RETRIAL QUEUEING SYSTEM WITH SEVERAL INPUT FLOWS

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ABSTRACT
We consider a single-server retrial queueing system with $K(K \geq 1)$ Poisson input flows. The service times have a common arbitrary distribution function $B_i(x)$ for customer of type i. An arriving customers of type i, $i = \overline{1,K}$, who finds the server free begins to get service inmediately and leaves the system after completion. Otherwise, if the server is busy, the customer with probability $1 - H_i$ leaves the system without service and with probability $H_i > 0$ joins an orbit of repeated customer but conserves its own type. The intervals separating two succesive repeated attempts of each customers from the orbit are exponentially distributed with rate γ. The orbit is finite or infinite. In case of a finite orbit an arriving customer who finds the server busy and the orbit completely full is lost. We derive the steady state probabilities of the multidimensional Markov process underlying the considered queueing system.

Key words: queueing system, several flows, repeated attempts, Markov process.

MSC: 60J25, 62J10

RESUMEN
Consideramos un sistema estable de colas con un solo servidor con $K(K \geq 1)$ flujos de entrada Poisson. Los tiempos de servicio tienen una función de distribución arbitraria $B_i(x)$ para usuarios del tipo i. Un usuario del tipo i, $i = \overline{1,K}$, que se incorpora y encuentra el servidor libre, para obtener servicio comienza a utilizarlo inmediatamente y abandona el sistema al obtenerlo. En otro caso, si el servidor está ocupado, el usuario abandona el sistema con probabilidad $1 - H_i$ sin obtener el servicio y con probabilidad $H_i > 0$ se une a una órbita de usuarios repetitivos pero conserva su propio tipo. Los intervalos separantes de dos intentos sucesivos de cada usuario de la órbita se distribuyen exponencialmente con tasa γ. La órbita puede ser finita o infinita. En caso de una órbita finita un usuario que arriba y encuentra el servidor ocupado y la órbita llena completamente se pierde. Nosotros derivamos las probabilidades de reintento del estado para el proceso multidimensional de Markov subyacente en el sistema de colas.

Palabras clave: sistemas de colas, flujos varios, esfuerzos repetidos, proceso de Markov.

1. INTRODUCTION

Queueing systems with repeated customers have wide use in the practice of designing of computer communications networks and telecommunication systems. There are many works considering different models of retrial single-server queueing systems with one Poisson input flow (see, for ex. Falin and Templeton (1997)). Nevertheless the single-server queueing systems with several independent Poisson input flows with retrials and without priorities were not investigated. For the case with no retrials the phenomenon of multitype Poisson input flow was investigated in Basharin (1965) for the $M_K/M_K/1/r$ queueing system with a buffer of capacity r, where r can be finite of infinite. In that work the steady state distribution of multidimensional queue lengths distribution was obtained. This result was extended in Bocharov (1985) for the $M_K/G_K/1/r$ queueing system with arbitrary service times distribution function $B_i(x)$ of customers of type i, $i = \overline{1,K}$.

The purpose of the present work is to study the multitype Poisson input flow phenomenon for a single-server queueing system without buffer for the primary customers arriving from outside but having an orbit for repeated customers where customers of different types are queueing separately. The maximum number of customers of all types waiting in the orbit to seek service again can be finite or infinite. The both cases are investigated.

It should be pointed out that the approach of Bocharov (1985) of the reduction of multidimensional limiting distribution of underlying linear Markov process to two-dimensional distribution, where the type of served customer and the total number of customers in the orbit are only marked, was developed for the queueing system under study.

2. DESCRIPTION OF THE QUEUEING SYSTEM

We consider a single-server queueing system without buffer with K ($K \geq 1$) independent Poisson flow of customers. The flow rate for the customers of type i (i-customers) in λ_i and their service times have a common arbitrary distribution function $B_i(x)$, $i = \overline{1,K}$. We assume that the distribution functions $B_i(x)$ are absolutely continuous and satisfy the conditions $B_i(0) = 0$ and $\int_0^\infty t dB_i(t) = 1/\mu_i < \infty, i = \overline{i,K}$.

