

PRICE DISCOUNT STRATEGIES: A REVIEW*

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ABSTRACT

This article reviews the literature on quantitatively oriented approaches for determining lot-sizes when price discounts are offered. We provide a review of existing literature, concentrating on descriptions of the types of problems that have been solved and important structural results. We identify a variety of shortcomings of the literature in addressing problems encountered in practice.

Key words: Price discount, all-unit quantity discount and incremental discount policies, economic batch quantity, quantity discount-pricing schedule, freight discount, warranty.

MSC: 91B24

RESUMEN

Este trabajo presenta una revisión de la literatura existente sobre los enfoques basados en criterios cuantitativos para determinar el tamaño de los lotes cuando se ofrecen precios de descuentos. Brindamos una revisión de la literatura, concentrándonos en las descripciones de los tipos de problemas que han sido resueltos y en importantes resultados de tipo estructural. Identificamos una variedad de deficiencias en la literatura en la señalización de problemas que se encuentran en la práctica.

1. INTRODUCTION

The numerous marketing policies are based on executive's decision. One such policy that every seller offers to the retailer is that governing the discounts, he grants to the purchasers who will order quantities larger than the actual order. Quantity price structure may be classified in four possible ways:

- 1.The minimum quantity at which discount is offered.
- 2.The number of "breaks" or additional discounts for large purchases, which are offered.
- 3.The maximum quantity qualifying for every additional discount.
- 4.The actual amount of discount offered at each quantity level.

The mathematical model should answer the following questions:

- a. What is to be accomplished by quantity discount? Or
- b. What policy is likely to be accomplished by a quality discount?

An efficient and balanced discount schedule will reflect costs at both the buyer's and the seller's ends of the business. The quantity discounting is more advantageous to buyers than to the sellers. Perhaps because of this, the economic attractiveness of quantity purchasing has been evaluated from the buyer's point of view. The mathematical model assumes two kinds of expenses, viz., ordering costs and inventory carrying costs, which may change with the size of an order. An increase in order size reduces one of the above costs while increases the other cost. Therefore, for every buyer, there is an optimum size order (EOQ) in the absence of quantity discount. Purchasing the quantities larger than EOQ, actually is a loss to the buyer. This loss must be counter balanced by any quantity discount, which force him to purchase more than computed EOQ. This will reduce the seller's cost. It adds savings to the seller. Thus, if the buyer EOQ is known, then we can determine the seller's and buyer's annual cost before any inducement in the form of a discount is offered for purchases in larger quantities than EOQ. Then for every large quantity purchased,

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1. the increased cost to the buyer;
2. the decreased cost to the seller; and
3. the net decrease in total cost to the two parties can be determined. Obviously, the first increase above the EOQ yields a substantial reduction in over-all cost to both the parties.

2. LITERATURE SURVEY

Crowther (1964) established the relationship between the prices on orders of different sizes after the seller has achieved a few fundamental decisions regarding the manner in which he wants a smooth and efficient business.

There are two general types of quantity discount schedules offered by supplies: the all-units discounts and the incremental discount. Purchasing large quantity in all-units discount schedule results in a lower unit price for the entire lot; whereas, in incremental discount schedule, lower unit facility is available only to units purchased above a specified quantity. The quantity at which price change is called price-break quantity. Gorham((1970) calculated break-even demand volumes to determine quantity discount desirability.

2.1. EOQ and Price discount

Based on the simple EOQ model, a model for determining the optimal stock replenishment strategy for temporary price reduction can be derived. Barman and Tersine (1995) extended the logic to a composite EOQ model that can be segmented into a family of hybrid models with broader operational flexibility. The composite EOQ provides malleability and flexibility to changing operational requirements by desegregating complexity. The resourcefulness of an expert system with its attendant economics is approached.

