A MODEL SPECIFICATION METHOD FOR TRANSFER FUNCTION MODELS BASED ON THE GENERALIZED EXTENDED SAMPLE AUTOCORRELATION FUNCTION AND A COMPARATIVE STUDY WITH THE EXTENDED SAMPLE AUTOCORRELATION FUNCTION FOR ARMA MODELS

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ABSTRACT
The identification of a (r,s,b)×(p,q) bivariate transfer function model is generally done through the sample cross-correlation behaviour analysis between the input and the output series. However, practice shows that this procedure as an identification instrument is not sufficient due the subjectivity related to the specification of the orders r and s associated to the output and the input polynomials. Following the papers by Oliveira C. and Müller D, (1998, 2000) and the PhD Thesis by Oliveira , C. (2001), where it is proposed a new identification methodology of these models based on the concept of the generalized extended sample autocorrelation function, in this work we investigate the behaviour of this methodology. To illustrate the procedure potentiality a simulation study is presented and comparisons are made from a simulation study for the extended sample autocorrelation function in the univariate ARMA model specification.

Key words: bivariate transfer function model; cross-correlation function; least-squares estimators; extended sample autocorrelation function.

RESUMEN
La identificación de un modelo de la función bivariada de transferencia (r,s,b)x(p,q) se hace generalmente a través del análisis de la correlación cruzada muestral entre la entrada y la salida de la serie. Sin embargo, la práctica muestra que este procedimiento, como un instrumento de identificación, no es suficiente dada la subjetividad relacionada con la especificación de los órdenes r y s asociados a los polinomios de la salida y la entrada. Siguiendo los trabajos de Oliveira C.- Muller D. (1998, 2000) y la tesis doctoral de Oliveira C (2001), donde se propuso una nueva metodología de identificación de esos modelos basado en el concepto de función de autocorrelación muestral generalizada, en este trabajo investigamos el comportamiento de esta metodología. Para ilustrar las potencialidades del procedimiento un estudio de simulación se presenta y se hacen comparaciones a partir del estudio de simulación para la función de autocorrelación extendida en la especificación de modelo ARMA univariado.

Palabras clave: modelo de la función de transferencia bivariada, función de correlación cruzada, estimadores mínimos cuadrados, función de autorrelación muestral extendida.

MSC: 60E05

1. INTRODUCTION

Let α_t and β_t be, respectively, the input and the output series, jointly stationary, of the (r,s,b)×(p,q) bivariate rational transfer function model,

$$\beta_t = \frac{\omega_s(B)}{\delta_r(B)}\alpha_{t-b} + \frac{\theta_q(B)}{\phi_p(B)}a_t \ , \tag{1.1}$$

where α_t is the prewhitened input series ($\phi_x(B)X_t = \theta_x(B)\alpha_t$),

β_t is the filtered output series $\left(\beta_t = \dfrac{\phi_x(B)}{\theta_x(B)}Y_t\right)$,