Any i-customer who finds the server busy at the time of his arrival joins with probability H_i a retrial group called "orbit" in order to be service again and with probability H_i a retrial group called "orbit" in order to be serviced again and with probability 1 - H_i leaves the system without service. The intervals separating two successive repeated attempts of each customer from the orbit are exponentially distributed with rate γ. The orbit is finite or infinite. An i-customer who become retrial conserves own type, i.e.his service time distribution function maintains $B_i(x)$. A customer (primary or retrial) who finds the server free is served immediatly. We consider two cases where the maximum number of repeated customers of all types waiting in the orbit to seek service again, which we call orbit size is limited by $s(1 \leq s < \infty)$ or can be unlimited ($s = \infty$). In the case of finite orbit an arriving customer who finds the server busy and s customers of all types being in the orbit is lost.

The queueing system with retrials, non-persistent customers and finite orbit we shall codify as $M_K/G_K/1/0/s/NP$ and this one with infinite orbit as $M_K/G_K/1/0/\infty/NP$.

3. THE $M_K/G_K/1/0/S/NP$ RETRIAL QUEUEING SYSTEM WITH FINITE ORBIT

In this section we consider the case where the maximum number of customers in the orbit is limited by $s < \infty$.

Let us denote $\vec{n} = (n_1,...,n_K), n_j \geq 0$, and $n = n. = \sum_{j=1}^{K} n_j$; here and later the subscript "." stands for summation over all values of the corresponding discrete argument.

The stochastic behaviour of the considered queueing system can be described by a linear Markov process $\{\xi(t), t \geq 0\}$ with the state space

$$\chi = \left\{ 0(\vec{n}); (x,i,\vec{n}), i = \overline{1,K}, n_j \geq 0, x \geq 0, j = \overline{1,K}, n = \overline{0,s} \right\}.$$

The state $0(\vec{n})$ of the process $\xi(t)$ at some instant time t means that the server is idle and there are n_j j-customers waiting for their service in the orbits, $i = \overline{1,K}$; the state $(x,i,\vec{n})$ corresponds to the situation which the number of j-customers in the orbit is n_j, $j = \overline{1,K}$, and a time x has elapsed since the beginning of the service of the i-customer.

Under these assumptions (Bocharov, Pechinkin, Albores (1997)) the process $\{\xi(t), t \geq 0\}$ is ergodic, there exists a unique stationary distribution of this process and the stationary probability densities $p(x,i,\vec{n})$ of the states $(x,i,\vec{n})$ can be represented in the form

$$p(x,i,\vec{n}) = [1 - B_i(x)]q(x,i,\vec{n}),$$

where $q(x,i,\vec{n})$ are bounded functions.

Let us denote the stationary probability of the state $(0,\vec{n})$ by $p_0(\vec{n})$ and the stationary probability of the state $(x,i,\vec{n})$ with no regard for the elapsed service time by $p(i,\vec{n})$:

$$p(i,\vec{n}) = \int_0^\infty p(x,i,\vec{n})dx.$$

Using the result in Bocharov, Pechinkin, Albores (1997) we obatin the system of differential equations

$$\frac{d}{dx} q(x,i,\vec{n}) = -u(s-n)\lambda^* q(x,i,\vec{n}) + \sum_{j=1}^{K} u(n_j)\lambda_j^* q(x,i,\vec{n}-\vec{e}_j), \quad i=\overline{1,K}, n=\overline{0,s}, \tag{1}$$

$$(\lambda + n\gamma)p_0(\vec{n}) = \sum_{i=1}^{K}\int_0^\infty q(x,i,\vec{n})dB_i(x), n=\overline{0,s}, \tag{2}$$

$$q(0,i,\vec{n}) = (n_i+1)\gamma p_0(\vec{n}+\vec{e}_i) + \lambda_i p_0(\vec{n}), i=\overline{1,K}, n=\overline{0,s-1}, \tag{3}$$

$$q(0,i,\vec{n}) = \lambda_i p_0(\vec{n}), n=s, \tag{4}$$

where $\lambda = \sum_{i=1}^{K}\lambda_i$, $\lambda_j^* = \lambda_j H_j, j=\overline{1,K}$, $\lambda^* = \sum_{j=1}^{K}\lambda_j^*$, $u(x)$ is the unit Heaviside function and $\vec{e}_i$ is the vector of a suitable size and consists of zeros, except unit at the ith place.

