilities of default, and that is why we justify the need of a new measure.

This paper introduces a new measure of credit risk that complements the traditional credit ratings, in that it is a continuous measure, for which reason it allows a finer separation of firms according to their credit worthiness. It is a measure computed from current bond data and, therefore, it is forward-looking and adjusts immediately to changes in the credit worthiness of firms. In essence, this measure has an economic meaning, because it may be interpreted as the risk neutral probability of immediate default.

We have to say that the measure introduced in this paper, being the risk neutral intensity of immediate default, focuses on the risk of default in the short term. In contrast, Moody's ratings usually refer to an entire business cycle and Merton's model typically produces one to two year default probabilities.

The comparison of these different measures reveals a high degree of correlation and they should be considered as complementary.

The data requirements to compute this measure are minimal. The only needs are the bonds issued by the firms under consideration and proxies for the risk-free bonds, which may be the treasury bond or the bonds issued by a firm of highest credit rating. However, to ensure sensible default intensities, we consider necessary to obtain the risk-free yield curves, with the condition that the risk-free yields lie below the yields of all corporate bond yields. The model is very simple, needing only the instantaneous forward rates and zero coupon bond prices, which play the key roles in determining the risk measures and default intensities. We stress that a firm's entire inter-temporal distribution of default intensities can be inferred from instantaneous forward rates and zero coupon bond prices, in order to extract the expected time to default.

Merton (1974) and Black and Sholes (1973) noted that the debt and the equity of a firm can be valued using options theory. Suppose a corporation has two sources of finance, debt, D , and equity E . If the value of the firm's assets " V " exceeds the amount of debt outstanding " F_B ", then the equity (share) holders obtain a positive payment since there are some assets left over, after paying off the corporate bond holders. On the other hand, if the firm's assets are valued at less than the face value of the debt, then the equity holders can leave, since they have limited liability. The payoff to equity holders is therefore like a call option. The call pays either the excess of value of the firm's assets over the amount owed to the bondholders ($V - F_B$), or it may have a payout of zero if the firm's assets are worth less than the amount owed to the bondholders. To consider this analysis it is further assumed:

F_B = face value of (zero coupon) bond (debt) which mature at time T .

D_t = value of bonds (debt) at time t ($\leq T$)

V_t = (market) value of the firm's assets at t ($\leq T$)

E_t = value of shareholder's equity at time t ($\leq T$)

Case A: Solvency at time T .

If $V_T \geq F_B$ Then $D_T = F_B$ and the equity holders receive $E_T = V_T - F_B$

Case B: Insolvency at time T .

Here $V_T < F_B \Rightarrow D_T = V_T$ and $E_T = 0$.

We summarise the two situations as follows:

$$E_T = \max[0, V_T - F_B]$$

$$D_T = F_B - \max[0, F_B - V_T]$$

The value of the equity is like a European call on the value of the firm's assets with a strike price of F_B . Merton used this correspondence to derive the market value and the volatility of the firm's underlying assets. More precisely, Merton used the Black and Scholes (1973) framework to solve for the asset value and volatility implied by the option price and the option price volatility.

The asset value and the asset volatility can be combined into a risk measure called "distance to default" that is directly related to the credit worthiness of the equity issuing firm.

Merton used a modified version of the Black-Scholes formula, that is:

$$V_E = V_A N(d_1) - e^{-r(T-t)} D N(d_2)$$

linking the market value of equity and the market value of assets, where

V_E = market value of the firm's equity,
 V_A = market value of the firm's assets,
 D = total amount of the firm's debt,
 $T - t$ = time of maturity of the firm's debt,
 r = risk free interest rate,

$$d_1 = \frac{\ln(V_A / D) + \left(r + \frac{1}{2}\sigma_A^2\right)(T - t)}{\sigma_A \sqrt{T - t}}$$

$$d_2 = d_1 - \sigma_A \sqrt{T - t}$$

$N(\cdot)$ = cumulative normal distribution.