a_t is a residual white noise series not correlated with the input series,

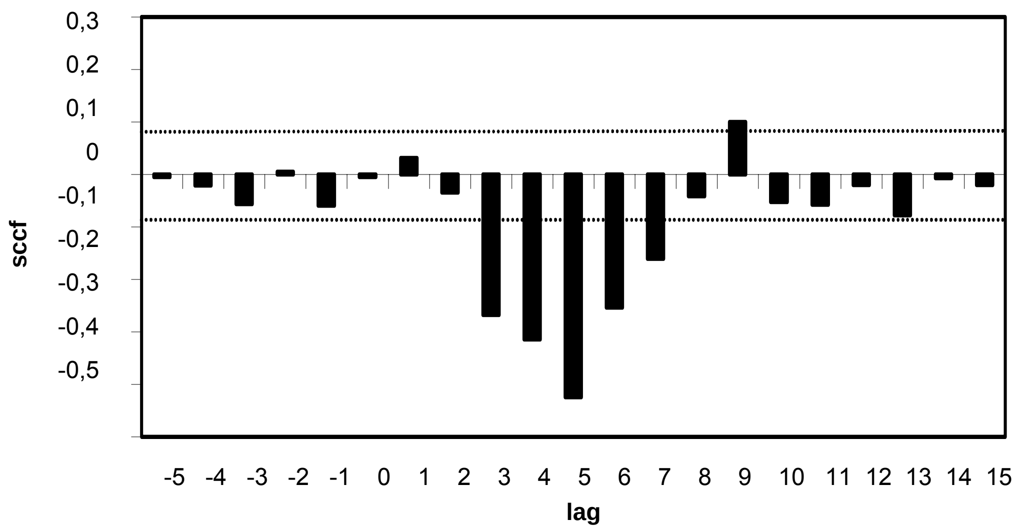
$b \in N_0$ is the delay parameter

and $\delta_r(B) = 1 - \delta_1 B - \cdots - \delta_r B^r$, $\omega_s = \omega_0 + \omega_1 B + \cdots + \omega_s B^s$, $\phi_p(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$

and $\theta_q(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$ with B as the backshift operator $(BX_t = X_{t-1})$.

The identification of the ARMA model polynomial orders p and q and of the delay parameter b are quite standard and well documented: p and q are identified from the sample autocorrelation, sample partial autocorrelation and extended sample autocorrelation functions of the residuals series and b is identified from the sample cross-correlation function (SCCF) between the α_t and the β_t and the series, $r_{\alpha\beta}$. However, the specification of the orders r and s associated to the output and the input polynomials, respectively, through the SCCF behaviour analysis is rather subjective and it is generally done through successive attempts as we can illustrate from the following example.

Let us considerer the two time series from the gas furnace data in Box **et al**. (1994) and its corresponding SCCF plot (bounds $\pm 1.5 \cdot \hat{\sigma}(r_{\alpha\beta})$).



The identification of the delay parameter, b = 3, is quite simple, since b is identifiable with the first lag where the cross-correlation is not null. Nevertheless, due to the structure presented by the SCCF plot, the specification of r and s is not so obvious, leading to the identification of several possible rational transfer functions for the two series under study such as (1,2,3), (1,3,3), (2,2,3), (2,3,3) and (0,4,3).

Following the paper by Müller, D. and Wei, W.W.S. (1997), concerning the iterated least-squares estimators consistency for the parameters of the bivariate transfer function models, Oliveira, C. and Müller, D. (1998, 2000) and Oliveira , C. (2001), proposed for any chosen b, p and q a new identification methodology of these models based on a generalization of the extended sample autocorrelation function concept introduced by Tsay, R.S. and Tiao, G.C. (1984). In this work we investigate the behaviour of this methodology with a simulation study and comparisons are made for the case of the extended sample autocorrelation function (ESACF) in the univariate ARMA model specification.

2. PRELIMINARY RESULTS[1]

The (r,s,b)×(p,q) bivariate rational transfer function model (1.1) can be written as

$$\delta^*_{p+r}(B)\beta_t = \omega^*_{p+s}(B)\alpha_{t-b} + \theta^*_{r+q}(B)a_t, \qquad (2.1)$$

where

$$\delta^*_{p+r}(B) = \delta_r(B)\phi_p(B) = 1 - \delta^*_1 B - \cdots - \delta^*_{p+r} B^{p+r},$$

[1]The preliminary results developments are in the paper “Generalized Extended Sample Autocorrelation Function: contribution to the identification of the Transfer Function Models”, Oliveira, C. and Müller, D. (1999), submitted to the *Journal of Time Series Analysis*

$$\omega^*_{p+s}(B) = \omega_s(B)\phi_p(B) = \omega^*_o + \omega^*_1 B + \cdots + \omega^*_{p+s} B^{p+s},$$

$$\theta^*_{r+q}(B) = \delta_r(B)\theta_q(B) = 1 - \theta^*_1 B - \cdots - \theta^*_{r+q} B^{r+q},$$

or, more explicitly, as

$$\beta_t = \sum_{i=1}^{p+r} \delta_i^* \beta_{t-i} + \sum_{l=0}^{p+s} \omega_l^* \alpha_{t-b-l} + a_t - \sum_{u=1}^{r+q} \theta_u^* a_{t-u}.$$