Lu and Qui (1994) derived the worst-case performance of a power-of-two policy in an all-unit quantity discount model with one price break point, which extends the sensitivity analysis for the classical EOQ model. The model showed that the worst-case performance will depend on the discount rate α and is within 7.66 % of optimality when $\alpha < 7.51$ % and approximately within 100α % of optimality when $\alpha \geq 7.51$ %.

Bastian (1992) developed a dynamic lot-size problem under discounting which allows a speculative motive for holding inventory. He derived a procedure that determined the first lot-size decision in a rolling horizon environment, using forecast data of the minimum possible number of future periods.

Chao (1992) generalised Bather's EOQ model with discounting. The model considers two cases: one with demand backlogging and the other, without backlogging. Important implications of discounting are investigated useful insights and formulas are provided.

Martin (1993) provides an alternative perspective on the quantity discount-pricing problem. He considered the multiple price breaks excluding the buyer's operating parameter from consideration, with the exception of price dependent demand.

Johnson (1971, (1975) described a graphical approach to price-break analysis. He argued that in any stock control situation the main problem is how many units of an item should be procured at any one time, the "economic batch quantity; (EBQ)" and having determined EBQ, the problem of quantity discount arises? Would it be profitable to order in quantities other (usually more) than the EBQ in order to take advantage of price discount. This approach provided:

- a. a rationale foundation for a very many real life purchasing decisions;
- b. a tool which is readily understood by the clever man in the business.

It suggested "Given a discount structure for a product group, find the most economical purchase quantities for all members of that group".

Weng (1995) presented the models for determining optimal all-unit and incremental quantity discount policies. He also investigated the effect of the quantity discounts on increasing demand and ensuring Pareto efficient transactions under general price-sensitive demand functions. Optimal quantity discount policies, their interrelationships and their benefits to supplier and the buyer were developed in this paper. The gains of

managerial insights for the scenarios of maximising the supplier's profit and the joint profit are illustrated in this paper. He developed a simple and efficient solution approach for determining the all-unit and the incremental optimal decision policies for general price-sensitive demand functions. The main findings were:

1. with price-sensitive demand there are two incentives in offering quantity discounts: increasing demand and ensuring Pareto-efficient transactions. In most cases increasing demand dominates in justifying the offering of quantity discounts,
2. using a single lot-size associated with all efficient transactions as with the constant demand does not hold with price sensitive demand case,
3. the optimal all-unit quantity policy is equivalent to the optimal incremental quantity discount policy function benefiting both supplier and the buyer.

Carlson, Milterburg and Rouseeau (1996) examined EOQ and optimal order quantity under both all-units and incremental quantity discounts when purchase cost, ordering cost and inventory holding cost are incurred on date-terms supplier credit. Payment dates for the three cost components need not be same. The traditionally used periodic cost minimisation methodology that is insensitive to differences in the timing of various within period cash flows is replaced with the discounted cash flow methodology. Differences in the characteristics of day-terms and date-terms solutions to the quantity discount case were highlighted.

Shinn, Hark and Park (1996) in their paper dealt with the problem of determining the retailer's optimal price and lot-size simultaneously under the conditions of permissible delay in payments. They also assumed that the ordering cost consists of a fixed set-up cost and a freight cost, where freight cost has a quantity discount offered due to the economics of scale. The constant price elasticity demand function is adopted which is a decreasing function of retail price. Insight of the properties of the optimal solution allows us to develop an algorithm and is illustrated with an example.

Determining a lot-size in the presence of either all-units or incremental discount has represented one of major interests of materials managers. It is a common practice that the suppliers offer quantity discount to entice the buyers to purchase more and to achieve economics of scale for transportation and processing costs. Benton and Park (1996) classified the literature on lot-sizing determination under several types of discount schemes in their paper. They also discussed some of the significant literature in this field over two decades with identifying the future research areas.

Federgruen and Lee proposed an optimal algorithm for the single-item dynamic lot-size model with all-unit discounts. Xu and Lu (1998) show that the optimal algorithm fails in some cases. They modified the algorithm to handle the failure.