We do not resolve these equations directly. For their solution we consider the following sets of states:

$$(0,n) = \bigcup_{n_1+\ldots+n_K=n} (0,\vec{n});$$

$$(x,i,n) = \bigcup_{n_1+\ldots+n_K=n} (x,i,\vec{n}); \ \ n=\overline{0,s},$$

and introduce the unknowns correspondings to these makrostates:

$$p_0(n) = \sum_{n_1+\ldots+n_K=n} p_0(\vec{n}), \ q(x,i,n) = \sum_{n_1+\ldots+n_K=n} q(x,i,\vec{n}),$$

$$p(x,i,n) = \sum_{n_1+\ldots+n_K=n} p(x,i,\vec{n}), \ p(i,n) = \sum_{n_1+\ldots+n_K=n} p(i,\vec{n}),$$

The following lemma shows that the unknown stationary probability distribution of the process $\{\xi(t), t \geq 0\}$ can be expressed in terms of the newly defined macrocharacteristics.

Lemma 1. For $i=\overline{1,K}$ we have the following relations:

$$q(x,i,\vec{n}) = q(x,i,j^n)M(\vec{n},j), \ x \geq 0, \ j=\overline{1,K}, n=\overline{0,s}, \tag{5}$$

$$p_0(\vec{n}) = p_0(j^n)M(\vec{n},j), \ j=\overline{1,K}, n=\overline{0,s}, \tag{6}$$

$$q(x,i,\vec{n}) = q(x,i,n)N(\vec{n}), \ x \geq 0, \ j=\overline{1,K}, n=\overline{0,s}, \tag{7}$$

$$p_0(\vec{n}) = p_0(n)N(\vec{n}), \ j=\overline{1,K}, n=\overline{0,s}, \tag{8}$$

where j^n is the vector $(n_1, n_2,...,n_j,...,n_k)$ with $n_j = n$ and $n_l = 0$ for $l \neq j$, and

$$M(\vec{n}, j) = \frac{n!}{(\lambda_j^*)^n} \prod_{l=1}^{K} \frac{(\lambda_l^*)^{n_l}}{n_l!}, \quad N(\vec{n}) = n! \prod_{l=1}^{K} \frac{(a_l^*)^{n_l}}{n_l!}$$

and $a_l^* = \frac{\lambda_l^*}{\lambda^*}$.

Proof. First of all, we consider the relation (5) for the case where x > 0. Substituting (5) into equation (1) we have

$$\frac{d}{dx} q(x,i,j^n) M(\vec{n}, j) = -u(s-n)\lambda^* q(x,i,j^n) M(\vec{n}, j) + \sum_{m=1}^{K} u(n_m)\lambda_m^* q(x,i,j^{n-1}) M(\vec{n} - \vec{e}_m, j).$$

It is easy to verify the equality

$$u(n_m)\lambda_m^* M(\vec{n} - \vec{e}_m, j) = \frac{\lambda_j^*}{n} n_m M(\vec{n}, j),$$

which implies that

$$\frac{d}{dx} q(x,i,j^n) = -u(s-n)\lambda^* q(x,i,j^n) + u(n)\lambda_j^* q(x,i,j^{n-1}).$$

The last is a particular case of the equation (1), therefore the relation (5) is true for x > 0.

The relation (6) is proved similarly bu iys substitution into equation (2).

Now we substitute (6) into equation (3). As a result we have

$$q(0,i,\vec{n}) = (n_i + 1)\gamma p_0(j^{n+1}) M(\vec{n} + \vec{e}_i, j) + \lambda_i p_0(j^n) M(\vec{n}, j)$$

$$= \left[(n+1)\gamma p_0(j^{n+1}) \frac{\lambda_i^*}{\lambda_j^*} + \lambda_i p_0(j^n) \right] M(\vec{n}, j)$$

$$= \left[\gamma p_0(j^n + \vec{e}_i) + \lambda_i p_0(j^n) \right] M(\vec{n}, j)$$

$$= q(0,i,j^n) M(\vec{n}, j).$$

Thus we proved the relation (5) for x = 0.