If we take

$$Y_{p+r,t} = \delta^*_{p+r}(B)\beta_t - \omega^*_{p+s}(B)\alpha_{t-b},$$

then, we obtain

$$Y_{p+r,t} = \theta^*_{r+q}(B)a_t, \tag{2.2}$$

that is, $Y_{p+r,t}$ follows a MA(r +q) process. Let $r_{Y_t}(k)$ be the sample autocorrelation function of Y_t at lag k. Then the sample autocorrelation function (SACF) of $Y_{p+r,t}$ is asymptotically null after the lag $k = r + q$,

$$\begin{aligned} r_{Y_{p+r,t}}(k) &\doteq 0 \;,\; k > r+q \\ &\neq 0 \;,\; k = r+q \end{aligned} \tag{2.3}$$

The model (2.1) can also be written as

$$\delta^*_m(B)\beta_t = \omega^*_{p+s'}(B)\alpha_{t-b} + \theta^*_{r+q}(B)a_t,$$

with $m \geq p+r$ and $s' \geq s$, where it is understood that $\delta_i^* = 0$ for $i > p+r$ and $\omega_l^* = 0$ for $l > p+s$.

Supposing that n observations of β_t and α_t are available satisfying model (2.1) and assuming that the value p is known, construct the iterated regressions of order m ($m = p, p+1, p+2, \ldots$), successively for $s' = 0,1,2,\ldots$, whose jth iterated regression is defined as follows,

$$\beta_t = \sum_{i=1}^{m} \delta^{s'(j)}_{i(m)} \beta_{t-i} + \sum_{l=0}^{p+s'} \omega^{s'(j)}_{l(m)} \alpha_{t-b-l} + \sum_{u=1}^{j} \theta^{s'(j)}_{u(m)} \hat{e}^{s'(j-u)}_{m,t-u} + e^{s'(j)}_{m,t}, \tag{2.4}$$
$$t = \max(m, b+p+s') + j + 1, \ldots, n \;;\; m = p, p+1, p+2, \ldots \;;\; j = 0,1,2,\ldots,$$

where $\hat{e}^{s'(\upsilon)}_{m,t} = \beta_t - \sum_{i=1}^{m} \hat{\delta}^{s'(\upsilon)}_{i(m)} \beta_{t-i} - \sum_{l=0}^{p+s'} \hat{\omega}^{s'(\upsilon)}_{l(m)} \alpha_{t-b-l} - \sum_{u=1}^{\upsilon} \hat{\theta}^{s'(\upsilon)}_{u(m)} \hat{e}^{s'(\upsilon-u)}_{m,t-u}$ is the estimated residual of the υth iterated regression of order m for s', and $\hat{\delta}^{s'(\upsilon)}_{i(m)}$, $\hat{\omega}^{s'(\upsilon)}_{l(m)}$ and $\hat{\theta}^{s'(\upsilon)}_{u(m)}$ are the corresponding least-squares estimates.

Lemma 1[2]: Let β_t and α_t be two time series related by the $(r,s,b)\times(p,q)$ rational transfer function model given in (1.1) and assume that the roots of $\delta^*_{p+r}(B)$ lie outside the unit circle. Then,

$$\hat{\delta}^{s'(j)}_{i(m)} = \delta_i^* + o_p(1) \;,\; i = 1,2,\cdots,m$$

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[2]In this work, it will be used the following notation (Brockwell and Davis (1995)): $X_n - Y_n \xrightarrow{p} 0$ is represented by $X_n - Y_n = o_p(1)$ or $X_n \doteq Y_n$ being X_n and Y_n asymptotically equivalent.

$$\hat{\omega}_{l(m)}^{s'(j)} = \omega_l^* + o_p(1) \ , \ l = 0,1,2,\cdots,p+s',$$

when

a) $s' = s$, $m = p+r$ and $j \geq \max(p+s, r+q)$,

b) $s' = s$, $m \geq p+r$ and $j = \max(p+s, r+q)$,

c) $s' > s$, $m = p+r$ and $j \geq \max(p+s', r+q)$,

d) $s' > s$, $m \geq p+r$ and $j = \max(p+s', r+q) = r+q$

where it is understood that $\delta_i^* = 0$ for $i > p+r$ and $\omega_l^* = 0$ for $s' > s$.

