AN $\in$-MINIMUM PRINCIPLE FOR MULTIOBJECTIVE STOCHASTIC OPTIMAL CONTROL PROBLEMS

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ABSTRACT
In this paper we consider a multiobjective stochastic control problem and derive necessary conditions for approximate solutions of the control problem using a multicriteria variational principle of Ekeland's type. The restrictions in the multiobjective stochastic control problem are formulated by dynamical equations. The solution of this dynamical equations can be obtained applying the Girsanov measure transformation. Furthermore, the objective functions are terminal costs g_i (x(1)) for which we consider the expected value of control u, i.e., E_u [g_i(x(1))] = F_i (u) (i = 1,...,l), where E_u denotes the expectation constructed from control u.

Key words: Martingales; variational principle, multicriteria

MSC: 34H05

RESUMEN
En este trabajo consideramos un problema multiobjetivo estocástico y derivamos condiciones necesarias para aproximar las soluciones del problema de control usando un principio variacional multicriterio del tipo Ekeland. Las restricciones del problema de control estocástico multiobjetivo son formuladas mediante ecuaciones dinámicas. La solución de estas ecuaciones dinámicas son obtenidas aplicando la medida transformada de Girsanov. Además las funciones objetivo son costos terminales g_i (x(1)) para los que consideramos el valor esperado del control u, i.e., E_u((g_i (x(1)) = F_i (u) (i = 1,...,l), donde E_u denota la esperanza construida desde el control u.

Palabras clave: Martingales; principio variacional, multicriterio

1. A MODEL OF A PARTIALLY OBSERVED STOCHASTIC CONTROL PROBLEM

Let w_t be a Brownian motion on a probability space (Ω, *A*, μ) taking values in R^m.

The σ-algebra of measurable states depending on t is denoted by $F_t = \sigma$ (x_s : s $\le$ t). In the following we assume

(C1) σ is an m x m–matrix–valued mapping $\sigma = (\sigma_{ij})$ defined on [0,1] x C with C = C [0,1], σ (t,x) is nonsingular, for $1 \le i, j \le m$, σ_{ij} (t, x) is F_t – measurable in its second argument and Lebesgue – measurable in its first; each σ_{ij} satisfies a uniform Lipschitz condition in x

$$\|x\|_s = \sup_{0\le t\le s}|x(t)| \,;$$

there is a constant $k_0 < \infty$ such that

$$\sum \int_0^1 \sigma_{ij}^2 dt \le k_0 \quad \text{a.s.P.}$$

Moreover, we suppose that

(C2) f : $[0,1]$ x C x U $\to R^m$ is measurable, continuous in its third component and f^u(t,x): = f(t,x,u(t,x)) is causal for every u $\in$U.

Consider the stochastic differential equation

$$\left.\begin{aligned} dx_t &= f(t,x,u)dt + \sigma(t,x)dw_t, \\ x(0) &= x_0 \in R^m, \end{aligned}\right\} \tag{1}$$

$$0 \le t \le 1,$$

where x_t splits into an observed component $y_t \in R^n$ and an unobserved component $z_t \in R^{m-n}$.

Furthermore, consider the observation σ - field Y_t generated by $Y_t := \{y_s : s \le t\}$.

Definition 1 An admissible partially observable feedback control is defined to be a Y_t – predictable mapping $u : [0,1] \times C \to U$, where U is a Borel subset of R, such that $E\,|\,u(\tau,.)\,| < \infty$.

The set of such controls is denoted by V. We define on V for $u_1, u_2 \in V$

$$d(u_1,u_2) = \tilde{P}(\{(t,x) : |u_1(t,x) - u_2(t,x)| > 0\}), \tag{2}$$

where $\tilde{P}$ is the product measure of λ and P(λ is the Lebesgue measure on $[0,1]$ and P is the probability measure on $C[0,1]$ induced by the Brownian motion).

We will see in Section 4 that (V,d) is a complete metric space. (cf. Elliott and Kohlmann (1980))

We recall that a mapping Φ: $[0,1] \times C \to R^m$ is called causal if Φ is optional, and

$$\left|\sigma^{-1}(t,x)\Phi(t,x)\right| \le M(1 + \|x\|_t) .$$

Under these assumptions it is possible to apply the *Girsanov theorem* to construct a probability measure which is absolutely continuous with respect to P and a given process is a Wiener Process with respect to the new probability measure.