At last we obtain the relation (7) by summing up $q(x,i,\vec{n})$ over all $\vec{n}$ such that n. = n:

$$q(x,i,n) = \sum_{n_1+...+n_K=n} q(x,i,\vec{n})$$

$$= q(x,i,j^n) \sum_{n_1+...+n_K=n} M(\vec{n}, j)$$

$$= q(x,i,j^n) \frac{(\lambda^*)^n}{(\lambda_j^*)^n} \sum_{n_1+...+n_K=n} N(\vec{n})$$

$$= \frac{1}{(a_j^*)^n} q(x,i,j^n).$$

This implies the relation (7). The relation (8) is proved by the similar way. Thus Lemma 1 is proved.

Now using Lemma 1, instead of equations (1) - (4) we have to resolve the more simple system of equations

$$\frac{d}{dx}q(x,i,n) = -u(s-n)\lambda^* q(x,i,n) + u(n)\lambda^* q(x,i,n-1),\ i = \overline{1,K},\ n = \overline{0,s}, \tag{9}$$

$$(\lambda + n\gamma)p_0(n) = \sum_{i=1}^{K}\int_0^\infty q(x,i,n)dB_i(x),\ \ n = \overline{0,s}, \tag{10}$$

$$q(0,i,n) = (n+1)a_i^*\gamma p_0(n+1) + \lambda_i p_0(n),\ i = \overline{1,K},\ n = \overline{0,s-1}, \tag{11}$$

$$q(0,i,s) = \lambda_i p_0(s), \tag{12}$$

The equations (9) - (12) are the equilibrium equations for underlying Marjov process for the similar retrial queueing system buth with only one Poisson arrival flow with rate λ^*, where the type i, $i = \overline{1,K}$, of service time distribution function for a customer is defined only at the beginning of his service. Such a queueing system we shall codify as $M/HG_K/1/0/s/NP$.

The equation (9) has the following solution:

$$q(x,i,n) = e^{-\lambda^* x}\sum_{m=0}^{n} q(0,i,m)\frac{(\lambda^* x)^{n-m}}{(n-m)!},\ n = \overline{0,s-1}, \tag{13}$$

$$q(x,i,s) = \sum_{m=0}^{s} q(0,i,m) - \sum_{m=0}^{s-1} q(x,i,m). \tag{14}$$

Substituting (13) into equation (10) we have

$$(\lambda + n\gamma)p_0(n) = \sum_{i=1}^{K}\sum_{m=0}^{n} q(0,i,m)\beta_{i,n-m},\ \ n = \overline{0,s-1}, \tag{15}$$

where

$$\beta_{i,k} = \int_0^\infty e^{-\lambda^* x}\frac{(\lambda^* x)^k}{k!}dB_i(x)$$

is the probability that k customers of any type arrived to the system $M/HG_k/1/0/s/NP$ during the service time of an i-customer.

Now we have to find the unknowns $p_0(n)$ and q(0,i,n). Let us denote $q(n) = q(0,\cdot,n) = \sum_{i=1}^{K} q(0,i,n)$.

Lemma 2. For the retrial $M/HG_k/1/0/s/NP$ queueing system the functions q(0,i,n) are given by

$$q(0,i,n) = a_i^* q(n) + p_0(n)[\lambda_i - \lambda a_i^*],\ n = \overline{0,s},\ \ (16) \tag{16}$$

The values of q(n) and $p_0(n)$ are defined by the recurrent relations

$$p_0(n) = \frac{1}{n\gamma\beta_0^*}\left[(\lambda + (n-1)\gamma)p_0(n-1) - \sum_{m=0}^{n-1} p_0(m)(m\gamma\beta_{n-m}^* + \lambda\beta_{n-m-1})\right],\ n = \overline{1,s}, \tag{17}$$

$$q(n) = (n+1)\gamma p_0(n+1) + \gamma p_0(n),\ n = \overline{0, s-1}, \tag{18}$$

$$q(s) = \lambda p_0(s), \tag{19}$$

where $\beta_k^* = \sum_{i=1}^{K} a_i^* \beta_{i,k},\ \beta_k = \sum_{i=1}^{K} a_i \beta_{i,k}^*,\ a_i = \frac{\lambda_i}{\lambda}$.