Under these circunstances, let be the series

$$Y_{m,t}^{s'(j)} = \hat{\delta}_m^{s'(j)}(B)\beta_t - \hat{\omega}_m^{s'(j)}(B)\alpha_{t-b} \ , \quad t = \max(m, b+p+s')+1,\ldots,n$$

defined from the jth iterated regression of order m for s' (2.4).

Therefore, the following result holds:

Theorem: Under the conditions of the Lemma 1 and (2.3), $Y_{m,t}^{s'(j)} \sim MA(r+q)$, that is the respective SACF is such that

$$r_{Y_{m,t}^{s'(j)}}(k) \doteq 0 \ , \ k > r+q \qquad \text{for } s' = s \ , \ m = p+r \text{ and } j \geq \max(p+s, r+q) \ . \tag{2.5}$$
$$\neq 0 \ , \ k = r+q$$

The SACF $r_{Y_{m,t}^{s'(j)}}(k)$ is designated by the mth generalized extended sample autocorrelation function (GESACF) of β_t and α_t for s'.

This function generalizes the concept of the extended sample autocorrelation function (ESACF) introduced by Tsay and Tiao (1984). In fact, if $\alpha_t \doteq 0$, a.s. , $r_{Y_{m,t}^{s'(j)}}(k)$ represents the ESACF of the β_t process when k = j.

One of the (r,s,b)×(p,q) bivariate rational transfer function model (1.1) fundamental hypothesis establishes the non existence of correlation between the input series and its corresponding residual series, that is, the cross-correlation function between these two series is always null,

$$\rho_{\alpha_t a_t}(k) = \frac{\operatorname{cov}(\alpha_t, a_{t+k})}{\left(\sigma_\alpha^2 . \sigma_a^2\right)^{\frac{1}{2}}} = 0 \ , \ \forall t,k.$$

Under these circunstances and considering (2.2),

$$r_{Y_{p+r,t}^{s}, \alpha_t}(k) \doteq 0 \ , \ \forall t,k \ , \tag{2.6}$$

where $r_{Y_{p+r,t}^{s}, \alpha_t}(k)$ denotes the sample cross-correlation at lag k.

Lemma 2: Under the conditions of Lemma 1 and (2.6),

$$r_{Y_{m,t}^{s'(j)} \alpha}(k) \doteq 0 \ , \forall k \ \text{ for } s' = s \ , \ m = p+r \text{ and } j \geq \max(p+s, r+q). \tag{2.7}$$

3. A MODEL SPECIFICATION METHOD

Transfer function model identification refers to the methodology to identify the values of the delay parameter b, the orders r and s associated to the output and the input polynomials and the ARMA orders, p and q, for the residuals series in (1.1).

Assume that the b, p and q values are identified by the traditional methods and based on the theoretical results exposed in Section 2, the orders r and s are specified evaluating, for each jth iterated regression, as described in (2.4), for $m \geq p$ and $j \geq \max(p,q)$ (minimum regression order and minimum iteration that verifies (2.5)), the GESACF values,

$$r_{Y^{s'(j)}_{m,t}}(k) = \frac{\sum_{t=t_1}^{n-k} Y^{s'(j)}_{m,t}\, Y^{s'(j)}_{m,t+k}}{\sum_{t=t_1}^{n} \left[Y^{s'(j)}_{m,t}\right]^2} \quad \text{for} \quad k = q, q+1, \ldots,$$

with $s' = 0,1,\ldots$, $m = p, p+1, \ldots$ and $t_1 = \max(m, b+p+s') + 1$, which are disposed in tables as shown in Table 1.

Table 1. The GESACF Table: $r_{Y^{s'(j)}_{m,t}}(k)$.