In order to formulate the multicriteria stochastic control problem we introduce the multiobjective function

$$J(u) := \begin{pmatrix} E_u\left[g_1(x(1))\right] \\ \cdots \\ E_u\left[g_l(x(1))\right] \end{pmatrix},$$

where E_u denotes the expectation (constructed from the control u) with respect to P_u,g_i are bounded F_1 – measurable functions, where P_u is the probability measure on (C,F_1) defined by

$$P_u(A) = \int_A \exp\left\{\int_0^1 f(t,x,u(t,x))(\sigma(t,x)\sigma'(t,x))^{-1}dx_t \; - \frac{1}{2}\int_0^1 f(t,x,u(t,x))(\sigma(t,x)\sigma'(t,x))^{-1} f(t,x,u(t,x))dt\right\} dP \;=: \int_A p_0^1(u)dP$$

In the following we denote the topological interior of a set $C \subset R^l$ by int C, the topological boundary of C by bd C and the topological closure of C by cl C.

We suppose

(C3) $K \subset R^l$ is a convex cone with int $K \neq \emptyset$, $B \subset R^l$ is a pointed, convex cone with int $B \neq \emptyset$ such that cl B +(K \ {0}) $\subset$ int B.

Now, we formulate the **multicriteria stochastic control problem** under the assumptions (C1) - (C3).

(P_C): Compute a feasible control $\bar{u}$ such that

$$J(u) \notin J(\bar{u}) - (K \setminus \{0\})$$

for all admissible controls u.

In this way we study an extension of the stochastic control problem introduced by Elliott and Kohlmann (1980).

The aim of our paper is to derive necessary conditions for approximate solutions of multicriteria stochastic control problems (P_c) using vector-valued variational principles given by Isac (1996) and Tammer (1992).

2. VARIATIONAL PRINCIPLES FOR MULTICRITERIA OPTIMIZATION PROBLEMS

In this section we recall several variationl principles given by Tammer (1992) for multicriteria optimization problems which are important for the proof of our main result in Section 5.

In Section 2 we formulated the multicriteria stochastic control problem (P_c) with an objective function which takes its values in the L-dimensional Euclidean space R^l.

In the following we introduce general concepts for optimal and suboptimal solutions of multicriteria optimization problems corresponding to the solution concept in the formulation of problem (P_c) and variational principles for such problems. Let us assume:

(A1): (V,d) is a complete metric space,
$K \subset R^l$ is a convex cone with $k^0 \in \text{int } K$,
$B \subset R^l$ is a pointed convex cone with $\text{int } B \neq \emptyset$ such that
$\text{cl } B + (K \setminus \{0\}) \subset \text{int } B$

(A2): $F : V \to R^l$
is lower semicontinuous with respect to k^0 and B in the following sense
$M_r = \{ v \in V \mid F(v) \in rk^0 - \text{cl } B \}$ is closed for each $r \in R$
and bounded from below, i.e., $F[V] \subset y + B$ for an element $y \in R^l$.

Now we consider the following vector optimization problem to determine the efficient point set of F [V] with respect to K:

(P): Compute the set Eff(F[V], K),

where

$$\text{Eff}(F[V],K) = \{F(\bar{v}) \mid \bar{v} \in V \text{ and } F[V] \cap (F(\bar{v}) - (K \setminus \{0\})) = \emptyset\}.$$

Furthermore, we will introduce approximately efficient elements of vector optimization problems.

The reason for introducing approximately efficient solutions is the fact that numerical algorithms usually generate only approximative solutions anyhow and moreover that the set of efficient points may be empty in the general noncompact case, whereas approximately efficient points always exist under very weak assumptions (see Tammer (1992), where existence results for approximate solutions of a vector optimization problem were shown, especially under the assumption that the objective function is bounded from below).

Definition 2 An element $F(v_\epsilon) \in F[V]$ is called an approximately efficient point of F[V] with respect to K, $k^0 \in \text{int } K$ and $\epsilon > 0$, if

$$F[V] \cap (F(v_\epsilon) - \epsilon k^0 - (K \setminus \{0\})) = \emptyset.$$

The approximately efficient point set of F [V] with respect to K, k^0 and ϵ is denoted by $\mathrm{Eff}(F[V],\ K_{\epsilon k^0})$, where $K_{\epsilon k^0} := \epsilon\, k^0 + K$.