Baker (1976) developed discount scenario of the placing special order at discounted price when the firm's regular ordering cycle coincides with the end of the discount period. Whybark (1977) described a mathematical model to establish trade-off between the potential for reducing unit cost and the cost of increased inventory for continuous demand. He used trade-off to measure the attractiveness of potential quantity discounts. This measure was the percentage change in the total costs that would result from taking the discount.

Laforge and Patterson (1985) developed an adjusted part-period algorithm (APPA) for determining lot-sizes with all-units discounts. They tested APPA using various demand data, ordering costs and price schedules, but it failed. Tersine and Price (1981), Tersine and Schwarzkopf (1989) derived decision problem for the retailer who buys units at the lower price from the start of the discount period, which may or may not coincide with regular ordering cycle. He buys units till the end of the discount period, which may or may not extend beyond the next regular replenishment period.

Kuzdrall and Britney (1982) developed a restrictive model that directs attention to the proper price-quantity interval. Rubin Dilts and Barron (1983) modified the approach of Whitin and Hadley (1963) to find the largest feasible EOQ with its corresponding total cost and compared it with the total cost of all price-break quantities larger than the feasible solution. Das (1984) developed a mathematical model to determine the optimal order quantity for a buyer when the supplier offers quantity discounts. Monahan (1984) investigated discount scenarios from both the buyer and sellers view points. Banerjee (1986) generalised Monahan's (1984) model by incorporating vendor's inventory cost.

Lal and Staelin (1984) formulated wherein the buyer is confronted with the decision to accept or reject the seller's discounting offer, under the framework of profit maximisation. Rosenblatt and Lee (1986) developed a mathematical model for the supplier's point of view and designed procedure to determine the optimal discount schedule for the supplier.

Sethi (1984) introduced a finite unit disposal cost in the EOQ model with all units quantity discounts and developed an algorithm to obtain optimal lot-sizes for an entire range of unit disposal costs.

Aucamp and Kuzdrall (1986) derived the order quantity which minimised the discounted cash flows for a one-time sale when the sale consummated the current inventory may be at or exceed the usual reorder point. In the later case, the company may decide to buy nothing, if a large minimum order quantity is required; in order to obtain price discount. Exact expressions for the order quantity, the associated cost savings, minimum acceptable percentage price discount and minimum vendor quantity requirements are derived. Srinivasan and Rao (1987) developed a joint profit maximisation framework (that is, profit maximisation for a seller and cost minimisation for a buyer) when discount is available for short duration of time.

2.2. Price discount policies considering Inventory costs

Ardalan (1988) analysed the effects of a special order during the sale period on inventory costs. Ardalan (1994) studied the joint effect of marketing and inventory policy on total profit by taking a general price demand relationship for determining retailer's optimal price and optimal ordering policies. Abad (1988) developed a procedure for finding the optimal selling price and lot-size by taking demand to be decreasing function of price, for a retailer purchasing a product for which the supplier offers all-unit quantity discounts.

Kim (1989) argued how the supplier can formulate the terms of a quantity discount-pricing schedule assuming that the buyer always behaves optimally. Formulas were derived for price and order-size, which maximise:

1. the economic gain of supplier resulting from revising price and order size;
2. the gain of the buyer ; and
3. the sum of gains of both the parties.

It was suggested that how the supplier can induce the buyer to pre-determined price and order-size, level of mutual benefit by utilising all-units and incremental quantity discounts in the system.

Min (1992) studied the profit maximising EOQ model for a monopolistic seller. A monopolistic seller determines both the order quantity and the quantity discount price schedule simultaneously where as buyers have their preferences on the purchase quantities.