Proof. Summing up the equation (11) over all i and multiplying the result by a_i^* we obtain (16). The relation (17) is given by substituting (13) in (10) and the relations (18), (19) are obvious.

Now using the results obtained above we can determine the stationary distributions of underlying Markov processes for the considered queueing systems.

Theorem 1. The stationary probabilities $p_0(n)$ are defined by the relation (17), and the stationary probabilities $p(i,n)$, $p(i,\vec{n})$ are defined by the following relations:

$$p(i,n) = \frac{1}{\lambda^*} \sum_{m=0}^{n} q(0,i,m)\gamma_{i,n-m},$$

$$p(i,s) = \frac{1}{\mu_i} \sum_{m=0}^{s} q(0,i,m) - \sum_{m=0}^{s-1} p(i,m) \tag{20}$$

and

$$p_0(\vec{n}) = p_0(n) N(\vec{n}),$$

$$p(i,\vec{n}) = p(i,n) N(\vec{n}),\ n = \overline{0, s}, \tag{21}$$

where the values $\gamma_{i,k}$ are determined by the recurrent formulas

$$\gamma_{i,0} = 1 - \beta_{i,0},\ i = \overline{1, K},$$

$$\gamma_{i,k} = \gamma_{i,k-1} - \beta_{i,k},\ i = \overline{1, K},\ k = \overline{1, s},$$

and

$$B_{i,k} = \int_0^\infty e^{-\lambda^* x} \frac{(\lambda^* x)^k}{k!} dB_i(x).$$

4. THE $M_K/G_K/1/0/\infty$/NP RETRIAL QUEUEING SYSTEM WITH INFINITE ORBIT

In this section we consider in brief the case where the orbit is infinite. Using the notations of the previous section under the assumption that there exists the stationary distribution of the underlying linear Markov process we shall write the following system of equilibrium equations:

$$\frac{d}{dx} q(x,i,\vec{n}) = -\lambda^* q(x,i,\vec{n}) + \sum_{j=1}^{K} u(n_j)\lambda_j^* q(x,i,\vec{n} - \vec{e}_j),\ i = \overline{1, K},\ n \ge 0, \tag{22}$$

$$(\lambda + n\gamma) p_0(\vec{n}) = \sum_{i=1}^{K} \int_0^\infty q(x,i,\vec{n}) dB_i(x),\ n \ge 0, \tag{23}$$

$$q(0,i,\vec{n}) = (n_i + 1)\gamma p_0(\vec{n} + \vec{e}_i) + \lambda_i p_0(\vec{n}),\ n \ge 0. \tag{24}$$

To find the steady state distribution we shall use the same approach which was demonstrated in the previous section. We do not repeat it again and remember only that all lemmas and theorem remain true but now for every $n \ge 0$.

To complete our investigation we shall derive the expressions for partial generating functions

$$P_0(z) = \sum_{n=0}^{\infty} p_0(n) z^n,\ Q_i(x,z) = \sum_{n=0}^{\infty} q(x,i,n) z^n,\ P_i(z) = \sum_{n=0}^{\infty} p(i,n) z^n,\ |z| \in [0,1].$$

The equations (9) - (11) for the case of infinite orbit, i.e. $s = \infty$ can be rewritten in terms of generating functions.

$$\frac{d}{dx} Q_i(x,z) = -Q_i(x,z)\lambda^*(1-z), \tag{25}$$

$$\lambda P_0(z) + \gamma z P_0'(z) = \sum_{i=1}^{K} \int_0^{\infty} Q_i(x,z) dB_i(x), \tag{26}$$

$$Q_i(0,z) = a_i^* \gamma P_0'(z) + \lambda_i P_0(z), i = \overline{1,K}. \tag{27}$$

The solution of the equation (25) has the following form:

$$Q_i(x,z) = Q_i(0,z)\exp\{-(\lambda^* - \lambda^* z)x\},\ i = \overline{1,K}. \tag{28}$$

Let us denote the Laplace-Stieltjes transform of $B_i(x)$ by $\beta_i(s)$ and put

$$k(z) = \sum_{i=1}^{K} a_i \beta_i(\lambda^* - \lambda^* z), \quad k^*(z) = \sum_{i=1}^{K} a_i^* \beta_i(\lambda^* - \lambda^* z).$$