Moreover, we will study approximately efficient elements with respect to the cone *B* from assumption (A1) instead of *K*.

Theorem 1 [Tammer (1996)] Let the assumptions (A1) and (A2) be fulfilled.

Then there exists for each $\epsilon > 0$, $\lambda > 0$ and for each $F(v_0) \in \mathrm{Eff}(F[V],\ B_{\epsilon k^0})$ an element $v_\epsilon \in V$ such that the following conditions hold

1. $F(v_\epsilon) \in F(v_0) - \lambda d(v_0,v_\epsilon)k^0 - \mathrm{cl}\ B$ and $F(v_\epsilon) \in \mathrm{Eff}(F[V],\ D_{\epsilon k^0})$, where D is an open subset of R^l with $K \setminus \{0\} \subset D$, $0 \in \mathrm{bd}\ D$ and $\mathrm{cl}\ D + (K \setminus \{0\}) \subset D$,

2. $d(v_0,v_\epsilon) \le \epsilon / \lambda$,

3. $F_{\lambda k^0}(v_\epsilon) \in \mathrm{Eff}(F_{\lambda k^0}[V], K)$,

where $F_{\lambda k^0}(v) := F(v) + \lambda d(v,v_\epsilon)\, k^0$.

The following theorem follows immediately from Theorem 1 regarding the fact that under the given assumptions there always exists an approximately efficient element $v_0 \in V$ with

$$F(v_0) \in \mathrm{Eff}(F[V],\ B_{\epsilon k^0})$$

(cf. Tammer [1992].

Theorem 2 Assume (A1), (A2).

Then for every $\epsilon > 0$ there exists some point $v_\epsilon \in V$ such that

1. $F[V] \cap (F(v_\epsilon) - \epsilon k^0 - (K \setminus \{0\})) = \emptyset$,

2. $F_{\epsilon k0}[V] \cap (F_{\epsilon k0}(v_\epsilon) - (K \setminus \{0\})) = \emptyset$,

where

$F_{\epsilon k0}(v) := F(v) + d(v,v_\epsilon) \in k^0$.

Remark 1: Theorem 1 is slightly stronger than Theorem 2. The main difference concerns condition 2. in Theorem 1, which gives the whereabouts of point v_ϵ in *V*.

Remark 2: The main result of the last theorems (statement 3 Theorem 1 and statement 2 in Theorem 2) says that v_ϵ is an efficient solution of a slightly perturbed vector optimization problem. This statement can be used in order to derive necessary conditions for approximately efficient elements. In our papers Tammer (1993a, 1993b) we have shown ϵ - Kolmogorov – conditions for approximately efficient solutions of abstract approximation problems applying the third condition in Theorem 1. In the next chapter we will use Theorem 2 in order to derive an ϵ - minimum-principle in the sense of Pontrjagin for suboptimal solutions of multicriteria stochastic control problems.

3. APPLICATION OF THE VARIATIONAL PRINCIPLE TO MULTICRITERIA STOCHASTIC CONTROL PROBLEMS

In this section we assume that for the multicriteria stochastic control problem (P_c) the assumptions of Theorem 2 are fulfilled.

Consider the space V of all partially observable admissible controls and the distance d on V introduced by (2). Then it holds

Lemma 1 (cf. Elliott and Kohlmann, 1989)

(V,d) is a complete metric space.

Furthermore, we introduce a vector-valued mapping F associated with the multicriteria control problem (Pc) for which the assumptions of the variational principle in Theorem 1 are fulfilled.

Lemma 2 (cf. Elliott and Kohlmann, 1989)

Suppose (C1) – (C3). Then the mapping $F : (V,d) \to (R^l, \|.\|_R^l)$ defined by

$$F(u) := \begin{pmatrix} E_u[g_1(x(1))] \\ \cdots \\ E_u[g_l(x(1))] \end{pmatrix},$$

is continuous.

Remark 3: Lemmata 1 and 2 show together with assumption (C3) that the assumptions (A1), (A2) of Theorem 2 are fulfilled for the multicriteria stochastic control problem (Pc).

4. AN $\in$-MINIMUM PRINCIPLE FOR A MULTICRITERIA STOCHASTIC CONTROL PROBLEM

Theorem 3 *Assume* $(C1) - (C3)$.