Arcelus and Srinivasan (1989a) considered price elastic demand and optimised net profit, return on investment and residual income rather than total cost. They considered demand to be function of the resale price, which in turn, is a function of a mark-up on the acquisition cost. They (1989b) extended the all-units quantity discount pricing model to consider the buyer and the seller, as maximises of their returns on their investments in the inventory. They gave upper limit on the quantity, the buyer may like to purchase and a lower limit on the amount the vendor is willing to sale, for a given discount rate. They established feasible solution to both the parties.

Goyal (1990) gave a procedure for determining the economic ordering policy for a product for which the supplier has offered reduction in price during given specified period by assuming demand to be constant over an intimate time horizon. Goyal and Gupta (1990) gave the simpler procedure requiring few EOQ calculations for determining the lot size. Goyal **et al.** (1991) reviewed industrial practice to offer special incentives to motivate the buyer; to order in larger than regular order quantities during a limited time only.

The active area of research in Inventory models is a model with temporary price discounts. Wee and Yu (1997) extended the fact that some commodities nay deteriorate during storage. Models for the exponentially deteriorating items with temporary price discount were developed under regular and non-regular replenishment time. The main goal was to maximise the total saving during the temporary price discount order cycle. This concept was illustrated with a numerical example.

Datta and Pal (1991) developed an inventory model with sale-price dependent demand rate for an infinite time horizon when the on-hand stock is larger than the optimal stock level. They adopted a price reduction special sale policy to increase vendor's profit. Aull-Hyde (1992) derived the special order quantity by comparing the benefits of a special order in terms of lower purchase price with the additional inventory holding cost.

Song (1994) explored qualitative effects of lead-time uncertainty in a basic continuous-time single-item inventory model with poisson demand and stochastic lead-times. The objective was to minimise the infinite horizon expected total discounted cost. Order costs are linear, so a base-stock policy is optimal. Traditionally and intuitively, people believed that a stochastically longer lead-time results in a larger lead-time demand and therefore that the system should have a higher optimal back-stock level. In fact, Song showed that the concept is true for models with the average cost criterion. He also showed that this intuition is not always validated for models with the discounted cost criterion. Certain systems were identified in which the lead-times do have a monotonic impact on the optimal base-stock levels. Systems with constant, geometric, gamma or uniform lead-time distributions were illustrated with examples. It was also found that a shorter lead-time does not necessarily result in a smaller optimal system cost.

Matta (1994) addressed problem encountered by retailers who experiences cyclical replenishment. Frequently, these retailers are provided with incentives for placing orders before the end of a cycle. The retailers have to "wrestle" with the questions of when, how many items and how many units to order. The complexity of the problem arises from having the balance between the reduction in the costs with the increase in ordering costs and in some case deal with storage and shortage costs. It was also proved that the decrease in the unit cost can be directly related to the enhancement of manufacturing efficiency due to longer production lead times while the increase in storage and shortage cost is a result of the increase in uncertainty due to a longer lead-time. The system was based on a single-echelon representation where the cost is minimised.

In the classical co-ordinated replenishment dynamic lot-sizing problem, the primary motivation for co-ordination is in the presence of the major and minor set-up costs, Chung, Hum and Kirka (1996) consider a separate element of co-ordination made possible by the offer of the quantity discounts in their paper. They provide a mathematical programming formulation for the extended problem under the all-units discount price structure and the incremental discount price structure. Then by using variable redefinition, tighter formulations are presented in order to obtain tight lower bounds for reasonable size problems. More significantly, as the problem is NP-hard, they represent an effective polynomial time heuristic procedure, for the incremental discount version of the problem that is capable of solving reasonably large size problems.

Arcelus and Srinivasan (1992a), (1992b) examined a firm's response to a temporary price reduction on the regularly ordering items. Responses vary depending upon whether the discount is offered:

1. on all purchases regardless of the order-size;
2. on all items purchased, if the order exceeds certain minimum; or
3. on only the amount in excess of the minimum.