Moreover, it is easily shown that the equity and asset volatility are related by the expression

$$\sigma_E = \frac{V_A}{V_E} N(d_1) \sigma_A \quad (2.2)$$

where

σ_E = volatility of the firm's equity returns,

σ_A = volatility of the firm's assets returns.

Solving the nonlinear system of equations gives V_A and σ_A , and the distance to default is defined by the expression

$$\gamma = \frac{\ln(V_A / D) + \left(r - \frac{1}{2}\sigma_A^2\right)(T - t)}{\sigma_A \sqrt{T - t}} \quad (2.3)$$

γ is simply the number of standard deviations that the firm value is from the limit point, and the smaller the value of γ the larger the (risk neutral) probability that the firm will default on its debt.

3. MOTIVATIONS FOR A NEW RISK MEASURE

To solve the drawbacks we mentioned about traditional credit ratings and scoring models, relying on accounting news, we try to introduce a measure which involves the extraction of information about credit risk from up to date market data. If credit risk is incorporated into market prices, we can manage to extract the component contained in these prices, attributed to credit risk.

Public information is likely to be instantaneously embedded into individual security prices in the stock and bond markets. Private information goes to different markets because of investor's preferences due to several reasons.

Data from the bond market can be extracted as the basis of getting credit risk information. We have to compare bonds with credit risk (corporate bonds) to those essentially without risk (treasury bonds or high quality corporate bonds). We construct the term structure of survival probabilities as with Jarrow, Lando and Turnbull (1997). There is a correspondence between the shape of these survival probability curves and the risk of default of the corresponding firms, and this paper introduces the slope at the short end of this curve as a simple proxy for the credit rating of the firm. Since the slope reduces to the difference between the instantaneous short rates implied by the corporate and the risk-free bonds, the problem of getting the risk measure is that of building reliable forward rate curves.

A new continuous credit risk measure

The new rate relies on bond market and has the advantage of being continuous and that it is based on daily market quotes. By being continuous it avoids the problems regarding widely different (expost) measures within the same rating class and has the interpretation of risk-neutral intensity of immediate default.

Our measure relies on markets being efficient (markets in which the prices reflect all available information). With this assumption, this measure will increase in reliability when bond markets increase around the world, with the help of banks, which will favour compulsory periodic emission of large quantities of subordinated debt, in order to impose discipline on these entities, as suggested by Calomiris (1999).

Another crucial assumption is the independence of the risk-free interest rate process and the default processes of the firms.

A related issue is the specification of a proxy for the risk free interest rate and the corresponding risk-free bonds from which risk-free interest rates should be obtained.

We construct here the risk-free forward rates curves with the condition that they lie below all the risky, or corporate forward rates curves. These risk-free forward rates are used to obtain the risk-free zero coupon bonds prices and the default intensities. This procedure ensures that all risky bonds are priced below their risk-free counterparts, considering that the differences, or the spreads, between the risky and risk-free bond prices are due primarily to credit risk.

We have to assume that the recovery rate (amount recovered by the bondholder in case of default) is given exogenously. We use here an average historical value quoted in Jarrow, Lando and Turnbull (1997).

The risk measures and the Role of Forward curves

The first step in determining the risk measure involves inferring the risk-neutral survival probabilities, or the probabilities that a firm will not default within given time horizons, from the bond market data.

Let $p(t,T)$ denote the time t price of a T -maturity risk-free zero coupon bond and let $v^\pi(t,T)$ be the corresponding price for bonds issued by firms with a certain external "credit rating π ". Then the assumption of independence of the risk free interest rate process and the default processes implies

$$v^\pi(t, T) = p(t, T)[\delta^\pi + (1 - \delta^\pi)F^\pi(T/t)] \quad (3.1)$$

where δ^π is the recovery rate for firms with external credit rating π , and $F^\pi(T/t)$ is the corresponding risk-neutral conditional survival probability to time T given that they have survived to time $t \leq T$. Rearranging this equation gives the expression,

$$F^\pi(T/t) = \frac{v^\pi(t,T)/p(t,T) - \delta^\pi}{1 - \delta^\pi} \quad (3.2)$$

for the risk-neutral conditional survival probabilities, which is a measure of the credit worthiness of firms. If it is assumed that the recovery rates are given exogenously, then the only task that remains in order to calculate survival probabilities, $F^\pi(T/t)$, is to extract zero coupon bond prices from traded coupon bond prices, and a detailed description of the method.