For any $\in > 0$ there exists an F_t-predictable process $\gamma_\in$, such that for any $t \in [0,1]$, any $A \in [0,1]$ and any admissible control $u \in V$ the following statements are true

1. For the control $u_\in \in V$ it holds

$$\begin{pmatrix} E_u[g_1(x(1))] \\ \cdots \\ E_u[g_l(x(1))] \end{pmatrix} \notin \begin{pmatrix} E_{u\in}[g_1(x(1))] \\ \cdots \\ E_{u\in}[g_l(x(1))] \end{pmatrix} - \in k^0 - (K \setminus \{0\}).$$

2. For $\tau > 0$ we get the following assertion

$$\begin{pmatrix} \int_t^{t+\tau}\int_A p_0^t(u_\in) p_t^{t+\tau}(u) \gamma_{1\in} \sigma^{-1}(f_s^u - f_s^{u_\in}) dP\, ds \\ \cdots \\ \int_t^{t+\tau}\int_A p_0^t(u_\in) p_t^{t+\tau}(u) \gamma_{l\in} \sigma^{-1}(f_s^u - f_s^{u_\in}) dP\, ds \end{pmatrix} \notin - \in k^0 \tau P(A) - (K \setminus \{0\}),$$

where $f_s^u := f(s,x(s), u(s,x))$.

Proof: We consider the vector-valued function $F: u \to J(u)$ and the space V of admissible controls $u: [0,T] \to U$.

From Lemma 1 we get that (V,d) is a complete metric space. Lemma 2 yields that F is lower semicontinuous with respect to k^0 and B and bounded from below on V. So we can conclude that the assumptions of Theorem 2 are fulfilled and we can apply Theorem 2 to the vector-valued function F. This yields an admissible control $u_\in \in V$ such that

(i) $F(u_\in) \in \mathrm{Eff}(F[V], K_{\in k0})$,

(ii) $F(u_\in) \in \mathrm{Eff}(F_{\in k0}[V], K)$,

where

$$F_{\in k0}(u) := F(u) + \in k^0 d(u, u_\in).$$

So we derive from (i)

$$J(u) \notin J(u_\in) - \in k^0 - (K \setminus \{0\})$$

for all feasible controls u, i.e., statement 1 is fulfilled.

Furthermore, the families of conditional expectations

$$G_i^t = E_{u_\in}\left[g_i(x(1)) \,\middle|\, F_t\right] \quad i = 1, \ldots, l,$$

are martingales and so they have the representation

$$G_i^t = F_i(u_\in) + \int_0^t \gamma_{i\in} dw_\in \quad i = 1, \ldots, l,$$

where $w_\in$ is the process defined by

$$dw_\in = \sigma^{-1}(dx - f^{u_\in} dt),$$

such that we can conclude from Girsanov's Theorem that $w_\in$ is a Brownian motion under the measure $P_{u_\in}$.

In order to prove statement 2 we take $t \in [0,1]$, $A \in Y_t$ and $u \in V$ and define $v_\tau \in V$ for $\tau > 0$ by:

$$v_\tau(s,x) := \begin{cases} u(s,x) : & (s,x) \in (t, t+\tau] \times A \\ u_\in(s,x) : (s,x) \in [0,t] \times C \cup (t, t+\tau] \times A' \cup (t+\tau, 1] \times C, \end{cases}$$

where $A' = \Omega \setminus A$.

The indicator function of $B = (t, t + \tau] \times A$, denoted by I_B is a Y_t - predictable map. Regarding that v_τ can be written as $I_B u + I_{B'} u_\in$ it follows that v_τ is predictable and an admissible control in V.

Now, we apply the martingale representations given above for t = 1 and i = 1,..., l:

$$g_i(x(1)) = F_i(u_\in) + \int_0^1 \gamma_{i\in} dw_\in.$$

So we get

$$E_{v_\tau}[g_i] = F_i(v_\tau) = F_i(u_\in) + E_{v\tau}\left[I_A \int_t^{t+\tau} \gamma_{i\in} \sigma^{-1}\left(f^u - f^{u_\in}\right) ds \right].$$