p + s´		**p**				**p + 1**				**...**
m	j^k	q	q +1	q + 2	...	q	q + 1	q + 2	...	...
p	0	$r_{Y^{0(0)}_{p,t}}(q)$	$r_{Y^{0(0)}_{p,t}}(q+1)$	$r_{Y^{0(0)}_{p,t}}(q+2)$	...	$r_{Y^{1(0)}_{p,t}}(q)$	$r_{Y^{1(0)}_{p,t}}(q+1)$	$r_{Y^{1(0)}_{p,t}}(q+2)$	...	...
	1	$r_{Y^{0(1)}_{p,t}}(q)$	$r_{Y^{0(1)}_{p,t}}(q+1)$	$r_{Y^{0(1)}_{p,t}}(q+2)$	...	$r_{Y^{1(1)}_{p,t}}(q)$	$r_{Y^{1(1)}_{p,t}}(q+1)$	$r_{Y^{1(1)}_{p,t}}(q+2)$	...	
	2	$r_{Y^{0(2)}_{p,t}}(q)$	$r_{Y^{0(2)}_{p,t}}(q+1)$	$r_{Y^{0(2)}_{p,t}}(q+2)$	...	...	...	...	...	
	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋱	
p + 1	0	$r_{Y^{0(0)}_{p+1,t}}(q)$	$r_{Y^{0(0)}_{p+1,t}}(q+1)$	$r_{Y^{0(0)}_{p+1,t}}(q+2)$	...	$r_{Y^{0(0)}_{p+1,t}}(q)$	$r_{Y^{0(0)}_{p+1,t}}(q+1)$	$r_{Y^{0(0)}_{p+1,t}}(q+2)$	...	...
	1	...	...	...	...	...	...	...	...	
	2	...	...	...	...	...	...	...	...	
	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋱	
⋮	⋮	⋮				⋮				⋱

Therefore, by (2.5) and for any previously identified p and q, the orders r and s can be determined as follows:

(a) If statistically null elements, i.e., $\left| r_{Y^{s'(j)}_{m,t}}(k) \right| \leq 1.96 \cdot \hat{\sigma}\left(r_{Y^{s'(j)}_{m,t}}(k) \right)$ according to the Bartlett's test, appear after lag k*, in the j*th line and following of the respective table, for some values of s* and m*, that is,

$$r_{Y^{s^*(j)}_{m^*,t}}(k) \doteq 0 \quad \text{for} \quad k > k^* \quad \text{and} \quad j \geq j^*,$$

it is assumed that s = s* and r + q = k*, that is, r = k* - q (Table 2).

b) After the values of *r* and *s* are found, it is necessary to check if the convergence condition in the Theorem holds, i.e., if the m* and j* values are such that m* = p + r and j* = max(p + s, r + q). If the convergence condition is satisfied for the identified model, we stop the process. Otherwise, the values of m and/or s' are increased and the whole identification procedure is repeated.

Table 2. The GESACF Table for a (r, s, b) × (p,q) model: $r_{Y^{s^*(j)}_{m^*,t}}(k) \doteq 0$ for $k > k^*$ and $j \geq j^*$ with $k^* = r + q$.

p + s´		...	p + s*									...
m	j^k	...	q	q +1	...	k*	k* + 1	k* + 2	k* + 3	k* + 4	...	...
...	...	...				...						
	0					⋮						
	1					⋮						
	⋮					⋮						
m	**j***	...	...	...	...	×	0	0	0	0	...	
	j* + 1					×	0	0	0	0	...	
	j* + 2					×	0	0	0	0	...	
	j* + 3					×	0	0	0	0	...	
	j* + 4					×	0	0	0	0	...	
	⋮					⋮	⋮	⋮	⋮	⋮		
...	...	...				...						...

(c) After a model has been specified, the non existence of correlation between the $Y^{s^*(j)}_{m^*,t}$ series, for $j \geq j^*$, and the α_t input series must be verified, according to (2.7). For practical purposes, the sample cross-correlation function is statistically not significant if

$$\left| r_{Y^{s^*(j)}_{m^*,t}\alpha_t}(k) \right| = \left| \frac{\sum_{t=t_1}^{n-k} Y^{s^*(j)}_{m^*,t}\,\alpha_{t+k}}{\left[\sum_{t=t_1}^{n} \left[Y^{s^*(j)}_{m^*,t} \right]^2 \cdot \sum_{t=t_1}^{n} \alpha_t^2 \right]^{\frac{1}{2}}} \right| \leq \frac{1.96}{\sqrt{(n - t_1 + 1) - k}}, \ \forall k \quad \text{for } j \geq j^*.$$

4. SIMULATION STUDY[3]

To test the bivariate transfer function model identification methodology based on the GESACF, a simulation study was conducted by generating, for each considered model, 1000 sets of series β_t and α_t of 300 observations according to the (r,s,b) × (p,q) rational transfer function model (1.1). The uncorrelated white noises α_t and α_t, used in this model, were generated and confirmed by Portmanteau's tests to the samples autocorrelation functions of α_t and α_t and to the sample cross-correlation function between these two series. Based on each set of series β_t and α_t we evaluated the iterated least squares estimates and the corresponding values of the GESACF. Then the proposed methodology was used to identify the original model.