Note that if π_1 represents a higher credit rating than π_2 , then it should be the case that

$$v^{\pi_1}(T,t) \geq v^{\pi_2}(T/t) \quad (3.3)$$

and we deduct that

$$F^{\pi_1}(T/t) \geq F^{\pi_2}(T/t) \quad (3.4)$$

for all $t \leq T$. This observation, of course, relies on the assumption of a constant recovery rate over the rating classes.

Now, provided that the bonds are priced correctly and consistently in the market, the increase in the risk of default, represented by decreasing π , should be reflected in the steepening of the corresponding curve for $F^\pi(T/t)$ at $T = t$ can be interpreted as a measure of a firm's short term default risk, or as a short term "credit rating". Since the slope of $F^\pi(T/t)$ at $T = t$ is negative, and the negative of the slope permits the convenient

economic interpretation as the intensity of immediate default, we have chosen to use the negative of the slope as the measure of risk. An important advantage of this measure compared to external ratings is that it is continuous and hence enables a much finer separation of firms according to their risk of default.

It follows from (3.2) that the slope of $F^\pi(T/t)$ at $T = t$ is given by the equation

$$\left. \frac{\partial F^\pi(T/t)}{\partial T} \right|_{T=t} = \frac{1}{1-\delta^\pi} \frac{\delta}{\delta T} \left[\frac{v^\pi(t,T)}{p(t,T)} \right] \Bigg|_{T=t} \quad (3.5)$$

Now, if we denote the risk-free forward rate curve by $r(t, T)$ and the forward rate curve for the firm π by $r^\pi(t, T)$, then by definition

$$v^\pi(t, T) = e^{-\int_t^T r^\pi(t,u) du} \quad \text{and} \quad p(t, T) = e^{-\int_t^T r(t,u) du},$$

and so we have

$$\begin{aligned} \left. \frac{\partial}{\partial T} \left[\frac{v^\pi(t, T)}{p(t, T)} \right] \right|_{T=t} &= \left. \frac{\partial}{\partial T} \left[e^{-\int_t^T (r^\pi(t,u) - r(t,u)) du} \right] \right|_{T=t} \\ &= -(r^\pi(t, T) - r(t, T)) e^{-\int_t^T (r^\pi(t,u) - r(t,u)) du} \Bigg|_{T=t} \\ &= -(r^\pi(t, T) - r(t, T)) \left[\frac{v^\pi(t, T)}{p(t, T)} \right] \Bigg|_{T=t}, \end{aligned} \quad (3.6)$$

and (3.5) becomes:

$$\left. \frac{\partial F^\pi(T/t)}{\partial T} \right|_{T=t} = -\frac{(r^\pi(t, t) - r(t, t))}{1-\delta^\pi} \left[\frac{v^\pi(t, t)}{p(t, t)} \right] = -\frac{(r^\pi(t, t) - r(t, t))}{1-\delta^\pi}$$

since $v^\pi(t, t) = p(t, t) = 1$. Consequently, the credit measure proposed in this paper is

$$m(\pi) = -\left. \frac{\partial F^\pi(T/t)}{\partial T} \right|_{T=t} = \frac{(r^\pi(t, t) - r(t, t))}{1-\delta^\pi} \quad (3.7)$$

Note that since the conditional probability of default, $\tilde{F}^\pi(T/t)$, satisfies the equation $\tilde{F}^\pi(T/t) = 1 - F^\pi(T/t)$, it follows that

$$m(\pi) = \left. \frac{\partial \tilde{F}^\pi(t, T)}{\partial T} \right|_{T=t} = \lim_{T \downarrow t} \tilde{f}^\pi(T/t), \quad (3.8)$$

where $\tilde{f}^\pi(T/t)$ represents the intensity of default. This shows that the measure, $m(\pi)$, can be interpreted as the intensity of "immediate" default.