We have from equation (26), (27) and (28),

$$\gamma(k^*(z) - z)P_0'(z) = \lambda(1 - k(z))P_0(z), \tag{29}$$

which has the solution

$$P_0(z) = P_0(1)\exp\left\{\frac{\lambda}{\gamma}\int_1^z \frac{1-k(u)}{k^*(u)-u} du\right\}. \tag{30}$$

Eliminating $P_0'(z)$ from equation (27) we get the following relation:

$$Q_i(0,z) = \frac{a_i^*(1-k(z)) + a_i(k^*(z)-z)}{k^*(z)-z} \lambda P_0(z). \tag{31}$$

This implies that

$$\begin{aligned} P_i(z) &= \frac{a_i^*(1-k(z)) + a_i(k^*(z)-z)}{k^*(z)-z} \lambda P_0(z) \int_0^{\infty} e^{-(\lambda^* - \lambda^* z)x} [1 - B_i(x)] dx \\ &= \frac{a_i^*(1-k(z)) + a_i(k^*(z)-z)}{k^*(z)-z} \frac{1-\beta_i(\lambda^* - \lambda^* z)}{\lambda^* - \lambda^* z} \lambda P_0(z),\ i = \overline{1,K}, \end{aligned} \tag{32}$$

where $\beta_i(s)$ is the Laplace-Stieltjes transform of $B_i(x)$.

Summing up equation (32) from $i = \overline{1,K}$, we have:

$$P.(z) = \sum_{i=1}^{K} P_i(z) = \frac{1-k(z)}{\lambda * [k*(z) - z]} \lambda P_0(z). \tag{33}$$

Putting z = 1 in the equation (33) we get

$$P.(1) = \frac{\sum_{i=1}^{K} \frac{a_i}{\mu_i}}{1 - \lambda * \sum_{i=1}^{K} \frac{a_i^*}{\mu_i}} \lambda P_0(1). \tag{34}$$

The unknown constant P.(1) and $P_0(1)$ can be found from the normalizing condition P.(1) + $P_0(1)$ = 1. Thus we get

$$P_0(1) = \frac{1 - \sum_{i=1}^{K} \frac{\lambda_i H_i}{\mu_i}}{1 + \sum_{i=1}^{K} \frac{\lambda_i (1 - H_i)}{\mu_i}}.$$

We will define

$$\rho = 1 - P_0(1) = \frac{\sum_{i=1}^{K} \frac{\lambda_i}{\mu_i}}{1 + \sum_{i=1}^{K} \frac{\lambda_i (1 - H_i)}{\mu_i}}.$$

Then we can rewrite relation (34) as

$$P.(1) = \frac{\rho}{1-\rho} P_0(1). \tag{35}$$

From the formula (35) it follows that

$$\rho < 1 \tag{36}$$

gives us a necessary condition for ergodicity of the process $\{\xi(t), t \geq 0\}$. Following Falin, Templeton (1997) it can be proved that the condition (36) is also a sufficient condition for ergodicity of this process.

From (30), (33) and (36) we get the generating function for stationary distribution of the common queue length in the orbit

$$\begin{aligned} P(z) &= P_0(z) + P.(z) = \frac{\lambda[1-k(z)] + \lambda * [k*(z) - z]}{\lambda * [k*(z) - z]} P_0(z) \\ &= (1-\rho)\left(\frac{\lambda[1-k(z)]}{\lambda * [k*(z) - z]} + 1 \right) \exp\left\{ \frac{\lambda}{\gamma} \int_1^z \frac{1-k(u)}{k*(u) - u} du \right\}. \end{aligned} \tag{37}$$

The mean common queue length in the orbit is

$$N = P'(1) = \frac{\rho}{(1-\rho)} \left[\frac{1}{\gamma} + \lambda * \frac{\sum_{i=1}^{K} \lambda_i \beta_i^{(2)} - \rho \sum_{i=1}^{K} \lambda_i (1-H_i) \beta_i^{(2)}}{2 \sum_{i=1}^{K} \frac{\lambda_i}{\mu_i}} \right].$$

We can expressed the stationary mean joint queue length N_i of i-customers in the orbit as:

$$N_i = a_i^* N, \quad i = \overline{1, K}$$

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