Then we may conclude from statement 2. in Theorem 2 $F(u_\in) \in E\,f\,f(F_{\in k^0}[V], K)$, such that

$$F(v_\tau) + \in k^0 d(u_\tau, u_\in) \notin F(u_\in) - (K \setminus \{0\}).$$

Regarding

$$d(v_\tau,u_\in) \leq \tau P(A)$$

it follows

$$F(v_\tau) + \in k^0 \tau P(A) \notin F(u_\in) - (K \setminus \{0\}).$$

Together with the definition of $P_{u\tau}$ and the properties given above we derive

$$\begin{pmatrix} \int_t^{t+\tau}\int_A p_0^t(u_\in)p_t^{t+\tau}(u)\gamma_{1\in}\sigma^{-1}(f_s^u - f_s^{u\in})dP\,ds \\ \cdots \\ \int_t^{t+\tau}\int_A p_0^t(u_\in)p_t^{t+\tau}(u)\gamma_{l\in}\sigma^{-1}(f_s^u - f_s^{u\in})dP\,ds \end{pmatrix} \notin -\in k^0\,\tau\,P(A) - (K \setminus \{0\}).$$

Using the martingal representation results given above we derive the following necessary condition which the approximate solution $u_\in$ must satisfy. This is a condition of the following kind: $u_\in$ must be a $\in k^0$ – weakly efficient element of the conditional expectation of a certain Hamiltonian.

Theorem 4 Consider the stochastic control problem (Pc) under the assumptions (C1) – (C3).

Then for each $\in > 0$ there exists an admissible control $u_\in$, such that

1. $$\begin{pmatrix} E_u[g_1(x(1))] \\ \cdots \\ E_u[g_l(x(1))] \end{pmatrix} \notin \begin{pmatrix} E_{u_\in}[g_1(x(1))] \\ \cdots \\ E_{u_\in}[g_l(x(1))] \end{pmatrix} - \in k^0 - (K \setminus \{0\})$$

for all feasible controls $u \in V$,

2. $$\begin{pmatrix} E[p_{1\in}\sigma^{-1}f_t^u|Y_t] \\ \cdots \\ E[p_{l\in}\sigma^{-1}f_t^u|Y_t] \end{pmatrix} \notin \begin{pmatrix} E[p_{1\in}\sigma^{-1}f_t^{u\in}|\gamma_t] \\ \cdots \\ E[p_{l\in}\sigma^{-1}f_t^{u\in}|\gamma_t] \end{pmatrix} - \in k^0 \text{-int } K$$

for all $u \in V$ and the F_t – predictable process

$$p_\in = \begin{pmatrix} p_0^t(u_\in)\gamma_{1\in} \\ \cdots \\ p_0^t(u_\in)\gamma_{l\in} \end{pmatrix}.$$

Proof: We will differentiate the left hand side in the vector-valued inequality of statement 2. in Theorem 3. So we derive for $(\tau > 0)$

$$\frac{1}{\tau}\begin{pmatrix} \int_t^{t+\tau}\int_A p_0^t(u_\in)p_t^{t+\tau}(u)\gamma_{1\in}\sigma^{-1}(f_s^u - f_s^{u\in})dPds \\ \cdots \\ \int_t^{t+\tau}\int_A p_0^t(u_\in)p_t^{t+\tau}(u)\gamma_{l\in}\sigma^{-1}(f_s^u - f_s^{u\in})dPds \end{pmatrix} \notin -\in k^0 P(A) - \frac{1}{\tau}(K \setminus \{0\}).$$

This yields regarding that K is a cone

$$\lim_{\tau\to 0}\frac{1}{\tau}\begin{pmatrix}\int_t^{t+\tau}\int_A p_0^t(u_\in)p_t^{t+\tau}(u)\gamma_{1\in}\sigma^{-1}(f_s^u - f_s^{u\in})dP\,ds\\ \cdots\\ \int_t^{t+\tau}\int_A p_0^t(u_\in)p_t^{t+\tau}(u)\gamma_{l\in}\sigma^{-1}(f_s^u - f_s^{u\in})dP\,ds\end{pmatrix}\notin -\in k^0P(A)-\operatorname{int}K.$$

Now, we compute the left hand side of this variational inequality. Y_t is countably generated for any rational number r, $0 \le r \le 1$ by sets $\{A_{nr}\}$, n = 1, 2,..., since the trajectories are continuous, almost surely.