Construction of Forward Rate Curves

Since the instantaneous forward rates, and in particular the instantaneous short rates, are key determinants of the risk measure introduced in this paper, it is important to ensure that the zero coupon bond prices are extracted in such a way that the corresponding forward rates are as accurate as possible. Moreover, since the shapes of forward rate curves play a key role in determining the risk neutral default intensities, it is also important to ensure that the curves of these forward rates are as smooth as possible. In practice, however,

these requirements cannot be satisfied simultaneously and, depending on the situation, the accuracy or smoothness must be sacrificed.

For the purposes of this paper, accurate short rates and smooth forward rate curves are required. Now, the only sure way to satisfy the latter requirement is to assume a parametric form for the forward rate curves, and the specification that provides a good compromise between flexibility and ease of implementation is the parametric form introduced in Nelson and Siegel (1987). The (T-t)-maturity instantaneous forward rate, $r(t, T)$, under their specification is assumed to be determined by the equation

$$r(t, T) = a_0 + a_1 e^{-k(T-t)} + a_2 (T-t) e^{-k(T-t)} \quad (3.9)$$

where a_0, a_1, a_2 are k parameters subject to constraints

$$a_0 > 0, \quad a_0 + a_1 > 0 \quad \text{y} \quad k > 0 \quad (3.10)$$

Note that $a_0 = \lim_{(T/t) \uparrow \infty} r(t, T)$, corresponds to the long rate and $a_0 + a_1 = r(0)$ corresponds to the short rate.

Now, for each firm and date of interest, the aim is to determine the four parameters in (3.9) from the corresponding bond data. In this paper, we have determined the parameters by minimizing the total weighted squared error between the model implied bond prices and their actual market quoted counterparts. That is, we have determined the parameters a_0, a_1, a_2 and k by minimising the quantity

$$\epsilon^2 = \sum_i w_i^2 (P_i(a_0, a_1, a_2, k) - \hat{P}_i)^2 \quad (3.11)$$

where P_i is the model implied price of the i -th bond, \hat{P}_i is the market observed price, and

$$w_i = \frac{1/D_i}{\sum_i 1/D_i}$$

with D_i being the duration of the i -th bond. Note that the introduction of the weighting factors, w_i , has the effect of reducing the natural tendency for the model to overfit the long dated bonds at the expense of the short dated bonds.

Numerical implementations of the above procedure require the input of initial starting values of the parameters in one form or another, and as observed for example in Bolder and Strésliński (1999), the parameters can be very sensitive to these initial values. The approach suggested in Bolder and Strésliński (1999) first approximates the parameters a_0 and a_1 by setting the short and the long rates equal to the yield on the shortest and the longest maturity bonds respectively. The remaining parameters are then determined by dividing the (a_2, k) space into several rectangular regions, computing the local minimum within each region, and then setting the parameters equal to the minimum among these local minima.

4. CONCLUSIONS

This paper presents a simple continuous model for measuring credit risk, computed from market bond data, which is known as the instantaneous rate default intensity and it is a very important and necessary measure of a company's risk of distress in the short term.

A comparison of the three measures studied here shows that they produce similar results, specially at the short end of the curve used as a proxy for the credit ratings of the firm.

There is a closer relation of rank estimates between the continuous measure and Moody's rating, taking the group in consideration.

The approach of this paper can be used to extract the whole intertemporal (risk-neutral) default intensity distribution of firms, which is useful for deducting the expected time of default and for the valuation, pricing and risk management of credit derivations.

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