To test the GESACF's "cutting-off" property we considered two criteria: the Bartlett´s test as referred earlier and the Portmanteau's test, at 5% significance level, to MA models, defined from the following known result, if

$$Y^{s'(j)}_{m,t} \dot{\sim} \text{ MA}(r+q) \quad \text{then} \quad \frac{n_{Y^{s'(j)}_{m,t}}}{1 + 2\sum_{k=1}^{r+q} r^2_{Y^{s'(j)}_{m,t}}(k)} \sum_{k=r+q+1}^{k'} r^2_{Y^{s'(j)}_{m,t}}(k) \dot{\sim} \chi^2_{k'-r-q},$$

where $n_{Y^{s'(j)}_{m,t}}$ denotes the size of the $Y^{s'(j)}_{m,t}$ series.

If at least one of the two criteria is satisfied we take the GESACF as having the "cutting-off" property.

[3]All programs have been developed in S-Plus Version 3.3 for Windows.

For the considered (r,s,b) × (p,q) rational transfer function models and the corresponding simulated series the rates of identified models from the GESACF methodology are presented in Table 3.

Table 3. Rate of identified generated (r, s ,b) × (p, q) rational transfer function models from the GESACF.

MODEL	PARAMETERS									SUCESSES (%)
	δ_1	δ_2	ω_0	ω_1	ω_2	ϕ_1	ϕ_2	θ_1	θ_2	
(0,1,3) × (0,1)			1,28	0,22				- 0,12		71,5
			- 1,24	- 0,74				- 0,59		92,6
			0,75	- 0,34				0,22		94,6
(0,2,3) × (1,0)			1,43	- 0,60	0,49	0,29				4,4
			- 0,60	- 0,75	- 0,25	- 0,65				7,8
			0,79	0,10	- 0,28	0,36				3,5
(1,0,3) × (0,2)	0,57		0,84					- 0,89	- 0,22	65,1
	0,65		- 0,66					- 0,15	0,25	69,0
	- 0,56		1,42					0,12	0,31	76,8
(2,0,3) × (1,2)	- 0,16	0,24	- 0,72			0,56		- 0,49	- 0,42	28,7
	0,81	- 0,42	- 1,15			0,43		- 1,28	- 0,42	40,5
	- 0,23	- 0,44	- 0,52			- 0,57		- 0,50	- 0,22	34,3
(1,1,3) × (2,0)	- 0,25		0,66	0,25		0,71	- 0,42			7,1
	0,56		0,88	0,35		- 0,12	0,30			12,2
	- 0,29		- 0,75	- 0,38		- 0,65	- 0,38			7,0
(1,2,3) × (1,1)	0,52		1,38	0,95	0,43	- 0,62		- 0,42		43,9
	0,31		- 1,14	0,33	- 0,19	0,65		0,49		35,7
	0,60		- 1,36	- 0,14	0,50	- 0,21		- 0,36		39,2
(2,1,3) × (2,1)	0,60	- 0,42	- 0,90	0,33		0,42	- 0,33	- 0,45		24,2
	0,20	- 0,42	1,49	0,67		- 0,89	- 0,39	- 0,65		32,7
	0,13	- 0,37	0,71	- 0,26		0,44	- 0,42	0,62		32,3
(2,2,3) × (2,2)	- 0,80	- 0,42	1,31	- 0,52	0,38	1,14	- 0,33	- 0,76	- 0,25	26,7
	- 0,94	- 0,37	- 1,39	- 0,21	0,35	- 0,11	0,37	- 0,14	0,31	33,8
	0,41	- 0,42	- 1,34	- 1,26	- 0,54	0,49	- 0,29	- 0,12	0,26	29,5

To compare the exposed procedure performance with the ESACF methodology for the univariate ARMA model identification, introduced by Tsay and Tiao (1984), a similar simulation study was performed and for the used models were generated 1000 series of 300 observations each according to the ARMA(p, q) process, $\phi_p(B)\,X_t = \theta_q(B)\,a_t$. The proportions of correctly specified generated models from the ESACF procedure are given in Table 4.