Different policies were developed, depending upon whether:

1. the sale period falls within a regular ordering cycle;
2. the end of the cycle coincides with the end of the discount period; or
3. one or more regular cycles fall within the discount period.

They concluded that more than one order might be optimal when the regular cycle falls within the discount period. They (1996) developed a general model of forward buying practices, which reflects price dependent demand characteristics and alternative pricing strategies available to a retailer, faced with the discount offer. This is accomplished through simultaneous determination of the most profitable:

1. stock level to be purchased by the retailer;
2. discount level to be passed on to the customers;
3. stock level qualifying for the retailer's discount; and;
4. time to initiate the discount.

Arcelus, Bhadury and Srinivasan (1996) provided a detailed g-t-analysis of the buyer-seller co-ordination problem embedded in the price-discount inventory model. Pure and mixed, co-operative and non-co-operative strategies were developed. The complete characterisation of Pareto optimal set, the determination of profit-sharing mechanisms for the co-operative case and the derivation of a set of parametric non-co-operative mixed strategies were highlighted in this paper.

Martin (1994) argued that the inventory holding cost in Aull-Hyde's (1992) formulation was incorrect even though the effect of the specification was small for lower discounts. Chen and Min (1995) integrated optimal inventory and disposal policies. Tersine and Barman (1995) discussed a desegregation of composite EOQ model into several traditional models to determine the optimal level of order quantity and backorder quantity in response to a temporary price discount.

Hall (1992) argued that the conventional approach for settling order quantities when prices change, does not incorporate a present value analysis, and does not differentiate inventory storage costs from the cost of money. He established that these factors cause optimal order quantities to be over estimated when prices are reduced by more than 30%. Following a present value analysis, he evaluated the relationship between optimal order quantity and the magnitude of the price change.

Joglekar and Kelly (1998) found that when volume discounts are available, in some cases, reliance on EOQ can induce purchasers to make wealth reducing decisions and the present value (PV) approach should be preferred. Though this finding seems to be theoretically correct, the magnitudes of wealth reduction in the numerical examples, explained in this paper seems questionable. Also, it finds that a purchaser, in some cases, using EOQ approach realises a net increase in current wealth compared to purchaser using the PV-approach. The logic suggests that such a finding cannot be correct, since by its very definition, if the PV-approach that seeks to maximise the current wealth. They offer an alternative frame of comparison and a modified model to show that, the EOQ approach can never realise a net increase in current wealth compared to the current wealth generated by the PV- approach. They show that when typical values of the relevant parameters prevail, the additional costs imposed by EOQ approach are not significant. They suggest that as PV approach requires greater administrative costs to implement, it may even be countered productive to the goal of wealth maximization.

Majewica and Swanson (1978) explained how the Wagner-Whitin algorithm can be applied for deterministic time varying demand over a finite time horizon when demand is non-standing and all-units discounts are offered. Jucker and Rosenblatt (1985) considered quantity discounts in the context of the single period inventory model known as the "Newsboy Problem". They gave marginal-cost-solution procedure by describing graphically. It demonstrated to handle complex discount schedules such as combined purchasing and transportation cost discount schedules.

Lin and Kroll (1997) considered the models for the single item news-boy problem with quantity discount and the following dual performance measure 'maximize the expected profit subject to a constraint that the probability of achieving a target profit level is no less than a predetermined risk level'. They also considered two types of quantity discount: all-unit and incremental. A closed form solution for determining the optimal order quantity was derived with zero shortage cost. However, models with positive shortage cost could be solved numerically.