Furthermore, u_{nr} can be considered as an admissible control over the time interval $(t,t+\tau]$ for $t \ge r$ and we can consider a perturbation of $u_\in$ by u_{nr} for $t \ge r$ and $x \in A \in Y_t$, as in the above section. Under the given assumptions the following limit exists and it holds

$$\lim_{\tau\to 0}\frac{1}{\tau}\begin{pmatrix}\int_t^{t+\tau}\int_{Anr} p_0^{t+\tau}(u_\in)\gamma_{1\in}\sigma^{-1}(f(s,x,u_{mr})-f(s,x,u_\in))dP\,ds\\ \cdots\\ \int_t^{t+\tau}\int_{Anr} p_0^{t+\tau}(u_\in)\gamma_{l\in}\sigma^{-1}(f(s,x,u_{mr})-f(s,x,u_\in))dP\,ds\end{pmatrix}=\begin{pmatrix}\int_{Anr} p_0^t(u_\in)\gamma_{1\in}\sigma^{-1}(f(s,x,u_{mr})-f(s,x,u_\in))dP\\ \cdots\\ \int_{Anr} p_0^t(u_\in)\gamma_{l\in}\sigma^{-1}(f(s,x,u_{mr})-f(s,x,u_\in))dP\end{pmatrix}$$

for almost all $t \in [0,1]$, i.e., there is a set $T_1 \subset [0,1]$ of zero measure, such that the equation given above is true for $t \notin T_1$ and all n,m,r.

Moreover, there is a set $T_2 \subset [0,1]$ of zero measure, such that if $t \notin T_2$

$$\lim_{\tau\to 0}\frac{1}{\tau}\begin{pmatrix}\int_t^{t+\tau}E_{u_\in}(\gamma_{1\in}^2)ds\\ \cdots\\ \int_t^{t+\tau}E_{u_\in}(\gamma_{l\in}^2)ds\end{pmatrix}=\begin{pmatrix}E_{u_\in}(\gamma_{1u_\in}^2)ds\\ \cdots\\ E_{u_\in}(\gamma_{lu_\in}^2)ds\end{pmatrix}$$

Then we can conclude applying Lemma 5.1 in Elliott and Kohlmann [4]

$$\lim_{\tau\to 0}\frac{1}{\tau}\begin{pmatrix}\int_t^{t+\tau}\int_{A_{nr}} p_0^t(u_\in)p_t^{t+\tau}(u_{mr})\gamma_{1\in}\sigma^{-1}(f(s,x,u_{mr})-f(s,x,u_\in))dPds\\ \cdots\\ \int_t^{t+\tau}\int_{A_{nr}} p_0^t(u_\in)p_t^{t+\tau}(u_{mr})\gamma_{l\in}\sigma^{-1}(f(s,x,u_{mr})-f(s,x,u_\in)dP\,ds\end{pmatrix}=\begin{pmatrix}\int_{Anr} p_0^t(u_\in)\gamma_{1\in}\sigma^{-1}(f(s,x,u_{mr})-f(s,x,u_\in))dP\\ \cdots\\ \int_{Anr} p_0^t(u_\in)\gamma_{l\in}\sigma^{-1}(f(s,x,u_{mr})-f(s,x,u_\in))dP\end{pmatrix}$$

for $t \notin T_1 \cup T_2$, all $r \le t$ and all n, m.

Finally, this implies that

$$\begin{pmatrix}E\left[p_{1\in}\sigma^{-1}f_t^u\middle|Y_t\right]\\ \cdots\\ E\left[p_{l\in}\sigma^{-1}f_t^u\middle|Y_t\right]\end{pmatrix}\notin\begin{pmatrix}E\left[p_{1\in}\sigma^{-1}f_t^{u\in}\middle|Y_t\right]\\ \cdots\\ E\left[p_{l\in}\sigma^{-1}f_t^{u\in}\middle|Y_t\right]\end{pmatrix}-\in k^0-\operatorname{int}K$$

for all $u \in V$ and a F_t – predictable process

$$p_{\in} = \begin{pmatrix} p_0^t(u_{\in})\gamma_{1\in} \\ \cdots \\ p_0^t(u_{\in})\gamma_{l\in} \end{pmatrix}.$$

Remark: Using martingale representation results and a variational principle for multicriteria optimization problems we obtain a necessary condition which $u_{\in}$ must satisfy.

In fact, almost surely $\tilde{P}$, $u_{\in}$ is a $\in$ - weakly minimal solution in the sense of multicriteria optimization for the conditional expectation of a certain Hamiltonian of the stochastic system. Here the expectation is taken with respect to the observed σ field.

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