Table 4. Rate of identified simulated ARMA models from the ESACF.

ARMA	PARAMETERS				SUCCESSES (%)
	ϕ_1	ϕ_1	θ_1	θ_2	
(0,1)			- 0,42		68,2
			0,53		67,5
			- 0,26		72,3
(0,2)			- 0,81	- 0,25	41,7
			0,19	- 0,31	75,7
			0,34	0,12	34,0
(1,0)	0,62				32,5
	- 0,34				13,4
	0,27				29,6
(2,0)	- 0,11	0,35			5,4
	0,61	-0,15			2,0
	0,80	-0,30			6,4
(1,1)	0,60		- 0,35		34,8
	- 0,41		0,19		18,6
	0,31		0,57		31,3
(1,2)	0,10		- 0,30	- 0,20	36,0
	- 0,60		- 0,10	0,20	56,0
	- 0,40		0,25	0,15	56,4
(2,1)	- 0,23	0,17	0,41		27,9
	- 0,92	-0,37	- 0,26		15,2
	0,58	-0,39	0,33		13,0
(2,2)	- 0,80	- 0,25	0,30	0,10	17,1
	- 0,11	0,35	0,19	- 0,31	13,7
	-1,20	- 0,40	- 0,18	- 0,29	26,4

These simulations results show that, in a general way, the success percentages obtained to the bivariate transfer function model identification methodology based on the GESACF are at the same level to those attained by the ESACF procedure to the univariate ARMA model specification. This fact confirms that the proposed methodology constitute a valid proposal to overtake the (r,s,b) $\times$ (p,q) rational transfer function model identification problem. It is also interesting to notice the existent parallelism between the several models behaviour relatively to both procedures, such as, the best performance of the (r,s,b)$\times$(0,q) models and the MA processes in opposition to the (r,s,b)$\times$(p,0) models and the AR processes, as well the instability presented by the bivariate transfer function models and the ARMA processes relatively to the parameters.

REFERENCES

BOX, G.E.P.; G.M. JENKINS and G.C. REINSEL (1994): **Time Series Analysis: Forecasting and Control,** Third Edition, Prentice Hall, New York.

BROCKWELL, P.J. and R.A. DAVIS (1995): **Time Series: Theory and Methods**, Second Edition, Springer-Verlag, Berlin.

JOHNSTON, J. (1984): **Econometrics Methods**, Third Edition, McGraw Hill, New York.

MÜLLER, D. and W.W.S. WEI (1997): "Iterative Least Squares and Estimation and Identification of the Transfer Function Model", **J. of Time Series Analysis** 18(6), 579-592.

MURTEIRA, B.J.F.; D. MÜLLER and K.F. TURKMAN (1993): **Análise de Sucessões Cronológicas**, McGraw Hill, New York.

OLIVEIRA, C. and D. MÜLLER (1998): "Função de Autocorrelação Estendida Generalizada: contributo para a identificação dos Modelos de Função Transferencia", **Working paper** 5, *CEMAPRE/ISEG/UTL.*

____________________________ (2000): "Generalized Extended Sample Autocorrelation Function: contribution to the identification of the Transfer Function Models", **Working paper** 12, *CEMAPRE/ISEG/UTL*, submitted to the **Journal of Time Series Analysis** in 1999.

OLIVEIRA, C. (2001): "Função de Autocorrelação Estendida Generalizada: contributo para a identificação dos Modelos de Função Transferencia", PhD Thesis, ISEG, Technical University of Lisbon, Portugal.

TSAY, R.S. and G.C. TIAO (1984): "Consistent Estimates of Autoregressive Parameters and Extended Sample Autocorrelation Function for Stationary and Nonstationary ARMA Models", **J. of Amer. Statist. Assoc***.* 79(385), 84-96.

WEI, W.W.S. (1990): Time Series Analysis: Univariate and Multivariate Methods, Addison Wesley, New York.