Johnsen and Thorstenson (1996) considered a cost function review (r, Q) inventory system with Poisson demand and at most one order outstanding. The replenishment lead-time is either constant or exponentially distributed. Demands, not covered immediately from inventory are lost. Costs includes a linear order cost with a fixed cost / order, and a fixed cost per unit lost sale. As regards inventory holding cost, the cost of capital often constitutes a major part. The paper focused on the interest related holding costs. In the undiscounted case, holding costs were linear and inventory performance was measured by the long-run average total cost incurred per unit time. In the discounted case, the performance measure was the expected present value of the ordering and lost sales costs. The cost associated with capital tie-up in inventory was accounted for by an appropriate discount rate. They formulated an exact model and design a policy-iteration algorithm for the discounted case. Results on the form of an optimal replenishment policy were derived and model was compared to a previously derived model for the undiscounted case. To evaluate the difference between the optimal solutions with and without discounting numerical examples were used. The effect of a stochastic lead-time on this difference was also considered by comparing solutions with constant and exponential lead-times. In general, the differences seem to be fairly small but exceptional cases exist when the service level is low.

Dave and Pandya (1985) extended Naddor's (1966) problem of returning or of selling the inventory excess to optimal stock level for deteriorating items. They developed mathematical models for the infinite and finite horizon by using deterioration of units to be a constant fraction of the on-hand inventory. Shiue (1990) derived an inventory model continuous in units and time for perishable items in a lot-size system by allowing partially back-logged and partially lost sales with quantity discount. Shah (1993a), Wee and Yu (1997) developed an EOQ model for exponentially decaying inventory under temporary price discount. Shah **et al.** (1994) derived a mathematical model under temporary price discount when the supply was random.

Benton and Whybark (1982) considered all-units discounts with stochastic demand. They compared the number of part periods of inventory that can be carried before the savings from a single discount are offset. Gupta and Dui (1991) formulated a problem of jointly setting order quantity of fresh items and discount price of one-period-old items as a stochastic dynamic program for commodities that perish in two periods. The problem was solved using a heuristic procedure that decouples inventory decisions in any two consecutive periods by assuming a salvage value of left over fresh items that is also the price paid for the entering stock of one-period old items. For an assumed salvage value, the single period expected profit maximising ordering discount policy was identified. The m-periods planning problem was then solved by sequentially solving m-single period problems.

Chao (1992) derived an EOQ with stochastic demand to the case with discounting by treating time to be continuous. Shah and Shah (1993b) extended Chao's (1992) model for the case of constant rate of deterioration of units in an inventory system. Shah (1998) derived model under the above-mentioned situation by treating cycle time to be discrete. Shah and Gor (1998) derived an order-level lot-size system when random chances of discounts are offered. Shah and Gor (1997) extended the same model for the deteriorating items. Shah and Shah (1997) formulated an EOQ model under random supply when random chance of discounts is offered.

2.3. Price discount policies under freight discounts

Previous studies have been on the procedures, which determine the optimal lot-size and selling price when demand depends on the price and all-unit quantity discounts are offered. The recent studies have been on the concepts of quantity and freight discounts for constant demand. Dave, Burwell, Fitzpatrick and Roy (1997) have incorporated quantity and freight discounts in inventory decision making when demand is considered to be dependent upon price (rather being constant). An algorithm was developed to determine the optimal lot-size and selling price for a class of demand functions, including constant price-elasticity and linear demand. A numerical example was illustrated to develop the model and computer program to implement the model.

Followill **et al.** (1990) studied managerial decision to accept a quantity discount, if total, per period inventory and acquisition costs are reduced. They developed an EOQ model within wealth maximisation framework, when volume discounts were available. They established that the traditional method of analysing volume discount opportunities may invoke wealth decreasing decisions.

Weng, Wang (1993), in this article, investigated the managerial insights related in using the all-unit quantity discount policies under various conditions. The models developed here were general treatments that deal with four major issues:

- a. one or multiple buyers;
- b. constant or price elastic demand;
- c. the relationship between the supplier's production schedule or ordering policy and buyer's ordering sizes;
- d. the supplier either purchasing or manufacturing the item.

The two main objectives of the developed models were the supplier's profit improvement or the supplier's increased profit share analysis. Algorithms were developed to find optimal decision policies. The present analysis provided the supplier with both optimal all-unit quantity discount policy and the optimal production or ordering strategy. The concept was illustrated with numerical examples.

Chaudhury, Forst and Zydiak (1993) studied the problem of vendor selection where a buyer must choose order quantities to place with the vendors in a multiple source networking. The influencing factors for selection are the price, delivery and quality objectives of the buyer as well as by the production or rationing constraints of the vendors. Vendors are assumed to offer price breaks which depend on the sizes of the order quantities.

Each price break schedule is characterised by two attributes: the representation is either quantity discounts or surcharges and is either all-unit or incremental. The presented models were linear and mixed binary integer programming. These models provided unifying frame works for models of vendor performance measures from the purchasing literature and models of price breaks from OR literature. They argued that the present models offer viable approach to vendor selection with price breaks because of the availability of the commercial software packages for personal computers that provide flexible and efficient computational tools for solving the models. This implied that vendor selection with price breaks could be performed at the fingertips of the purchasing managers.

Benton (1991) considered quantity discount procedures under conditions of multiple items, resource limitations and multiple suppliers. He offered an efficient heuristic programming procedure for evaluating alternative discount schedules. The article provided encouraging findings for the managers. Gaither and Park (1991) developed optimal ordering policies for a group of inventory items when a supplier offers a one time discount for the group by minimising total inventory costs within a firm's ware house space or funds availability constraints.

Carlson and Miltenburg (1993) examined order quantities for families of items when regular and special discount schedules are available, and the objective is to minimise the present value of the relevant cash flows. Order quantities can be large when special discounts are available, traditional cost minimisation models which ignore the effects of the timing of cash flows are less accurate than the discounted cash flow models. They extended the earlier research work on families of items, special quantity discounts and discounted cash flows. Order cost, invoice cost, physical inventory carrying cost and financial carrying cost were identified. The amount and the timing of the cash out flow for each cost were described. Regular as well as two special types of discounts was also considered. Regular discounts were schedules of discounts and break point order quantities that were always available. The special discounts are an opportunity to order before a price increase and a 'sale' or special one-time price reduction.

Sorger (1994) derived a condition that must be satisfied by a cost function $h: X \rightarrow X$ in order to be the optimal function of a strictly concave deterministic dynamic programming problem which defined on the state space X and which has a given discount factor ζ . This condition was used to show that there was no such DPP on a one-dimensional state space that generates optimal solutions that are periodic with minimal period three unless the discount rate exceeds 82%. This bound held uniformly for all strictly concave problems and all period three cycles.

In many practical situations, co-ordination between replenishment orders for a family of items can be cost saving. A well-known class of strategies for the case where cost savings are due to reduced joint ordering costs is the class of can-order strategies. However, these strategies, which are simple to implement do not take price discount possibilities into consideration. Duyn, Van and Heuts (1994) proposed a method to incorporate discounts in the framework of can-order strategies. A continuous review multi-item inventory system was considered with independent compound Poisson demand processes for each of the individual items. The supplier as a percentage of the total dollar value offers discounts whenever this value exceeds a given threshold. They developed a simple heuristic to evaluate these discount opportunities taking the can-order strategy as a basic rule. The performance of the can-order strategy with discount evaluation was compared with that of the another class of discount evaluation rules as proposed by Miltenburg and Silver.

Russell (1992) addressed the issues relating in determining whether products should be ordered independently and shipped as a single product order or co-ordinated and shipped as a group, or multi-product, order from a single source. The factors which might influence the decision were the level or volume of demand, the distribution of demand across products, the weight of items and the attractiveness of quantity discounts offered. An optimal inventory-theoretic model was used which incorporates transport weight breaks and quantity discounts, assessment of when the product orders should be combined and what products should be ordered separately. The effects of these decisions were examined on the interval, the number of order groupings, the proportion of items ordered independently, the proportion of attractive discounts foregone in favour of consolidation, and the relative cost savings. Russell used an extensive set of stimulated data on a firm in the automobile industry supply chain.

2.4. Price discount policies under JIT environment

Most manufacturers rarely use formal price break (quantity discount) analysis while placing purchase orders; and too often discounts are totally ignored because of lot-sizing rules in an MRP-II system. Grant (1993) presented the issue of price breaks how it was dealt with in the JIT environment, how policy was

changed, the program written to analyse the opportunities, and the implementation results of the new policy and program. During the last two years, asking for and taking advantage of price breaks had saved the present company several hundred thousand dollars.

Companies have traditionally relied on taking advantage of price-quantity discount (PQD) on large purchases. With the adoption of JIT philosophy companies are encouraged to purchase materials in small lots to synchronise production deliveries, raising a question whether PQD is applicable in a JIT purchasing environment. Gupta and Kini (1995) argued that though seemingly inconsistent, JIT and PQD could coexist. They developed an integrated JIT-PQD model to allow a buyer to decide how much to purchase and how many shipments should be placed per order.

The inventory costs of purchasing under EOQ model with a quantity discount is determined and compared to the costs under JIT. Fazel, Fischer and Gilbert (1998) show that at low levels of demand JIT is the preferred model, whereas, EOQ has the cost advantage for an item with a high demand. The proposed model predicts that the higher value of the item, the carrying cost, or the ordering cost associated with the EOQ model, and the smaller the quantity discount rate, the wider will be the range of demand for which JIT remains preferable.

In supply chain literature, an increasing body of work, studies how suppliers can use incentive schemes such as quantity discounts to influence buyers ordering behaviour, thus reducing the supplier's costs. Various function forms for such incentive schemes have been proposed but a critical assumption always made is that the supplier has the full information about the buyer's cost structure. Corbett and Groote (2000) derive the optimal quantity discount under asymmetric information and compare it to the situation where the supplier has the full information.

2.5. Price discount policies under different environments

Mamer studied the expected costs to producers and benefits to consumers of three types of product warranty:

- a. the ordinary free replacement warranty;
- b. the unlimited free replacement warranty;
- c. Pro-data warranty.

In addition to assuming a random product lifetime, Mamer's models allow for randomness in consumer re-purchase behaviour and for the possibility of independent damage process acting on products sold under warranty. The criteria for evaluating warranty policies are discounted costs and net revenues over infinite horizon, and per unit costs and not revenues. Kao and Smith (1993), in their paper, considered the case of phase-type product lifetimes. Under the above assumption, they simplified the computation of the expected cost and revenue functions. Since most distributions can often be represented by the phase distributions the present simplifications enhance the applicability of Mamer's useful but computationally unwisely results. They also established an interesting relation pertaining to expected net revenues as a function of consumer loyalty. They found under the per-unit-net-revenue criterion, from the producer's perspective, repeated purchases by the same customer are not profitable as would be intuitively conjectured. Such a counter-intuitive relation makes the need for being able to compete various costs and revenue functions more compelling- particularly when lifetime are not exponential.

Ghandforoush((1992) presented procurement model for multinational Oil Company with affiliated plants located world-wide. The model used a quantity discount relationship to reduce the total procurement cost. They used a non-linear programming method as a solution and compare with current approach used by the Oil Company. The proposed approach had distinct advantages, which assure in minimising the cost, ease of data management, flexibility in design and accuracy in results.

Fuglseth and Gronhaug (1997) argued that business redesign and decision support could be useful complementary approaches to the redesign of business processes handling complex tasks in an ever-changing environment. They focussed on the possibilities of IT:

- a. to enhance consistency between strategy and operation;
- b. to improve the quality of empowered employee's decisions;
- c. to facilitate organisational learning.

3. CONCLUSIONS

The goal of this paper is to bring out a complete and up-to-date review of published articles on price discount strategies. The available relevant models have been classified and quantified into a number of categories and their principal features have been discussed.

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